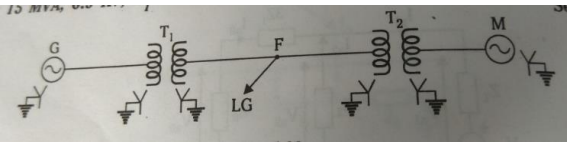


Internal Assessment Test – III  
A Section

Sub:	Power System Analysis	Code:	15EE62
Date:	21/05/2018	Duration:	90 mins
		Max Marks:	50
		Sem:	6
		Branch:	EEE
Answer Any FIVE FULL Questions			

		Marks	OBE	
			CO	RBT
1	A double line to ground fault occurs at the terminals of an unloaded generator. Derive an expression for the fault currents. Draw the sequence networks.	[10]	CO4	L2
2a	For two conductor open fault, derive the expressions for currents and show the connections of sequence networks.	[5]	CO4	L2
2b	Define stability as applied to power system studies and distinguish between 1) Steady state stability and 2) Transient stability	[5]	CO5	L2
3	Explain the concept of equal area criterion when there is a fault in the middle of one of the parallel lines.	[10]	CO5	L3
4	A Synchronous motor is receiving 10 MW of power at 0.8 pf lag at 6 kV. An LG fault takes place at the middle of the transmission line as shown in fig. Find the fault current. The ratings of the generator, motor and transformer are as under: Generator: 20 MVA, 11 kV, $X_1=0.2$ pu, $X_2=0.1$ pu, $X_0=0.1$ pu. Transformer T1: 18 MVA, 11.5 / 34.5 kV YY, $X=0.1$ pu. Transformer T2: 15 MVA, 6.9 / 34.5 kV YY, $X=0.1$ pu. Transmission line: $X_1=X_2=5 \Omega$ , $X_0=10 \Omega$ Motor: 15 MVA, 6.9 kV, $X_1=0.2$ pu, $X_2=X_0=0.1$ pu. Take generator rating as base value.	[10]	CO4	L3
				
5a	Define swing curve and its use.	[5]	CO5	L2
5b	Define critical clearing angle and time	[5]	CO5	L2
	An alternator operating at 50 Hz delivers 1 pu of power to an infinite bus through a transmission line. A fault takes place reducing the maximum power transferred to 0.5 pu where as before the fault it was 2.0 pu and after the fault it is 1.5 pu. Calculate the critical clearing angle.		CO5	L3

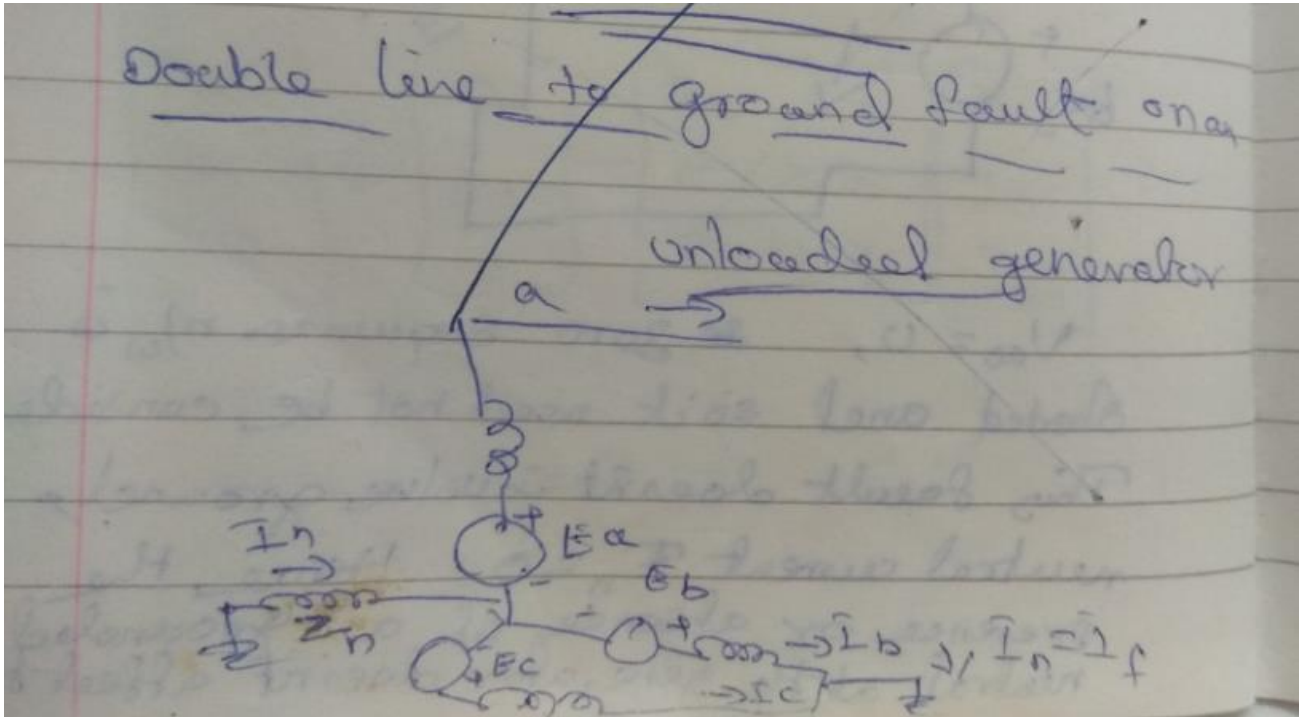
Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Relate the power system network to network topology.	2	-	-	-	-	-	-	-	-	1	-	-
CO2:	Recognize the network and form the matrix.	3	-	-	-	-	-	-	-	-	1	-	-
CO3:	Use the algorithms to calculate the load flow in the power system.	3	-	1	-	-	-	-	-	-	1	-	-
CO4:	Analyse the different algorithms for the load flow in the power system.	3	-	-	-	-	-	-	-	-	1	-	-
CO5:	Apply the economic scheduling algorithm for the load dispatch in power system.	3	-	-	-	-	-	-	-	-	1	-	-
CO6:	Apply different mathematical methods to solve the swing equation.	3	-	-	-	-	-	-	-	-	1	-	-

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 - *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

