1a)

N-point DFT of $x(n)$,

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad , \quad k = 0, 1, \ldots N-1. \qquad \ldots\ldots (1)$$

where $W_N = e^{-j\frac{2\pi}{N}}$.

where $W_N = e^{-j\frac{2\pi}{N}}$

The N-point DFT given by (1), can be split into 2 - $\frac{N}{2}$ point DFTs corresponding to even numbered and odd numbered samples of $x(n)$, as follows.

$$X(k) = \sum_{n\text{-even}} x(n) W_N^{kn} + \sum_{n\text{-odd}} x(n) W_N^{kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2kn} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{k(2n+1)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_{\frac{N}{2}}^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{k} \cdot W_{\frac{N}{2}}^{kn}$$

$$\left[\because W_N^2 = W_{\frac{N}{2}}\right]$$

$$= F_1(k) + W_N^k F_2(k) \quad , \quad k=0,1,\ldots N-1 \quad \ldots\ldots (2)$$

where $F_1(k) = \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_{\frac{N}{2}}^{kn}$ .... (3)

& $F_2(k) = \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_{\frac{N}{2}}^{kn}$ .... (4)

Both $F_1(k)$ and $F_2(k)$ are $\frac{N}{2}$- point DFTs.

$\therefore \quad F_1(k+\frac{N}{2}) = F_1(k)$ ..... (5)

$F_2(k+\frac{N}{2}) = F_2(k)$ ..... (6)

$\therefore$ (2) can be written as,

$$X(K) = F_1(K) + W_N^k F_2(K) \quad , \quad K = 0, 1, \cdots \frac{N}{2}-1 \quad \cdots\cdots (7)$$

$$X\left(K+\frac{N}{2}\right) = F_1\left(K+\frac{N}{2}\right) + W_N^{\left(K+\frac{N}{2}\right)} F_2(K)$$

$$= F_1(K) + W_N^{\frac{N}{2}} W_N^k F_2(K)$$

$$= F_1(K) - W_N^k F_2(K) \quad \cdots\cdots (8)$$

$$K = 0, 1, \cdots \frac{N}{2}-1$$

Thus, we have divided an N-point DFT into two $\frac{N}{2}$-point DFTs.

We observe that the direct computation of $F_1(K)$ requires $\left(\frac{N}{2}\right)^2$ complex multiplications.

The same applies to the computation of $F_2(K)$.

Furthermore, there are additional $\frac{N}{2}$ complex multiplications required to compute $W_N^k F_2(K)$.

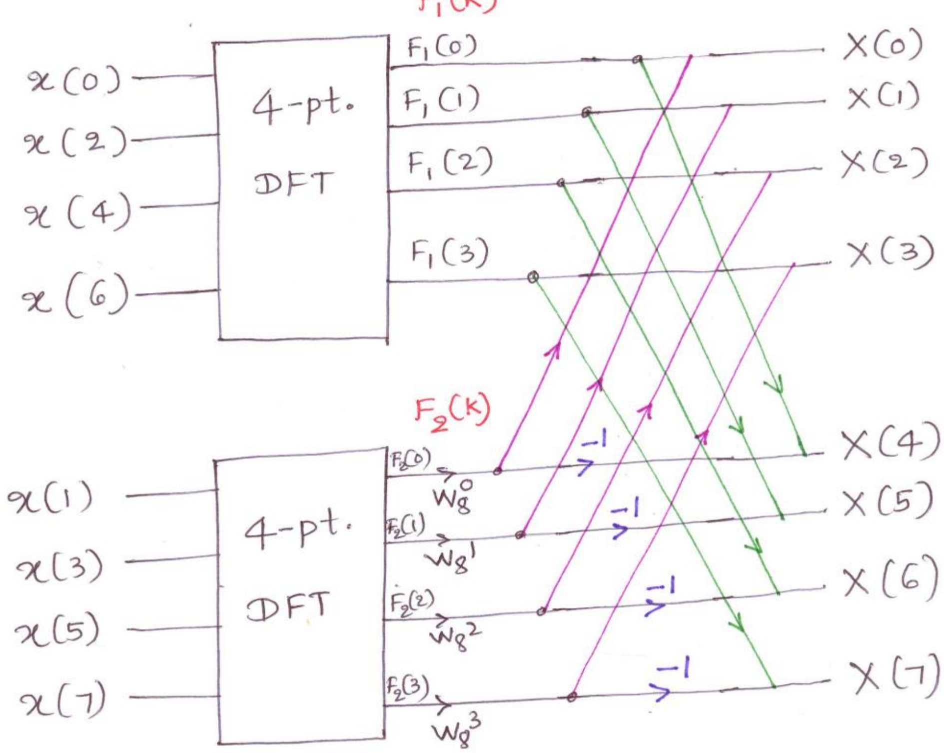
Hence, the computation of $X(K)$ requires

$2 \times \left(\frac{N}{2}\right)^2 + \frac{N}{2} = \frac{N^2}{2} + \frac{N}{2}$ complex multiplications

Thus, the first stage of decimation results in the reduction of number of multiplications from N^2 to $\frac{N^2}{2} + \frac{N}{2}$, which is about a factor of 2 for N large.

Let $N=8$.

After 1st stage of decimation,



Here, we have made use of (7) to find $X(0), X(1), X(2), X(3)$.

