

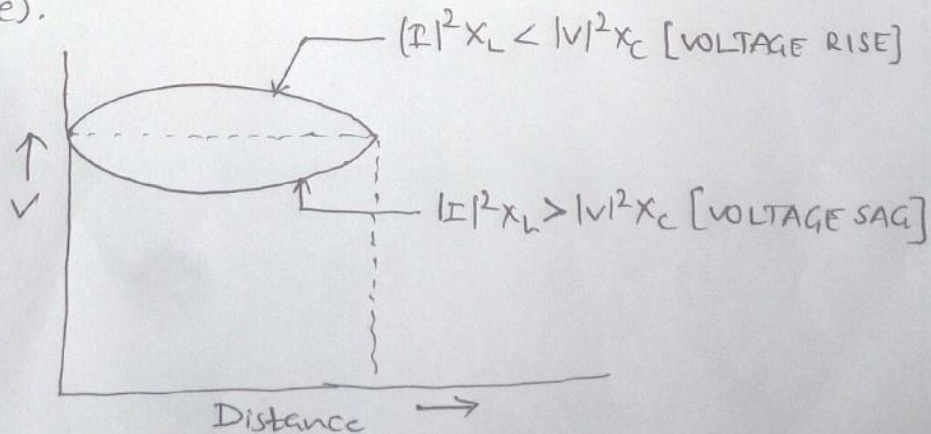
1. Explain different sources of reactive power generation and absorption of reactive power in a power system.

* TRANSMISSION LINES:

1. The loading condition in which the VARs absorbed are equal to VARs generated by the line is called the surge impedance loading (SIL), and it is where the voltage throughout the length of the line is same.

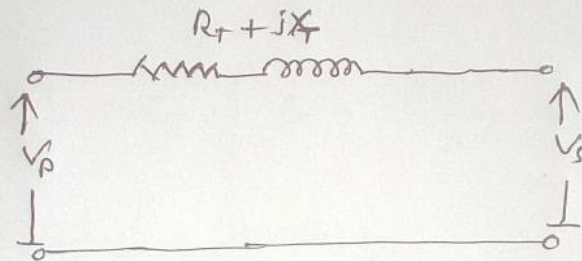
2. Normally the loading is greater than SIL and therefore, the condition $(I^2 X_L > |V|^2 X_C)$ exists and the net effect of the line will be to absorb (sink) the reactive power (VARs).

3. Under light load conditions the effect of shunt capacitors is predominating and the line will work as VARs generator (source).



* TRANSFORMERS: —

The equivalent circuit of a transformer for power frequency is:



R_T = per unit resistance

X_T = per unit reactance

By definition, Per unit reactance (X_T) = $\frac{\text{Actual Reactance (X)}}{\left(\frac{V}{I}\right)}$

$$\text{Actual reactance, } X = X_T \cdot \left(\frac{V}{I}\right)$$

$$I = \frac{\text{KVA}}{\sqrt{3} \text{KV}}$$

$$\therefore X = \frac{\sqrt{3} X_T \cdot \text{KV}^2 \cdot 1000}{\text{KVA}}$$

The reactive power absorbed by the transformer,

$$3 I^2 X = \frac{3 \text{KVA}^2}{3 \text{KV}^2} \cdot \frac{\sqrt{3} X_T \text{KV}^2 \cdot 1000}{\text{KVA}}$$

$$\boxed{3 I^2 X = \sqrt{3} \text{KVA} \cdot X_T} \text{ KVARs}$$

Transformer always absorb reactive power.

* SYNCHRONOUS MACHINES:

It is known that the power transmitted from a generator bus to an infinite bus-bar is given by,

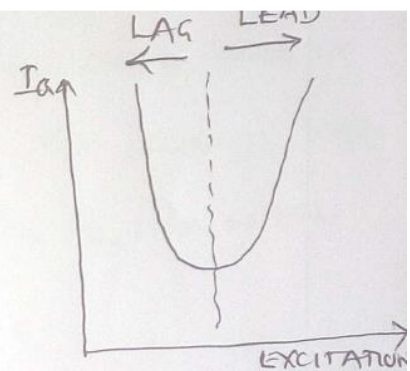
$$P = \frac{|E||V|}{X} \sin \delta \quad \Bigg| \quad Q = \frac{|V||E|}{X} \cos \delta - \frac{|V|^2}{X}$$

Where, E = Generator voltage

V = Infinite bus bar voltage

X = Reactance of the unit

δ = Angle between E and V



The above formula tells that if $|E| \cos \delta > |V|$, then $Q > 0$ and the generator produces reactive power i.e. it acts as a capacitor. Therefore, it can be said that an over-excited synchronous machine produces reactive power and acts as a shunt capacitor.