# Solutions

1



The fault current  $I_f = \frac{E_a}{Z_a + Z_b + Z_c}$   
 $I_a = 0, V_b = V_c = 0.$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$$V_{a0} = V_{a1} = V_{a2} = V_a/3$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_g \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$V_{a1} = E_g - I_{a1} Z_1$$

$$V_{a0} = V_{a1} = V_{a2}$$
~~$$\begin{bmatrix} E_g - I_{a1} Z_1 \\ E_g - I_{a1} Z_1 \\ E_g - I_{a1} Z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ E_g \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$~~

$$\begin{cases} E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \end{cases} = \begin{pmatrix} 0 \\ E_a \\ 0 \end{pmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{pmatrix} 0 \\ E_a \\ 0 \end{pmatrix} = \begin{cases} E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \end{cases}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{pmatrix} 0 \\ E_a \\ 0 \end{pmatrix} = \begin{bmatrix} 0 \\ E_a/Z_1 \\ 0 \end{pmatrix} = \begin{cases} E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \\ E_a - I_{a1}Z_1 \end{cases}$$

$$\therefore \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a/Z_1 \\ 0 \end{bmatrix} = \begin{cases} \frac{E_a - I_{a1}Z_1}{Z_0} \\ \frac{E_a - I_{a1}Z_1}{Z_1} \\ \frac{E_a - I_{a1}Z_1}{Z_2} \end{cases}$$

$$I_{a0} = -\frac{(E_a - I_{a1}Z_1)}{Z_0} = \frac{-E_a + I_{a1}Z_1}{Z_0}$$

$$I_{a1} = \frac{E_a - I_{a1}Z_1}{Z_1} = I_{a1}$$

$$I_{a2} = -\frac{(E_a - I_{a1}Z_1)}{Z_2} = \frac{-E_a + I_{a1}Z_1}{Z_2}$$

Praktikum

$I_a = 0$

$I_a = I_{a0} + I_{a1} + I_{a2} = 0$

$$\frac{-E_a + I_{a1}Z_1}{Z_0} + \frac{E_a}{Z_1} - \frac{E_a + I_{a1}Z_1}{Z_2} = 0$$

$$I_{a1} \left[ \frac{Z_1 + Z_1 + 1}{Z_0 Z_2} \right] = \frac{E_a}{Z_0} + \frac{E_a}{Z_2}$$

$$I_{a1} \left[ 1 + Z_1 \left( \frac{1}{Z_0} + \frac{1}{Z_2} \right) \right] = E_a \left[ \frac{Z_0 + Z_2}{Z_0 Z_2} \right]$$

$$I_{a1} \left[ 1 + Z_1 \frac{(Z_0 + Z_2)}{Z_0 Z_2} \right] = \frac{E_a (Z_0 + Z_2)}{Z_0 Z_2}$$

X by  $\frac{Z_0 Z_2}{Z_0 + Z_2}$

$$I_{a1} \left[ \frac{Z_0 Z_2 + Z_1 (Z_0 + Z_2)}{Z_0 + Z_2} \right] = E_a$$

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}}$$

$I_{a2} = \frac{-I_{a1} \times Z_0}{Z_2 + Z_0}$

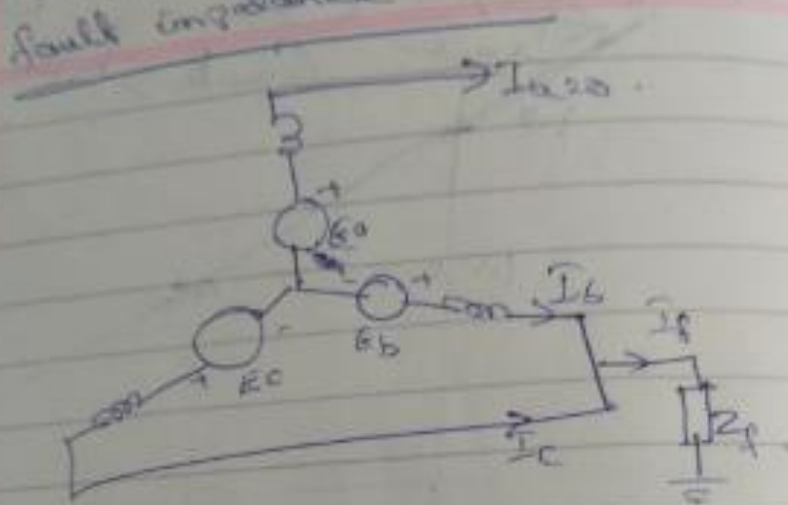
$I_{a0} = \frac{-I_{a1} \times Z_2}{Z_2 + Z_0}$

$I_f = I_b + I_c = I_{a0} + a^2 I_{a1} + a I_{a2}$   
 $+ I_{a0} + a I_{a1} + I_{a2}$   
 $= 2I_{a0} + (a + a^2) I_{a1} + (a + 1) I_{a2}$   
 $= 2I_{a0} - I_{a1} - I_{a2}$   
 $= 2I_{a0} - C (I_{a1} + I_{a2}) = 3I_{a0}$

$$I_f = 3I_{a0} = 3I_{a1} \left( \frac{Z_2}{Z_2 + Z_0} \right)$$



Double line to ground fault through fault impedance



$$I_a = 0, \quad I_f = I_b + I_c$$

$$V_b = V_c = I_f Z_f = (I_b + I_c) Z_f$$

$$\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} \quad V_b = V_c$$

$$V_{a0} = \frac{1}{3} [V_a + 2V_b]$$

$$V_{a1} = \frac{1}{3} [V_a + (a + a^2)V_b] = \frac{1}{3} [V_a - V_b]$$

$$V_{a2} = \frac{1}{3} [V_a + (a^2 + a)V_b] = \frac{1}{3} [V_a - V_b]$$

$$a) V_{a1} = V_{a2}$$

~~$$\text{also } V_{a0} - V_{a2} = \frac{1}{4} V_b = \frac{1}{4} (I_b + I_c) Z_f = \frac{1}{4} Z_f$$~~

$$V_{a0} - V_{a2} = 3 I_{a0} Z_f$$

$$\# \boxed{V_{a0} = V_{a2} + 3 I_{a0} Z_f}$$

$$I_0 = 0 \quad a) I_{a0} + I_{a1} + I_{a2} = 0$$



$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_2(3Z_f + Z_0)}{Z_0 + Z_2 + 3Z_f}}$$

$$I_{a2} = -I_{a1} \frac{Z_0 + 3Z_f}{Z_2 + Z_0 + 3Z_f}$$

$$I_{a0} = \frac{-I_{a1} \times Z_2}{Z_2 + Z_0 + 3Z_f}$$

$$I_f = I_b + I_c = 3I_{a0}$$

$$= -3.335 - j3$$

$$= 4.485 \angle -138^\circ \text{ p.u.}$$

$$(I_b)_{M_1} \text{ in amperes} = 4.485 \times \left( \frac{1.25 \times 10^6}{\sqrt{3} \times 0.6 \times 10^3} \right)$$

$$= 5394 \text{ A.}$$

### 5.5 Series type of Faults

We have so far discussed the various shunt type of faults that occur in a power system. But unsymmetrical faults in the form of open conductors (series type) also do take place in power system. It is required to determine the sequence components of line currents and the voltages across the broken ends of the conductor.