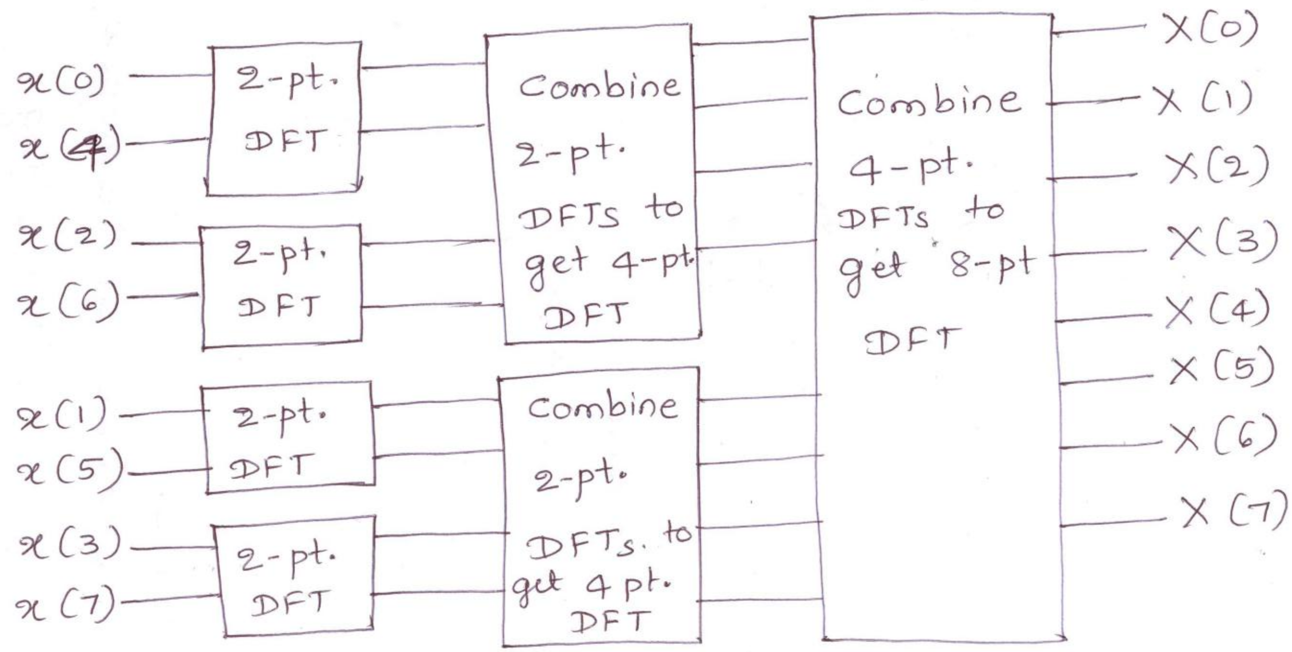
We have made use of (8) to find $X(4), X(5), X(6), X(7)$.

In the 2nd stage of decimation, each $\frac{N}{2}$-point DFT is divided into two $\frac{N}{4}$-point DFTs.

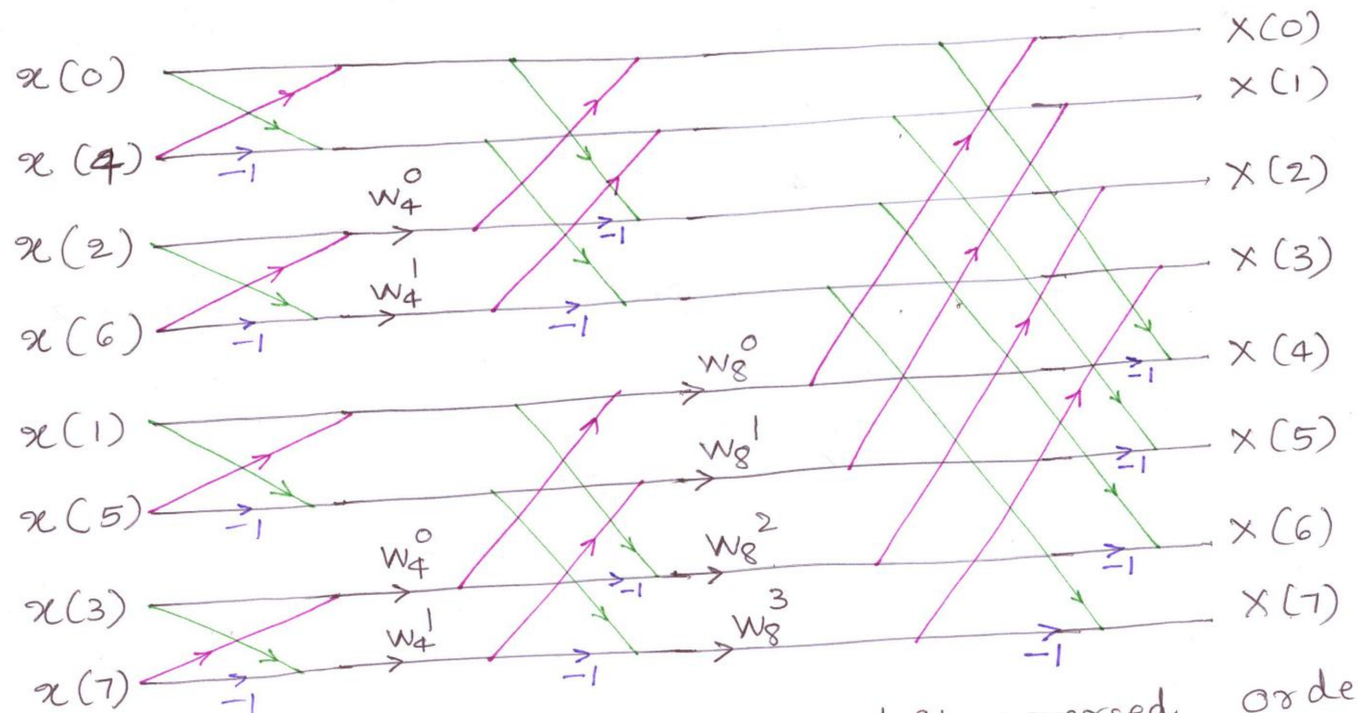
This procedure can be repeated again and again until the resulting sequences are reduced to one-point sequences.