Similarly when $|E| \cos \delta < |V|$, $Q < 0$ and the machine consumes reactive power. Consequently an under-excited machine acts as a shunt coil.

* SHUNT CAPACITORS AND REACTORS:

1. Shunt capacitors are used across an inductive load, to supply part of the reactive power (VARs) required by the load. Thereby the voltage across the load is maintained within certain desirable limits.

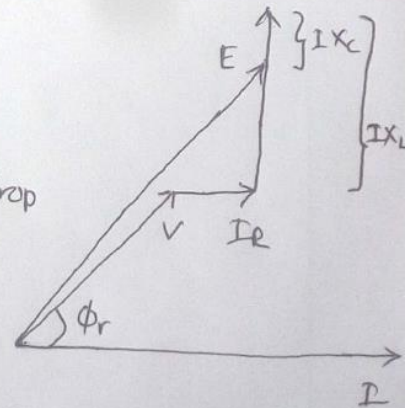
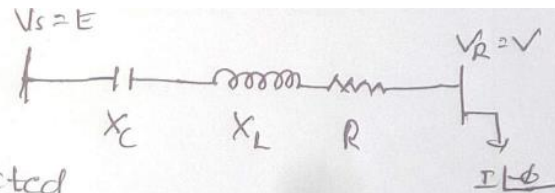
2. The shunt reactors are used across capacitive loads or lightly loaded lines to absorb some of the leading VARs again to control the voltage across the load to within certain desirable limits.

* SERIES CAPACITORS:

If a static capacitor is connected in series with the line, it reduces the inductive reactance between the load and the supply point and the voltage drop is approximately

$$IR \cos \phi_r + I(X_L - X_C) \sin \phi_r$$

It is clear from the vector diagram, that the voltage drop produced by an inductive load can be reduced particularly when the line has a high X/R ratio.



CABLES:

Cables are generators of reactive power owing to their high shunt capacitance.

2. Explain with suitable block diagram, the mathematical modeling of AVR.

OBJECTIVES:—

- * To maintain the static accuracy of the terminal voltage.
- * For better transient response.

AMPLIFIER MODEL:—

Let the transfer function of Amplifier;

$$G_A(s) = \frac{K_A}{1 + sT_A} = \frac{\Delta V_R(s)}{\Delta e(s)}$$

K_A = Gain of Amplifier

T_A = Time constant of Amplifier.