Fig. 5.56 shows a system wherein an open conductor fault takes place.

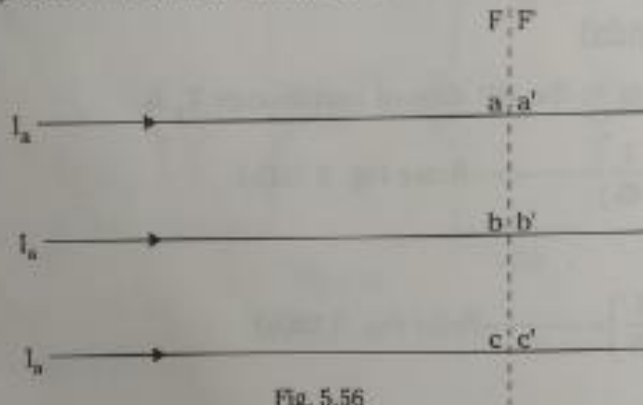


Fig. 5.56

The ends of the system on the sides of the fault are identified as F, F', while the conductor ends are denoted by aa', bb' and cc'. The voltage across the conductors are denoted by  $V_{aa'}$ ,  $V_{bb'}$  and  $V_{cc'}$ . The symmetrical components of these voltages are  $(V_{aa'})_1$ ,  $(V_{aa'})_2$  and  $(V_{aa'})_0$ . The sequence networks as seen from the two ends FF' of the system are schematically shown in fig. 5.57.

These are suitably interconnected depending on the type of fault (one or two conductors open).



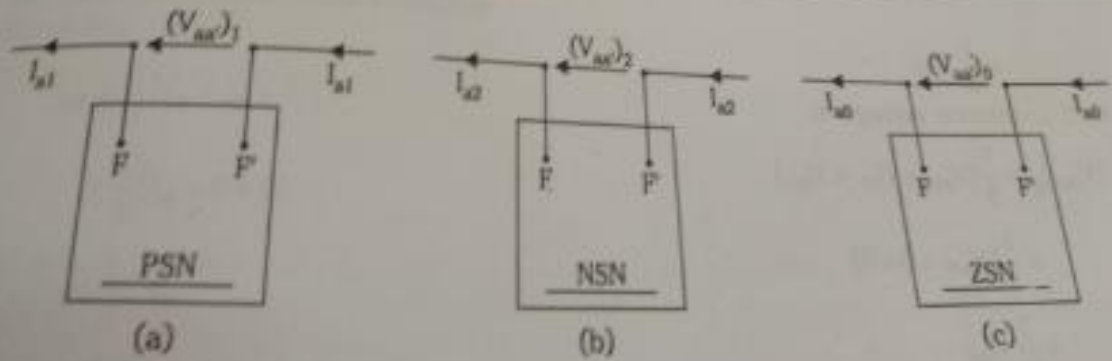


Fig. 5.57

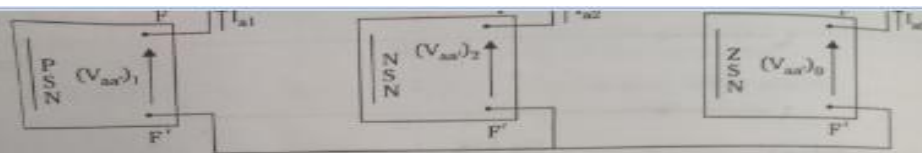


Fig. 5.59

(ii) Two conductors open fault

Let us assume that the two conductors b and c get open at the points F, F' as shown in fig. 5.60.

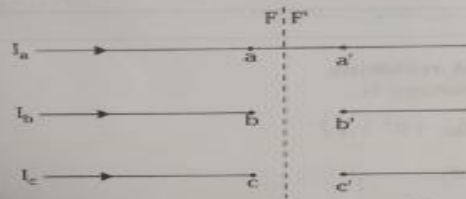


Fig. 5.60

Terminal conditions

As seen from the points F and F', the terminal conditions that are applicable to this fault are :

$I_b = 0$  ..... (5.99)

$I_c = 0$  ..... (5.100)

$V_{aa'} = 0$  ..... (5.101)

**Symmetrical component relations**

Consider,

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c)$$

$$= \frac{1}{3}(I_a + 0 + 0)$$

$$= \frac{1}{3} \cdot I_a$$

$$I_{a1} = \frac{1}{3}(I_a + a \cdot I_b + a^2 \cdot I_c)$$

$$= \frac{1}{3}(I_a + 0 + 0)$$

$$= \frac{1}{3} \cdot I_a$$

$$I_{a2} = \frac{1}{3}(I_a + a^2 \cdot I_b + a \cdot I_c)$$

$$= \frac{1}{3}(I_a + 0 + 0)$$

$$= \frac{1}{3} \cdot I_a$$

Thus  $I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a$  ..... (5.102)

The condition  $V_{aa'} = 0$  gives the result

$$(V_{aa'})_0 + (V_{aa'})_1 + (V_{aa'})_2 = 0$$
 ..... (5.103)

These conditions are similar to those of a line - to - ground fault and suggest that the three sequence networks be connected in series and shorted as shown in fig. 5.61

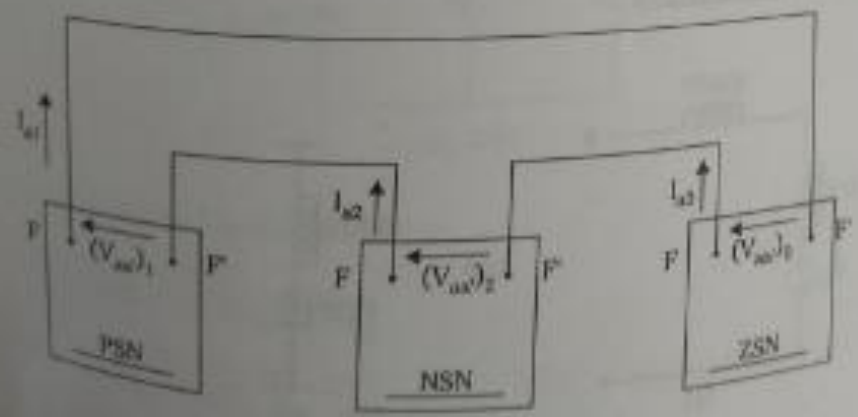


Fig. 5.61

## 6.1 Introduction

Stability of a large interconnected power system is its ability to return to normal or stable operation after having been subjected to some form of disturbance. Conversely, instability denotes a condition of loss of synchronisation in the system. This will result in wild fluctuation of currents and voltages within the power system network which is obviously undesirable. Hence, stability considerations form an important aspect in the study of power systems.