For $N = 2^p$, where p is an integer, $(p \in \mathbb{Z}^+)$, this decimation can be performed $p = \log_2 N$ times.

For $N = 8$, computation is done in 3 stages as shown below.



The complete signal flow graph for the computation of 8-pt. DFT is as shown below.



Note that input is arranged in bit reversed order and output appears in natural order.

[Note that 2-point DFT of a length-2 sequence can be computed by the addition and subtraction of the sample values as proved below:

Let $g(n) = (g(0), g(1))$

$$G(k) = \sum_{n=0}^{1} g(n) e^{-j\frac{2\pi}{2}kn} , \quad K = 0, 1$$

$$G(0) = \sum_{n=0}^{1} g(n) e^{0}$$

$$= \sum_{n=0}^{1} g(n)$$

$$= g(0) + g(1)$$

$$G(1) = \sum_{n=0}^{1} g(n) e^{-j\frac{2\pi}{2}n}$$

$$= \sum_{n=0}^{1} g(n) e^{-j\pi n}$$

$$= g(0) - g(1)$$]

8 Derive the Radix-2 DIF-FFT algorithm to compute DFT of an N=8 point sequence. Draw the complete signal flow graph.

N-point DFT of $x(n)$ is given by

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn} , \quad \ldots\ldots (1)$$

$$K = 0, 1, \ldots N-1$$

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Let $g(n) = (g(0), g(1))$

$$G(k) = \sum_{n=0}^{1} g(n)\, e^{-j\frac{2\pi}{2}kn}, \quad k = 0,1$$

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$$k = 0, 1, \ldots N-1$$

where $W_N = e^{-j\frac{2\pi}{N}}$.

$X(k)$ can be written as,

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + \sum_{n=\frac{N}{2}}^{N-1} x(n) W_N^{kn}, \quad k=0,1,\dots N-1$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\tfrac{N}{2}\right) W_N^{k\left(n+\frac{N}{2}\right)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\tfrac{N}{2}\right) W_N^{k\frac{N}{2}} W_N^{kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\tfrac{N}{2}\right) (-1)^k W_N^{kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^k x\left(n+\tfrac{N}{2}\right)\right] W_N^{kn} \quad \dots\dots (2).$$

$$k=0,1,\dots N-1$$

Let us split $X(k)$ into even and odd numb-ered samples.

$$X(2k) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + x\left(n+\tfrac{N}{2}\right)\right] W_N^{2kn}, \quad k=0,1,\dots \tfrac{N}{2}-1$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + x\left(n+\tfrac{N}{2}\right)\right] W_{\frac{N}{2}}^{kn} \quad \dots\dots (3)$$

$$k=0,1,\dots \tfrac{N}{2}-1$$

$$X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) - x\left(n+\tfrac{N}{2}\right)\right] W_N^{(2k+1)n} \quad k=0,1,\dots \tfrac{N}{2}-1$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) - x\left(n+\frac{N}{2}\right)\right] W_N^{n} W_N^{2kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) - x\left(n+\frac{N}{2}\right)\right] W_N^{n} W_{\frac{N}{2}}^{kn} \quad \ldots (4)$$

$K = 0, 1, \ldots \frac{N}{2}-1$

Let $g_1(n) = x(n) + x\left(n+\frac{N}{2}\right) \quad \ldots (5)$

and $g_2(n) = \left[x(n) - x\left(n+\frac{N}{2}\right)\right] W_N^{n} \quad \ldots (6)$

$n = 0, 1, \ldots \frac{N}{2}-1$

$\therefore$ From (3) and (4), we can write

$$X(2k) = \sum_{n=0}^{\frac{N}{2}-1} g_1(n) W_{N/2}^{kn}, \quad \ldots (7)$$

$K = 0, 1, \ldots \frac{N}{2}-1$

$$X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} g_2(n) W_{\frac{N}{2}}^{kn} \quad \ldots (8)$$

$K = 0, 1, \ldots \frac{N}{2}-1$

Thus, we have split $X(k)$ into two $\frac{N}{2}$-point DFTs.

We observe that, direct computation of $X(2k)$ requires $\left(\frac{N}{2}\right)^2$ complex multiplications. The same applies to the computation of $X(2k+1)$.

Furthermore, there are additional $\frac{N}{2}$ complex multiplications required to compute $g_2(n)$.

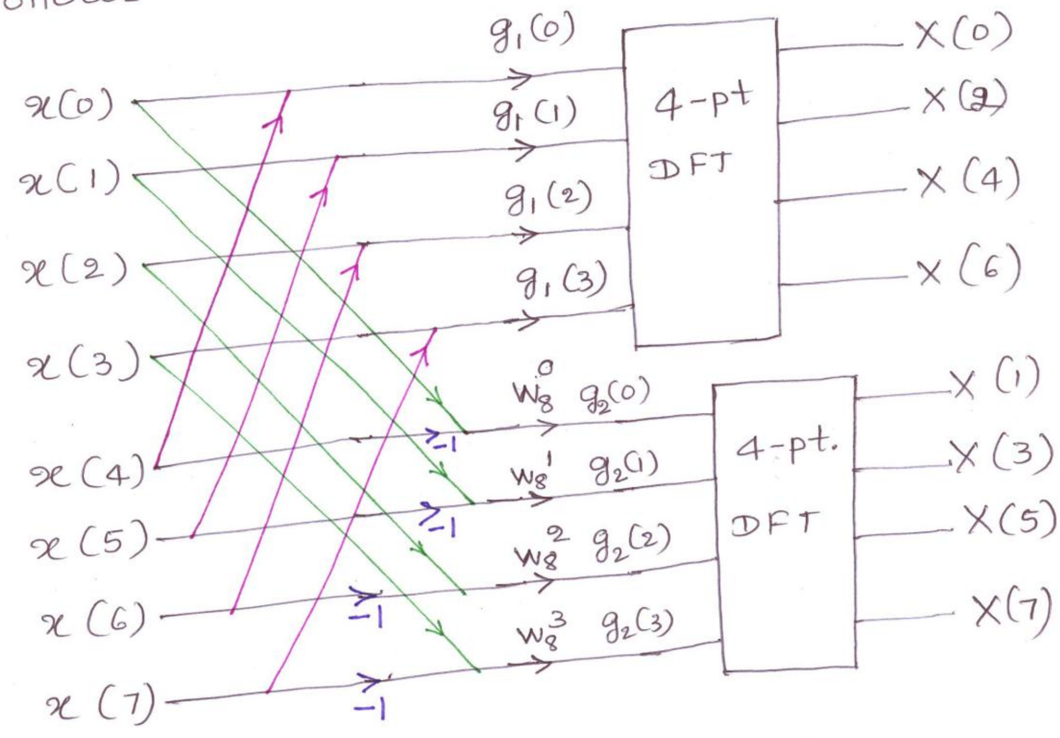
Hence, the computation of $X(K)$ requires