$$\therefore \Delta V_R(s) = G_A(s) \cdot \Delta e(s)$$

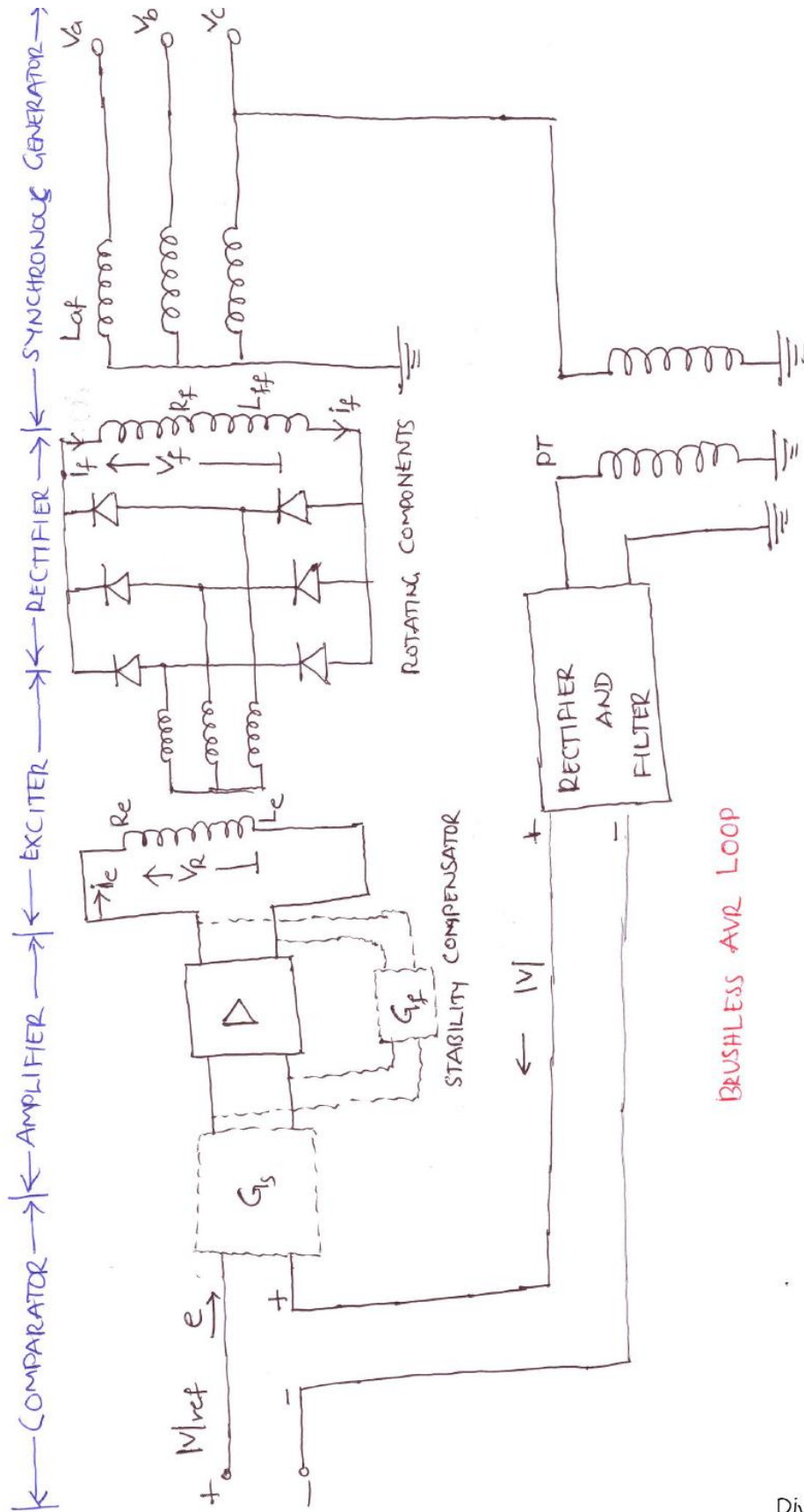
EXCITER MODELING:—

Define R_e = Exciter Field Resistance (Ω)

L_e = Exciter Field Inductance (H)

$$\therefore \Delta V_R = R_e \cdot \Delta i_e + L_e \frac{d(\Delta i_e)}{dt}$$

(b)



(4)

Taking Laplace Transform; $\Delta V_R(s) = R_e \cdot \Delta I_e(s) + s L_e \Delta I_e(s)$

$$\Delta I_e(s) = \frac{\Delta V_R(s)}{[R_e + s L_e]}$$

From above AVR loop it is clear that;

$$\Delta V_f \propto \Delta I_e$$

$$\therefore \Delta V_f(s) = k_1 \Delta I_e(s)$$

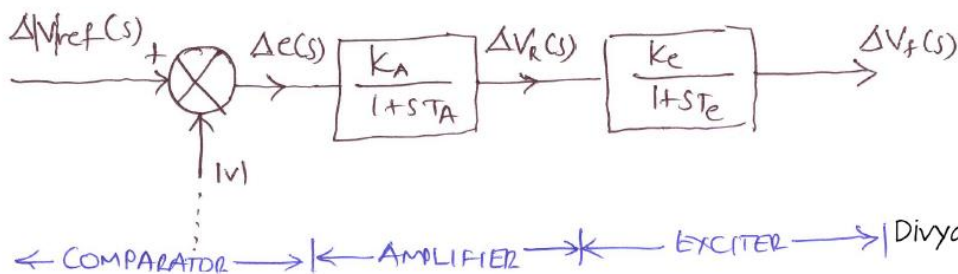
$$\Delta V_f(s) = k_1 \cdot \left[\frac{\Delta V_R(s)}{R_e + s L_e} \right]$$