## 6.2 Some Definitions

**Stability** : Stability, when used with reference to a power system, is that attribute of the system, or part of the system, which enables it to develop restoring forces between the elements thereof equal to or greater than the disturbing forces so as to restore a state of equilibrium between the elements. This is the standard definition of AIEE.

**Steady State Stability** : This is the stability of the system under consideration subjected to a gradual or relatively slow change in load.

**Transient State Stability** : This refers to the stability of the system subjected to a sudden large disturbance. The large disturbance may be brought about by a sudden large change in load, faults in systems or loss of generation in the system.

**Dynamic Stability** : This denotes the artificial stability given to a system by the action of automatic control devices like fast acting voltage regulators and governors.

**Steady State Stability Limit (SSSL)** : This refers to the maximum flow of power possible through a particular point in the system without loss of stability when the power is increased gradually.

**Transient Stability Limit (TSL)** : This refers to the maximum flow of power possible through a particular point without the loss of stability when a sudden disturbance occurs.

**Infinite bus** : A system having a constant voltage and a constant frequency regardless of the load on it is called an Infinite bus-bar system or an Infinite bus. Physically, it is impossible to have an infinite bus-bar system. This is just considered for the purpose of analysis.

**Case (ii) Short circuit away from line ends**

When a three-phase fault occurs away from line ends (say in the middle of a line), there is some impedance between the paralleling buses & the fault. Therefore, some power is transmitted while the fault is still on the system. The one-line diagram of the system is shown in fig. 6.38.

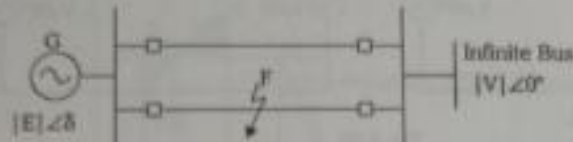


Fig. 6.38

The equivalent circuit before occurrences of fault is shown in fig. 6.39.

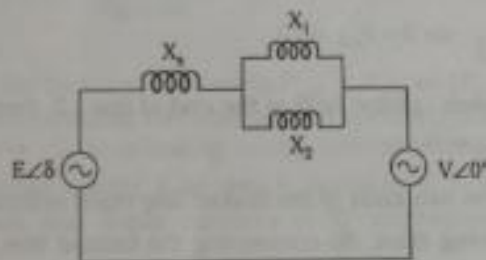


Fig. 6.39

The power angle curve is given by

$$P_{e1} = \frac{|E||V|}{X_s + (X_1 \parallel X_2)} \cdot \sin \delta = P_{m1} \sin \delta$$

Circuit model of the system during fault is shown in fig. 6.40.

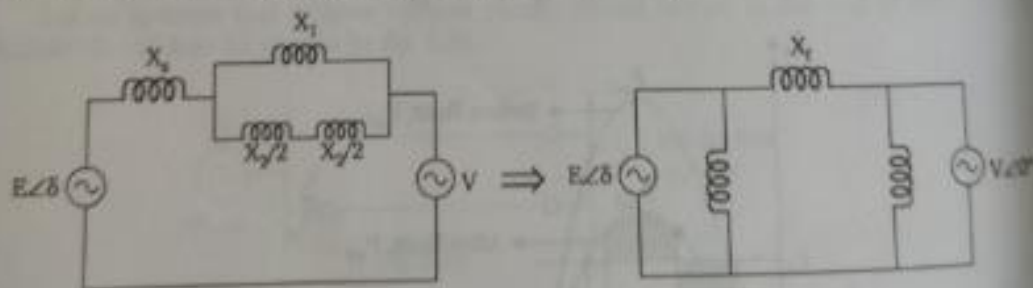


Fig. 6.40

Here,  $X_f$  = transfer reactance of the system. The power angle curve during fault is therefore given by

$$P_{e2} = \frac{|E||V|}{X_f} \cdot \sin \delta = P_{m2} \sin \delta$$

After the clearing of the fault by opening of the circuit breakers, the equivalent circuit is as shown in fig. 6.41.

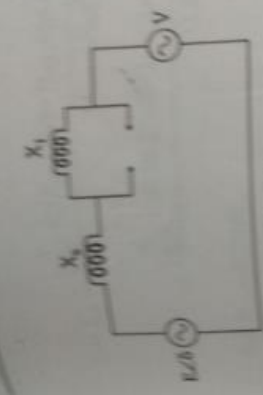


Fig. 6.41

The power angle curve is given as

$$P_{e3} = \frac{|E||V|}{X_2 + X_1} \sin \delta = P_{m3} \sin \delta$$

The power angle curves corresponds to  $P_{e1}$ ,  $P_{e2}$  &  $P_{e3}$  are shown in fig. 6.42.

The system is stable only if it is possible to find an area  $A_2$  equal to  $A_1$ .

### 6.4.6 Critical clearing angle & Critical clearing time

In the previous case, if  $P_g$  is increased, then  $\delta_1$  increase, area  $A_1$  increases and to find  $A_2 = A_1$ ,  $\delta_2$  is increased till it has a value  $\delta_{m'}$ , the maximum allowable limit for stability. Then the system is said to be critically stable. The angle  $\delta_1$  is then called as the critical clearing angle ( $\delta_{cc}$ ). The time corresponding to this is called the critical clearing time ( $t_{cc}$ ). The critical clearing angle can be determined from the EAC. However, the critical clearing time cannot be obtained from EAC. It is possible to estimate critical clearing time using the swing curve. This time is very much essential in designing the protective circuit breakers for the system.

The case of critical stability of a system is shown in fig. 6.43.