$$2 \times \left(\frac{N}{2}\right)^2 + \frac{N}{2} = \frac{N^2}{2} + \frac{N}{2}$$ complex multiplications.

Thus, the first stage of decimation results in the reduction of number of multiplications from N^2 to $\frac{N^2}{2} + \frac{N}{2}$, which is about a factor of 2 for N-large.

Let $N = 8$.

By separating even and odd indexed DFT samples, we can obtain $X(K)$ as follows.



In the above figure, $g_1(0), g_1(1), g_1(2), g_1(3)$ are obtained from (5).

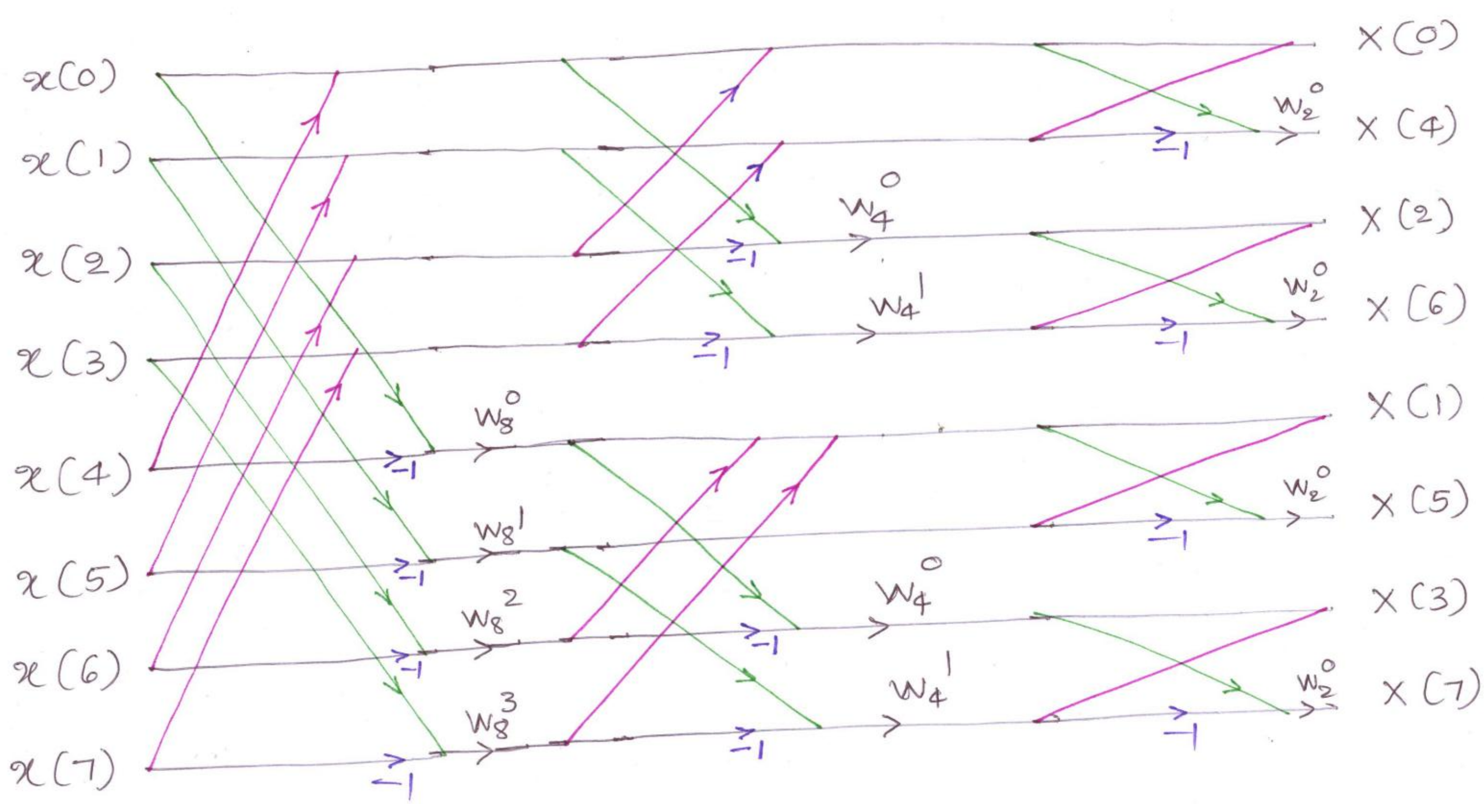
$g_2(0), g_2(1), g_2(2), g_2(3)$ are obtained from (6).