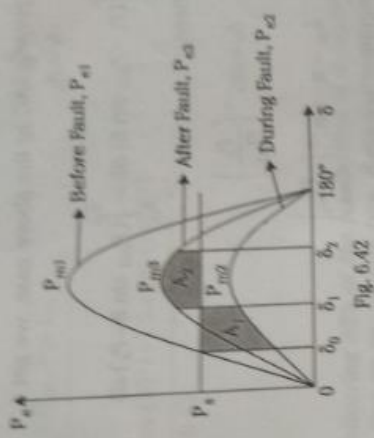


Fig. 6.42

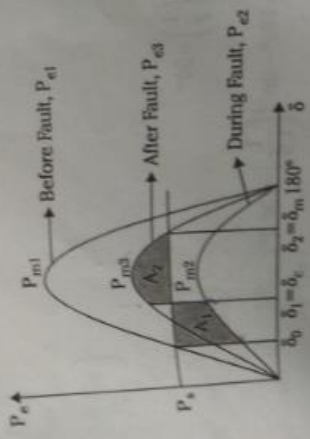


Fig. 6.43



Applying EAC to the above case, we get

$$A_1 = A_2$$

$$\int_{\delta_0}^{\delta_{cc}} (P_s - P_{m2} \sin \delta) d\delta = \int_{\delta_{cc}}^{\delta_m} (P_{m3} \sin \delta - P_s) d\delta$$

where,

$$\delta_0 = \sin^{-1} \left( \frac{P_s}{P_{m1}} \right)$$

$$\delta_m = \pi - \sin^{-1} \left( \frac{P_s}{P_{m3}} \right)$$

Integrating, we get

$$(P_s \cdot \delta + P_{m2} \cdot \cos \delta) \Big|_{\delta_0}^{\delta_{cc}} = (-P_{m3} \cos \delta - P_s \delta) \Big|_{\delta_{cc}}^{\delta_m}$$

$$\text{or } P_s (\delta_{cc} - \delta_0) + P_{m2} (\cos \delta_{cc} - \cos \delta_0) + P_s (\delta_m - \delta_{cc}) + P_{m3} (\cos \delta_m - \cos \delta_{cc}) = 0$$

$$\text{or } \cos \delta_{cc} = \frac{P_s (\delta_m - \delta_0) - P_{m2} \cos \delta_0 + P_{m3} \cdot \cos \delta_m}{(P_{m3} - P_{m2})} \quad (6.32)$$

The angles in the above equation are in radians. If the angles are in degrees, the equation modifies as below.

$$\cos \delta_{cc} = \frac{\frac{\pi}{180^\circ} P_s (\delta_m - \delta_0) - P_{m2} \cdot \cos \delta_0 + P_{m3} \cdot \cos \delta_m}{(P_{m3} - P_{m2})} \quad (6.33)$$

Base kilovolt on the generator = 11 kV.

Base kilovolt on the transmission line =  $11 \times \frac{34.5}{11.5} = 33 \text{ KV}$ .

Base kilovolt on the motor =  $33 \times \frac{6.9}{34.5} = 6.6 \text{ KV}$ .

**Sequence reactances of generator**

$$X_1 = 0.2 \times \frac{(20)}{(20)} \times \frac{(11)^2}{(11)^2} = 0.2 \text{ p.u.}$$

$$X_2 = 0.1 \times \frac{(20)}{(20)} \times \frac{(11)^2}{(11)^2} = 0.1 \text{ p.u.}$$

$$X_0 = 0.1 \times \frac{(20)}{(20)} \times \frac{(11)^2}{(11)^2} = 0.1 \text{ p.u.}$$

**Sequence reactances of transformer T<sub>1</sub>**

$$\begin{aligned} X_1 = X_2 = X_0 &= X \times \frac{(MVA)_{B, \text{new}}}{(MVA)_{B, \text{old}}} \times \frac{(KV)_{B, \text{old}}^2}{(KV)_{B, \text{new}}^2} \\ &= 0.1 \times \frac{(20)}{(18)} \times \frac{(11.5)^2}{(11)^2} \\ &= 0.12 \text{ p.u.} \end{aligned}$$

**Sequence reactances of transmission line**

$$\begin{aligned} X_1 = X_2 = X_0 &= X \text{ in } \Omega \times \frac{(MVA)_{B, \text{new}}}{(KV)_B^2} \\ &= 5 \times \frac{20}{(33)^2} = 0.092 \text{ p.u.} \end{aligned}$$

$$X_0 = 10 \times \frac{20}{(33)^2} = 0.184 \text{ p.u.}$$

**Sequence reactances of transformer T<sub>2</sub>**

$$\begin{aligned} X_1 = X_2 = X_0 &= X \times \frac{(MVA)_{B, \text{new}}}{(MVA)_{B, \text{old}}} \times \frac{(KV)_{B, \text{old}}^2}{(KV)_{B, \text{new}}^2} \\ &= 0.1 \times \frac{(20)}{(15)} \times \frac{(6.9)^2}{(6.6)^2} \\ &= 0.146 \text{ p.u.} \end{aligned}$$

Sequence reactances of motor :

$$X_1 = 0.2 \times \frac{(20)}{(15)} \times \frac{(6.9)^2}{(6.6)^2}$$

$$= 0.29 \text{ p.u.}$$

$$X_2 = X_0 = 0.1 \times \frac{(20)}{(15)} \times \frac{(6.9)^2}{(6.6)^2}$$

$$= 0.145 \text{ p.u.}$$

### Positive Sequence Network (PSN)

Using the calculated values of positive sequence impedances, the PSN is drawn as in fig. 5.40.

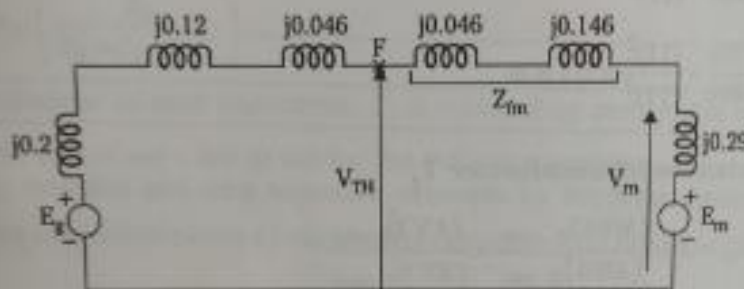


Fig. 5.40

To find the voltage at the fault point ( $V_{FF}$ )

$$\text{The current drawn by the motor } I_m = \frac{10 \times 10^6}{\sqrt{3} \times 6 \times 10^3 \times 0.8} \angle -\cos^{-1} 0.8$$

$$= 1202.8 \angle -36.87^\circ \text{ A.}$$

$$\text{The base current in the motor } (I_m)_B = \frac{20 \times 10^6}{\sqrt{3} \times 6.6 \times 10^3} = 1749.55 \text{ A.}$$

$$\therefore I_m \text{ in p.u.} = \frac{I_m}{(I_m)_B} = \frac{1202.8}{1749.55} \angle -36.87^\circ = 0.687 \angle -36.87^\circ \text{ p.u.}$$

$$V_m \text{ in p.u.} = \frac{6}{6.6} = 0.909 \angle 0^\circ \text{ p.u.}$$

Hence the voltage at the fault point is

$$V_{FF} = V_m + I_m \cdot Z_{fm}$$

$$= 0.909 + (0.687 \angle -36.87^\circ) \times (0.192 \angle 90^\circ)$$

$$= 0.909 + 0.132 \angle 53.13^\circ$$

$$= 0.909 + 0.0792 + j0.106$$

$$= 0.9882 + j0.106$$

$$= 0.994 \angle 6.1^\circ \text{ p.u.}$$

To find the Thevni's impedance  $Z_{1TH}$   
 The Thevni's impedance as seen from point 'F' is

$$Z_{1TH} = j[(0.2 + 0.12 + 0.046) \parallel (0.046 + 0.146 + 0.29)]$$

$$= j[0.366 \parallel 0.482]$$

$$= j0.208 \text{ p.u.}$$

Hence the equivalent PSN of the system is as shown below :

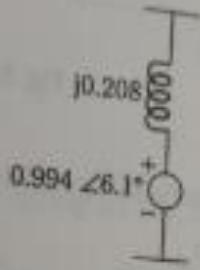


Fig. 5.41

**Negative Sequence Network (NSN)**

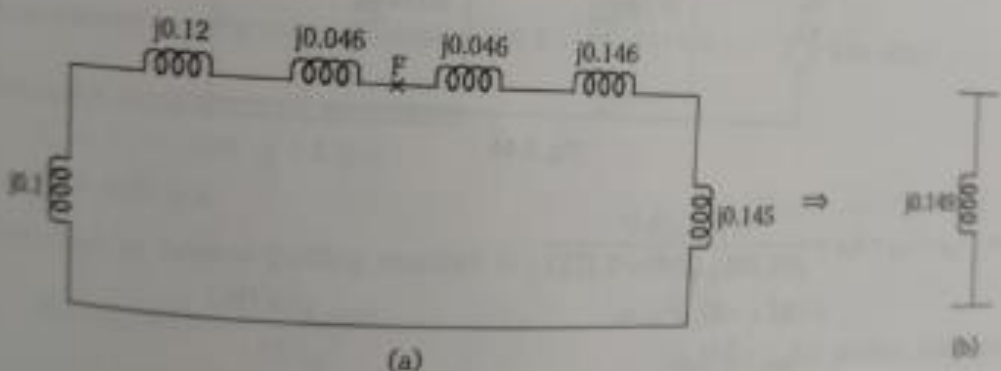


Fig. 5.42

The Thevni's equivalent impedance with respect to the fault point is

$$Z_{2TH} = j[0.1 + 0.12 + 0.046] \parallel (0.046 + 0.146 + 0.145)$$

$$= j[(0.266) \parallel (0.337)]$$

$$= j0.149 \text{ p.u.}$$

**Zero Sequence Network (ZSN)**

The Thevni's zero sequence impedance is

$$Z_{0TH} = j[(0.1 + 0.12 + 0.092) \parallel (0.092 + 0.146 + 0.145)]$$

$$= j[(0.312) \parallel (0.383)]$$

$$= j0.172 \text{ p.u.}$$

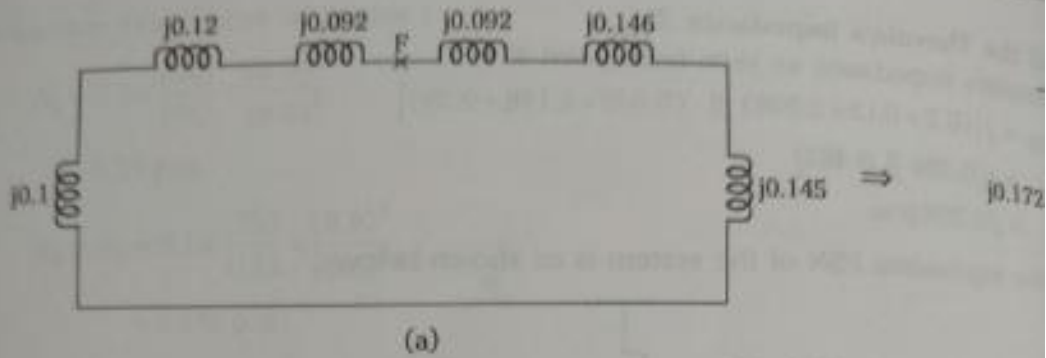


Fig. 5.43

**Interconnection of sequence networks**

The sequence networks are connected as shown in fig. 5.44 to represent LG fa

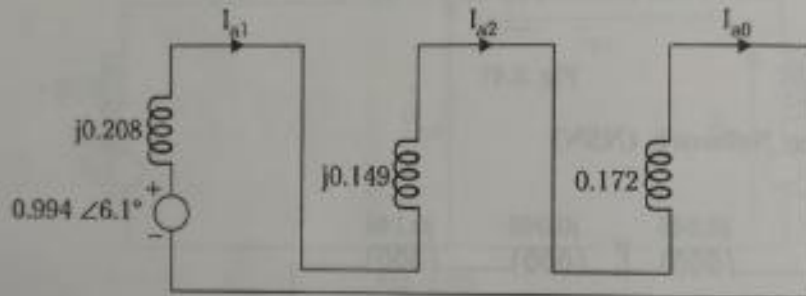


Fig. 5.44

$$\text{Here, } I_{a1} = I_{a2} = I_{a0} = \frac{0.994 \angle 6.1^\circ}{j(0.208 + 0.149 + 0.172)}$$

$$= 1.88 \angle -83.9^\circ \text{ p.u.}$$

$$\text{Hence fault current } |I_f|_{p.u.} = 3 |I_{a0}|$$

$$= 3 \times (1.88)$$

$$= 5.64 \text{ p.u.}$$

$$\text{Fault current in amperes is } |I_f|_{p.u.} \times (I_{TL})_B$$

$$= 5.64 \times \left( \frac{20 \times 10^6}{\sqrt{3} \times 33 \times 10^3} \right)$$

$$= 1973.49 \text{ A.}$$

**Example 5.9 :** A 25 MVA, 11 kV, three phase generator has a subtransient reactance of 20%. The generator supplies two motors over a transmission line.