Further, each $\frac{N}{2}$-point DFT can be divided into two $\frac{N}{4}$-point DFTs.

This process can be repeated $\log_2 N$ times till we reach N one-point DFTs.

The complete signal flow graph for $N=8$ is shown below.

Note that output appears in bit-reversed order and input is in natural order.


x(0)
x(1)
x(2)
x(3)
x(4)
x(5)
x(6)
x(7)
W_8^0
W_8^1
W_8^2
W_8^3
W_4^0
W_4^1
W_4^0
W_4^1
W_2^0
W_2^0
W_2^0
W_2^0
-1
X(0)
X(4)
X(2)
X(6)
X(1)
X(5)
X(3)
X(7)

2) b) $x(n) = \cos\left(\frac{\pi}{4}n\right)$ $N = 8$

$x(0) = 1$

$x(1) = 1/\sqrt{2}$

$x(2) = 0$

$x(3) = -1/\sqrt{2}$

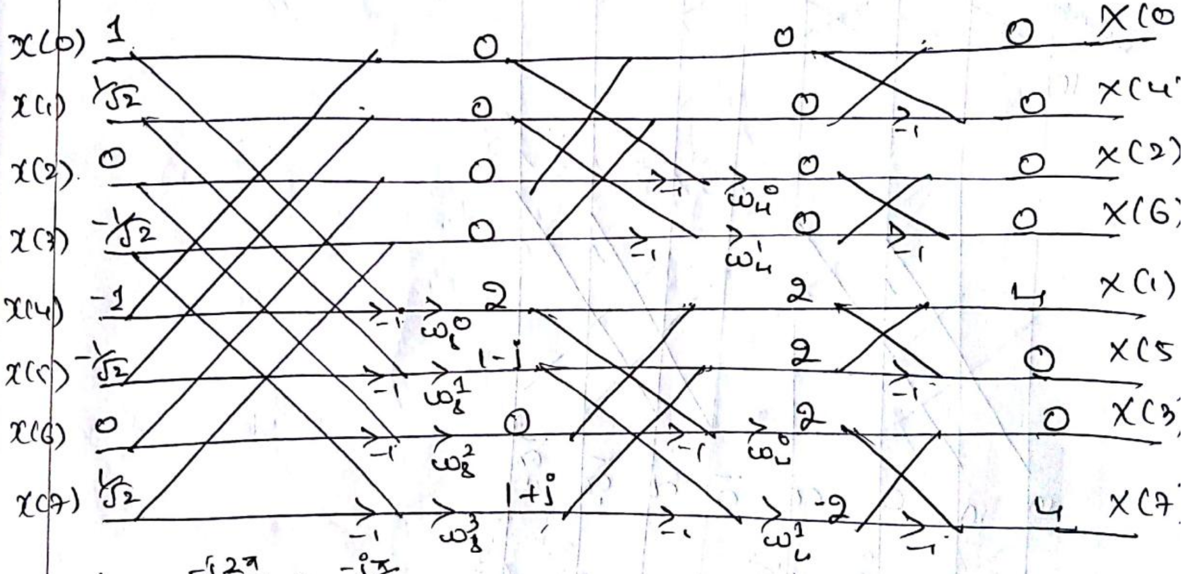
$x(4) = -1$

$x(5) = -1/\sqrt{2}$

$x(6) = 0$

$x(7) = 1/\sqrt{2}$

~~x(8) =~~



$\omega_4^1 = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}}$

$\omega_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$\omega_8^0 = \cancel{0}\ 1$

$\omega_8^3 = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$\omega_8^2 = -j$

$\therefore X(k) = [X(0), X(1), X(2), X(3), X(4), X(5), X(6), X(7)]$

$X(k) = [0, 4, 0, 0, 0, 0, 0, 4]$

3.

a

$$X(k) = [10, 1+j, -j, -1-j, 5, -1+j, j, 1-j]$$

$$N = 8$$

$$W_8^0 = 1$$

$$W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$W_8^2 = -j$$

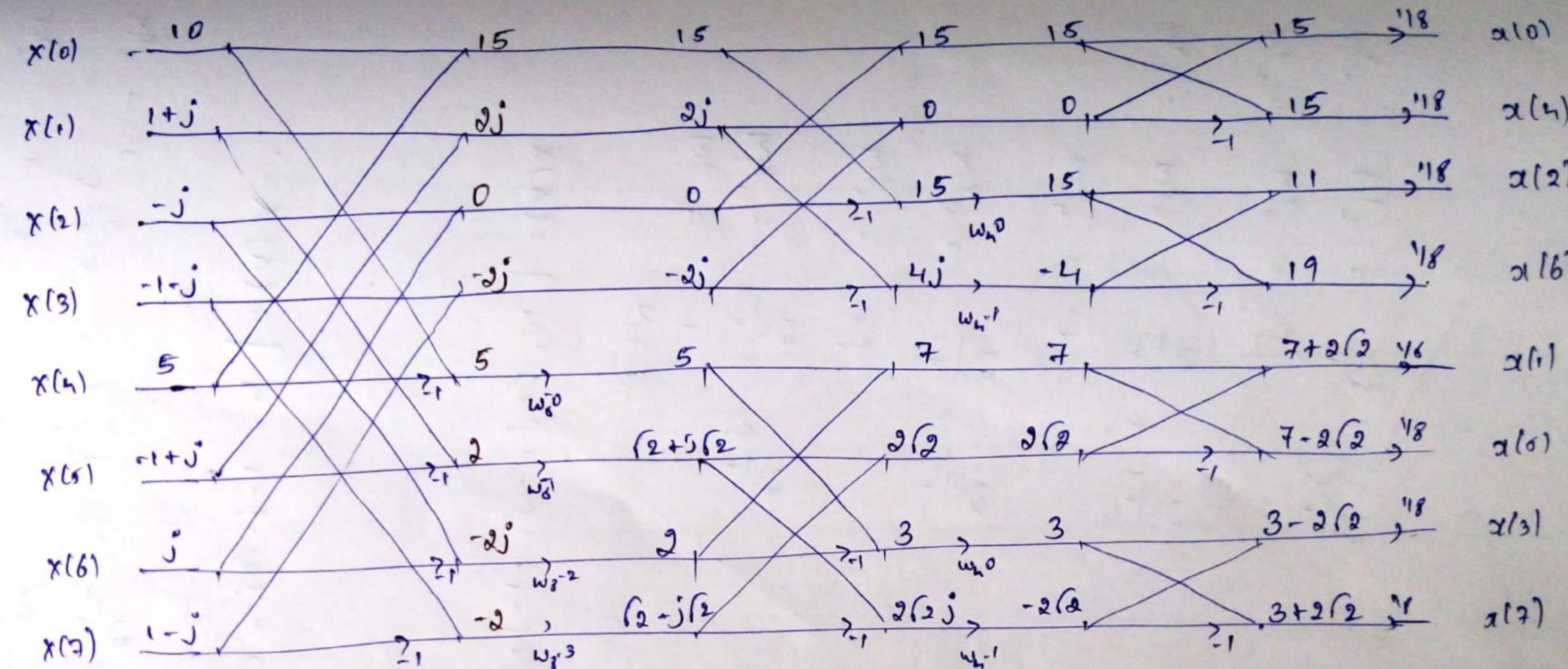
$$W_8^3 = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

$$W_8^4 = -1$$

$$W_8^{-3} = W_8^5 = \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$W_8^{-2} = W_8^6 = j$$

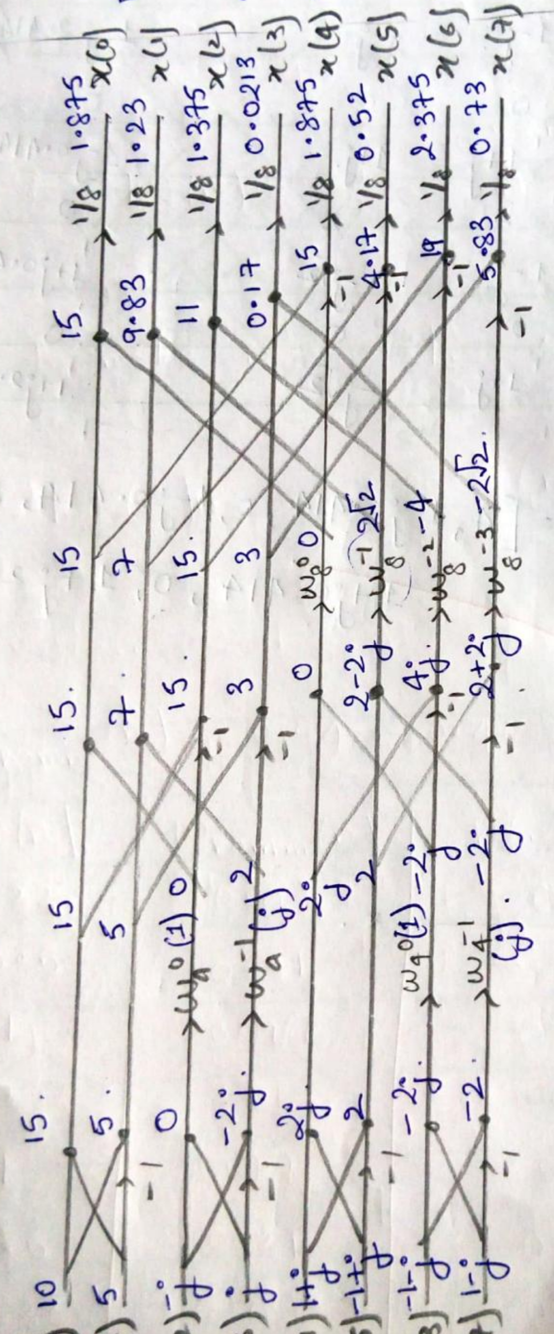
$$W_8^{-1} = W_8^7 = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$



$$x(n) = \frac{1}{8}\left[15,\ 7+2\sqrt{2},\ 11,\ 3-2\sqrt{2},\ 15,\ 7-2\sqrt{2},\ 19,\ 3+2\sqrt{2}\right]$$

Q3) b) IDFT DIF FFT   i/p: X (deci)   o/p: x (natural).

$X(k) = [10, 1+j, -j, -1-j, 5, -1+j, j, 1-j]$.

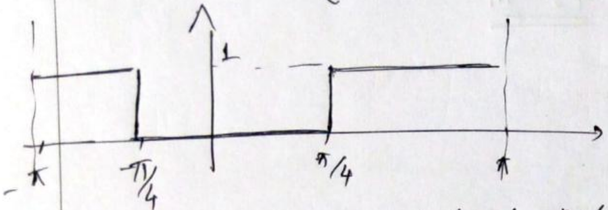


$w_8^0 = 1$; $w_8^{-1} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$; $w_8^{-2} = j$; $w_8^{-3} = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$.

$x(n) = [1.875, 1.23, 1.375, 0.0213, 1.875, 0.52, 2.375, 0.73]$.