### 6.4.2 Swing Equation

The load angle or the torque angle  $\delta$  depends upon the loading of the machine. Larger the loading, larger is the value of the torque angle. If some load is added or removed from the shaft of the synchronous machine, the rotor will decelerate or accelerate respectively with respect to the synchronously rotating stator field and a relative motion begins. It is said that the rotor is swinging with respect to the stator field. The equation describing the relative motion of the rotor (load angle  $\delta$ ) with respect to the stator field as a function of time is called as **swing equation**.

Consider the generator shown in fig. 6.26. It receives mechanical power  $P_s$  at torque  $T_s$  and rotor speed  $\omega$  via shaft from the prime mover. It delivers electrical power  $P_e$  to the power system network via the bus bars. The generator develops electromechanical torque  $T_e$  in opposition to  $T_s$ .

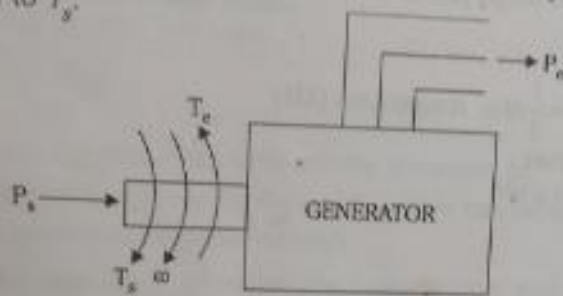


Fig. 6.26

Assuming that winding and friction losses to be negligible, the accelerating torque on the rotor is given by

$$T_a = T_s - T_e \quad (6.22)$$

Multiplying by ' $\omega$ ' on both sides, we get

$$\omega \cdot T_a = \omega \cdot T_s - \omega \cdot T_e$$

but

$$\omega \cdot T_a = P_a = \text{accelerating power}$$

$$\omega \cdot T_s = P_s = \text{mechanical power input}$$

$$\omega \cdot T_e = P_e = \text{electrical power output assuming that power loss is negligible.}$$

Therefore, we get

$$P_a = P_s - P_e \quad (6.23)$$

Under steady state conditions,  $P_s = P_e$ , so that  $P_a = 0$ .

When  $P_s \cdot P_e$  balance is disturbed, the machine undergoes dynamics governed by

$$P_a = T_a \omega = I \cdot \alpha \cdot \omega = M \cdot \frac{d^2 \theta}{dt^2} \quad (6.24)$$

where  $\alpha = \frac{d^2 \theta}{dt^2}$  is the angular acceleration of the rotor.

Since the angular position  $\theta$  of the rotor is continually varying with time, it is more

convenient to measure the angular position and velocity with respect to a synchronously rotating axis. (Fig. 6.27).

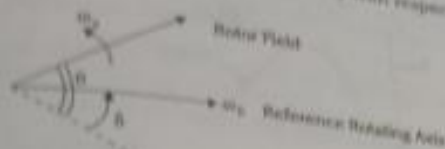


Fig. 6.27

From the fig. 6.25, it can be inferred that

$$\delta = \theta - \omega_0 t \tag{6.25}$$

where,  $\omega_0$  = angular velocity of the reference rotating axis.

$\delta$  = rotor angular displacement with respect to the stator field.

Taking time derivatives of Eq. (6.25),

$$\frac{d\delta}{dt} = \frac{d\theta}{dt} - \omega_0$$

$$\text{and } \frac{d^2\delta}{dt^2} = \frac{d^2\theta}{dt^2} \tag{6.26}$$

Combining equation (6.23), (6.24) and (6.26), we get

$$M \frac{d^2\delta}{dt^2} = P_a = P_s - P_e \tag{6.27}$$

This equation is called as the swing equation of the synchronous machine. When the machine is connected to the infinite bus bars, then  $P_e = \frac{|E| |V|}{X} \sin \delta = P_m \sin \delta$ .

$$\text{or } M \frac{d^2\delta}{dt^2} = P_s - P_m \sin \delta \tag{6.28}$$

### 6.4.3 Swing Curve

The solution of swing equation gives the relation between rotor angle ' $\delta$ ' as a function of time ' $t$ '. The plot of ' $\delta$ ' versus ' $t$ ' is called as swing curve. The exact solution of the swing equation is however a very tedious task. Normally, step-by-step method or any other numerical solution techniques like Euler's method, Runge-Kutta's method are used for solving the swing equation. The swing curve is used to determine the stability of the system. In case  $\delta$  increases indefinitely, it indicates instability. Whereas if it reaches a maximum and starts decreasing, it shows that the system will not lose stability since the oscillations will be damped out with time. A sample swing curve is shown in fig. 6.28.

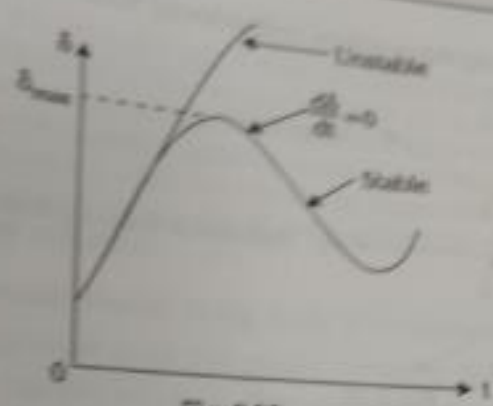


Fig. 6.28

The stability of the system,  $\frac{d\delta}{dt} = 0$  ..... (6.29)

System will be unstable if  $\frac{d\delta}{dt} > 0$  for a sufficiently long time (normally more than 1

Example 6.9 : A two pole, 50 Hz, 11 kV turbo alternator has a rating of 100 MW, power factor 0.85 lagging. The rotor has a moment of inertia of 10,000 kg.m<sup>2</sup>. Calculate H and M.

Solution :

Rating of the alternator =  $G = \frac{100}{0.85} = 117.65$  MVA.