4.

a) $$H_d(\omega) = \begin{cases} 0 \,, & -\pi/4 \le |\omega| \le \pi/4 \\ e^{-j2\omega} \,; & \pi/4 \le |\omega| \le \pi \end{cases}$$



$\therefore$ It is a high pass filter ; $\alpha = 2$

$\therefore \frac{M-1}{2} = 2$

$M = 5$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{-\pi/4} \Big(\underbrace{e^{-j2\omega} e^{j\omega n}}_{e^{j\omega(n-2)}} \Big) d\omega + \int_{\pi/4}^{\pi} \left(e^{-j\omega 2} e^{j\omega n} \right) d\omega \right]$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega(n-2)}}{j(n-2)} \Bigg|_{-\pi}^{-\pi/4} + \frac{e^{j\omega(n-2)}}{j(n-2)} \Bigg|_{\pi/4}^{\pi} \right]$$

$$= \frac{1}{j2\pi(n-2)} \left[e^{j(n-2)(-\pi/4)} - e^{j(-\pi)(n-2)} + e^{j\pi(n-2)} - e^{j\pi/\ldots} \right.$$

$e^{-j\pi} = \cos \pi - j \cancel{\sin \pi} = (-1)^k$; $e^{j\pi} = (-1)^k$

$$= \frac{1}{2\pi j(n-2)} \left[e^{-\pi/4 \, j(n-2)} - e^{j\pi/4(n-2)} - \cancel{(-1)^k} + \cancel{(-1)^k} \right]$$

$$= \frac{1}{2\pi j(n-2)} \left[-\left(e^{j\pi/4(n-2)} - e^{-\pi/4 \, j(n-2)} \right) \right]$$

$$hd(n) = \frac{-\sin\left(\frac{\pi}{4}(n-2)\right)}{(n-2)\pi} \quad ; \quad n \neq 2$$

For $n = 2$,

$$hd(2) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} d\omega + \int_{\pi/4}^{\pi} d\omega\right] = \frac{1}{2\pi}\left[-\frac{\pi}{4} + \pi + \pi - \frac{\pi}{4}\right]$$

$$= \frac{1}{2\pi}\left[2\pi - \frac{\pi}{2}\right] = 1 - \left(\frac{\pi}{2}\right)\frac{1}{2\pi} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\therefore hd(n) = \begin{cases} -\dfrac{\sin\left(\frac{\pi}{4}(n-2)\right)}{\pi(n-2)} & ; \ n \neq 2 \\ \dfrac{3}{4} & ; \ n = 2 \end{cases}$$

Blackman window $= 0.42 - 0.5\cos\left(\frac{2\pi n}{M-1}\right) + 0.08\cos\left(\frac{4\pi n}{M-1}\right)$

n	hd(n)	$\omega_{bl}(n)$	$h(n) = hd(n) \times \omega_{bl}(n)$
0	−0.159	0	0
1	−0.225	0.34	−0.0765
2	3/4	1	3/4
3	−0.225	0.34	−0.0765
4	−0.159	0	0

↓
odd, symm
∴ Type 1

(P.T.O)

For a Type 1 FIR filter,

$$H(e^{jw}) = e^{-jw\alpha}\left[h(\alpha) + \sum_{n=0}^{\frac{M-3}{2}} 2h(n) \cos(\omega(n-\alpha))\right]$$

$$\alpha = \frac{m-1}{2} = 2$$

$$H(e^{jw}) = e^{-j2w}\left[3/4 + \sum_{n=0}^{1} 2h(n) \cos(\omega(n-2))\right]$$

$$= e^{-jw2}\left[3/4 + \cancel{2h(0)\cos(w(-2))} + 2h(1)\cos(w(-1\right.$$

$$= e^{-jw2}\left[3/4 + 2\times(-0.0765)\cos w\right]$$

$$= e^{-j2w}\left[3/4 + (-0.153)\cos w\right]$$

Frequency response of the ideal digital differentiator,

$$H(\omega) = j\omega \ , \quad -\pi < \omega \leq \pi .$$

and $H(\omega + 2\pi) = H(\omega)$.

Impulse response,

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} j\omega \, e^{j\omega n} \, d\omega .$$

$$= \frac{j}{2\pi} \int_{-\pi}^{\pi} \omega \frac{d}{d\omega} \left\{ \frac{e^{j\omega n}}{jn} \right\} d\omega .$$

$$= \frac{j}{2\pi}\left[\frac{\omega e^{j\omega n}}{jn}\Bigg|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{e^{j\omega n}}{jn}\,d\omega\right]$$

$$= \frac{j}{2\pi}\left[\frac{\omega e^{j\omega n}}{jn}\Bigg|_{-\pi}^{\pi} - \frac{e^{j\omega n}}{j^2 n^2}\Bigg|_{-\pi}^{\pi}\right]$$

$$= \frac{j}{2\pi}\left[\frac{\pi e^{j\pi n} + \pi e^{-j\pi n}}{jn} + \frac{e^{j\pi n} - e^{-j\pi n}}{n^2}\right]$$

$$= \frac{j}{2\pi}\left[\frac{\pi\, 2\cos(\pi n)}{jn} + \frac{\cancel{2j\sin(\pi n)}}{n^2}\right]$$

$$= \frac{\cos(\pi n)}{n}, \quad n \neq 0$$

$$h(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \cdot \int_{-\pi}^{\pi} j\omega\, d\omega$$

$$= 0. \qquad \left[\because H(\omega) = j\omega \text{ is an odd function.}\right]$$

$$h(n) = \begin{cases} \dfrac{\cos(\pi n)}{n}, & n \neq 0 \\ 0, & n = 0 \end{cases}$$

ideal

$\alpha = \frac{M-1}{2}$

$\alpha = \frac{7-1}{2}$

$\alpha = 3$

$$\therefore h_d(n-\alpha) = \begin{cases} \frac{\cos(\pi(n-\alpha))}{(n-\alpha)} & n \neq \alpha \\ 0 & n = \alpha \end{cases}$$

$$h_d(n-3) = \begin{cases} \frac{\cos(\pi(n-3))}{(n-3)} & , n \neq 3 \\ 0 & n = 3 \end{cases}$$

using Hamming window $\Rightarrow 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)$

n	$h_d(n)$	haming window	$h(n) = h_d(n) \times$ Haming
0	0.33	0.08	0.0264
1	-0.5	0.31	-0.155
2	1	0.77	0.77
3	0	1	0
4	-1	0.77	-0.77
5	0.5	0.31	0.155
6	-0.33	0.08	-0.0264.

Since h(n) is odd and antisymmetric type 3 - FIR filter can be designed

$$H(e^{j\omega}) = e^{-j\omega\left(\frac{M-1}{2}\right)} \left[\sum_{n=0}^{\frac{M-3}{2}} 2j h(n) \sin\left(\omega\left(n - \frac{M-1}{2}\right)\right) + h\left(\frac{M-1}{2}\right) \right]$$

$$= e^{-j\omega 3} \left[\sum_{n=0}^{2} 2j h(n) \sin(\omega(n-3)) + h(3) \right]$$

$$= e^{-j3\omega} \left[h(3) + 2jh(0)\sin(\omega(-3)) + 2jh(1)\sin(\omega(-2)) + 2jh(2)\sin(\omega(-1)) \right]$$

$$= e^{-j3\omega} \left[h(3) - 2jh(0)\sin(3\omega) - 2jh(1)\sin(2\omega) - 2jh(2)\sin(\omega) \right]$$

$$H(e^{j\omega}) = e^{-j3\omega} \left[1 - 0.0528j\sin(3\omega) + 0.31j\sin(2\omega) - 1.54j\sin(\omega) \right]$$

5a

Given: $\Omega_c = 30\pi$ rad/s $\quad \omega_{SB} = \frac{\Omega_{SB}}{f_s} = 0.45\pi$ rad

$A_{SB} = 50$ dB

$\Omega_{SB} = 45\pi$ rad/s $\quad \omega_c = \frac{\Omega_c}{f_s} = 0.30\pi$ rad

1. $\omega_c = \frac{\omega_{PB} + \omega_{SB}}{2} = 0.3\pi$ rad.

$\therefore \omega_{PB} = 2 \times 0.3\pi - \omega_{SB} = 0.15\pi$ rad.

2. $\Delta\omega = |\omega_{PB} - \omega_{SB}| = 0.3\pi$ rad

3. Since A_{PB} is not given, we choose the window based on A_{SB}.

For $A_{SB} = 50$ dB, Hamming window can be chosen.

4. $N = \frac{6.6\pi}{\Delta\omega} = 22 \simeq 23.$

5. $\alpha = \frac{N-1}{2} = 11$

6. Impulse response,

$$h(n) = h_d(n-\alpha)\, \omega(n), \quad 0 \le n \le 22$$

$$h_d(n) = \begin{cases} \dfrac{\sin(\omega_n n)}{\pi n}, & n \neq 0 \\ \dfrac{\omega_c}{\pi}, & n = 0 \end{cases}$$

$0 \le n \le 22$

$\omega_c = 0.15\pi$ rad

$$\omega(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right), \quad 0 \le n \le 22$$

5b

5 b.

$$H_d(\omega) = \begin{cases} e^{-j2\omega}, & 0 \le |\omega| \le \pi/2 \\ 0, & \pi/2 \le |\omega| \le \pi \end{cases}$$

$$\omega \to \frac{2\pi}{M}k$$

$$H_d\left(\frac{2\pi}{M}k\right) = \begin{cases} e^{-j2\frac{2\pi}{M}k}, & 0 \le \left|\frac{2\pi}{M}k\right| \le \pi/2 \\ 0, & \pi/2 \le \left|\frac{2\pi}{M}k\right| \le \pi \end{cases}$$

$$\alpha = \frac{M-1}{2}$$

$$2 = \frac{M-1}{2} \Rightarrow \underline{\underline{M=5}}$$

$$H_d\left(\frac{2\pi}{5}k\right) = \begin{cases} e^{\frac{-j4\pi k}{5}}, & 0 \le |k| \, 1 \\ 0, & 2 \le |k| \, 3 \end{cases}$$

$$h_d(n) = \frac{1}{M}\left[H(0) + \sum_{k=1}^{\frac{M-1}{2}} 2R_e\left(H_d(k)\, e^{j\frac{2\pi}{M}kn}\right)\right]$$

$$= \frac{1}{5}\left[1 + \sum_{k=1}^{2} 2R_e\left(H_d(k)\, e^{j\frac{2\pi}{5}kn}\right)\right]$$

$$= \frac{1}{5}\left[1 + 2R_e\left(e^{\frac{-j4\pi}{5}}\, e^{\frac{j2\pi}{5}}\right) + 0\right]$$

$$= \frac{1}{5}\left[1 + 2R_e\left(e^{j\frac{2\pi}{5}(n-2)}\right)\right]$$

$$h_d(n) = \frac{1}{5}\left[1 + 2\cos\left(\frac{2\pi}{5}(n-2)\right)\right]$$

n	$h_d(n)$	$h_{rect}(n)$	$h(n)$
0	-0.123	1	-0.123
1	0.3236	1	0.3236
2	0.6	1	0.6
3	0.3236	1	0.3236
4	-0.123	1	-0.123

It is a Type (1) FIR filter

$$H_d(e^{j\omega}) = e^{-j\omega\left(\frac{M-1}{2}\right)}\left[h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\left(\omega\left(n-\frac{M-1}{2}\right)\right)\right]$$

$$= e^{-j2\omega}\left[h(2) + 2\sum_{n=0}^{1} h(n)\cos\left(\omega(n-2)\right)\right]$$

$$= e^{-j2\omega}\left[h(2) + 2h(0)\cos(2\omega) + 2h(1)\cos(\omega)\right]$$

$$= e^{-j2\omega}\left[0.6 - 0.246\cos(2\omega) + 0.647\cos(\omega)\right]$$