Therefore

Kinetic Energy =  $GH = \frac{1}{2} I \omega^2$

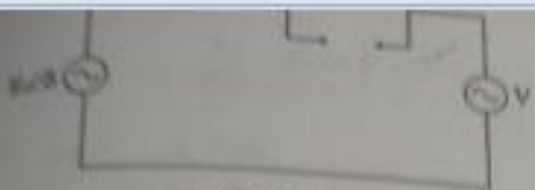


Fig. 6.41

The power angle curve is given as

$$P_{e3} = \frac{|E||V|}{X_s + X_1} \sin \delta = P_{m3} \sin \delta$$

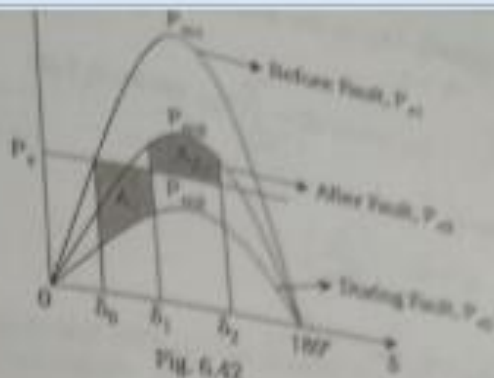


Fig. 6.42

The power angle curves corresponds to  $P_{e1}$ ,  $P_{e2}$  &  $P_{e3}$  are shown in fig. 6.42. The system is stable only if it is possible to find an area  $A_2$  equal to  $A_1$ .

### 6.4.6 Critical clearing angle & Critical clearing time

In the previous case, if  $P_s$  is increased, then  $\delta_1$  increase, area  $A_1$  increases and to find  $A_2 = A_1$ ,  $\delta_2$  is increased till it has a value  $\delta_m$ , the maximum allowable limit for stability. Then the system is said to be critically stable. The angle  $\delta_1$  is then called as the critical clearing angle ( $\delta_{cc}$ ). The time corresponding to this is called the critical clearing time ( $t_{cc}$ ). The critical clearing angle can be determined from the EAC. However, the critical clearing time cannot be obtained from EAC. It is possible to estimate critical clearing time using the swing curve. This time is very much essential in designing the protective circuit breaker for the system.

The case of critical stability of a system is shown in fig. 6.43.

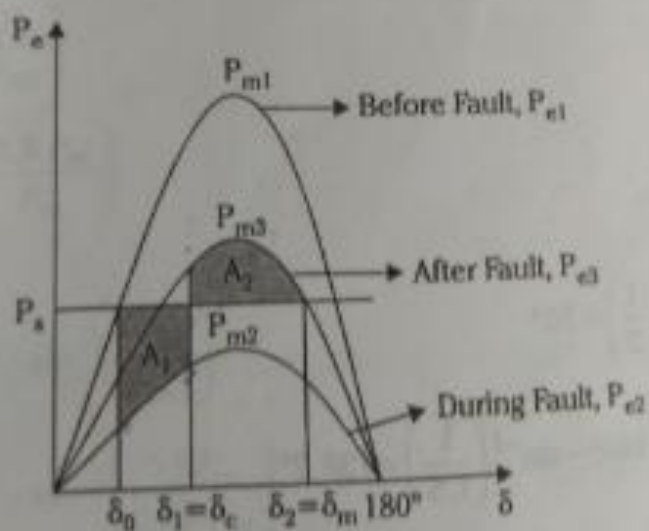


Fig. 6.43

Applying EAC to the above case, we get

$$A_1 = A_2$$

$$\int_{\delta_0}^{\delta_m} (P_s - P_{m2} \sin \delta) d\delta = \int_{\delta_{cc}}^{\delta_m} (P_{m3} \sin \delta - P_s) d\delta$$

where,

$$\delta_0 = \sin^{-1} \left( \frac{P_s}{P_{m1}} \right)$$

$$\delta_m = \pi - \sin^{-1} \left( \frac{P_s}{P_{m3}} \right)$$

Integrating, we get

$$(P_s \delta + P_{m2} \cos \delta) \Big|_{\delta_0}^{\delta_m} = (-P_{m3} \cos \delta - P_s \delta) \Big|_{\delta_{cc}}^{\delta_m}$$

$$\text{or } P_s (\delta_m - \delta_0) + P_{m2} (\cos \delta_m - \cos \delta_0) + P_s (\delta_m - \delta_{cc}) + P_{m3} (\cos \delta_m - \cos \delta_{cc}) = 0$$

$$\text{or } \cos \delta_{cc} = \frac{P_s (\delta_m - \delta_0) - P_{m2} \cos \delta_0 + P_{m3} \cos \delta_m}{(P_{m3} - P_{m2})} \quad (6.32)$$

The angles in the above equation are in radians. If the angles are in degrees, the equation modifies as below.

$$\cos \delta_{cc} = \frac{\frac{\pi}{180^\circ} P_s (\delta_m - \delta_0) - P_{m2} \cos \delta_0 + P_{m3} \cos \delta_m}{(P_{m3} - P_{m2})} \quad (6.33)$$



**Example 6.12 :** An equivalent generator connected to a 50 Hz infinite bus power limits before, during & after a fault is cleared as 2.0 pu, Calculate the critical clearing angle if the initial load is 1.0 p.u.

**Solution :**

Given :  $P_{m1} = 2$  pu

$$P_{m2} = 0.5 \text{ pu}$$

$$P_{m3} = 1.5 \text{ pu}$$

$$P_s = 1 \text{ pu}$$

$$\therefore \delta_0 = \sin^{-1} \left( \frac{P_s}{P_{m1}} \right) = \sin^{-1} \left( \frac{1}{2} \right) = 30^\circ$$

$$\delta_m = 180^\circ - \sin^{-1} \left( \frac{P_s}{P_{m3}} \right) = 180^\circ - \sin^{-1} \left( \frac{1}{1.5} \right) = 138.2^\circ.$$

We have,

$$\cos \delta_{cc} = \frac{\frac{\pi}{180^\circ} P_s (\delta_m - \delta_0) - P_{m2} \cdot \cos \delta_0 + P_{m3} \cdot \cos \delta_m}{(P_{m3} - P_{m2})}$$
$$= \frac{\frac{\pi}{180^\circ} \times 1 (138.2^\circ - 30^\circ) - 0.5 \times \cos (30^\circ) + 1.5 \times \cos (138.2^\circ)}{(1.5 - 0.5)}$$

$$\cos \delta_{cc} = 0.3388.$$

$$\therefore \delta_{cc} = 70.2^\circ.$$

This is the desired result.