$$\frac{\Delta V_f(s)}{\Delta V_R(s)} = \frac{k_1 / R_e}{1 + s \left(\frac{L_e}{R_e} \right)} = \frac{k_e}{1 + s T_e} = G_e(s)$$

↳ Transfer Function of Exciter

Where; $k_e =$ Gain constant of Exciter

$T_e =$ Time constant of Exciter.



We need to close the above loop, if the voltage drop across armature winding is neglected we can write;

$$|E| \approx |V|$$

↖ Terminal voltage/ph

$$\text{Induced EMF/ph in armature}$$

GENERATOR FIELD MODELING:—

Define R_f = Generator field resistance (Ω)

L_{ff} = Generator field inductance (H)

L_{af} = Mutual inductance between rotor and stator fields.

$$\therefore \Delta V_f = R_f \Delta i_f + L_{ff} \frac{d}{dt} (\Delta i_f)$$

Taking Laplace Transform; $\Delta V_f(s) = (R_f + sL_{ff}) \Delta I_f(s)$

$$\Delta I_f(s) = \frac{\Delta V_f(s)}{(R_f + sL_{ff})}$$

$$\Delta |E|(s) = \Delta |V|(s) = \frac{\omega L_{af}}{\sqrt{2}} \Delta I_f(s)$$

$$\Delta |V|(s) = \frac{\omega L_{af}}{\sqrt{2}} \cdot \frac{\Delta V_f(s)}{(R_f + sL_{ff})}$$

$$\Delta |V|(s) / \Delta V_f(s) = \frac{[\omega L_{af} / \sqrt{2} R_f]}{[1 + s[L_{ff}/R_f]]}$$

$$\boxed{\frac{\Delta |V|(s)}{\Delta V_f(s)} = \frac{k_f}{1 + sT_{df}} = G_f(s)}$$

→ TF of Generator field.

$$\phi_{fa} = \phi_m \sin \omega t$$

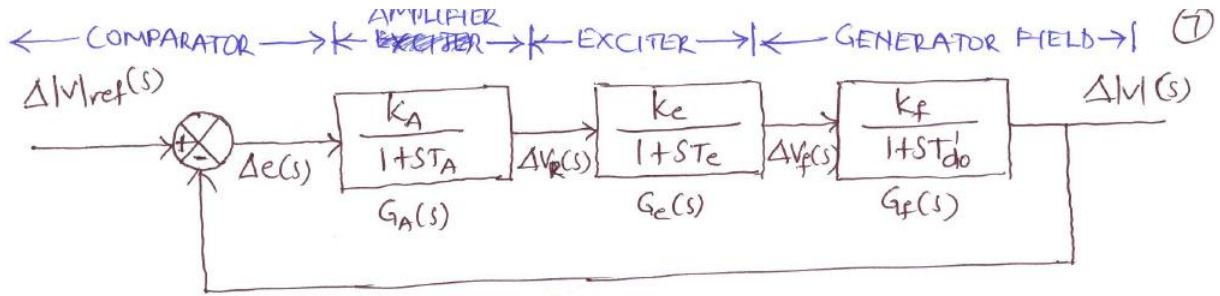
$$e(t) = \frac{d\phi_{fa}}{dt} = \omega \phi_{fa} \cos \omega t$$

$$E_m = \omega \phi_{fa}$$

$$E_{rms} = |E| = \frac{E_m}{\sqrt{2}} = \frac{\omega \phi_{fa}}{\sqrt{2}}$$

$$\phi_{af} = L_{af} \cdot i_f$$

$$|E| = \frac{\omega L_{af}}{\sqrt{2}} \cdot i_f$$

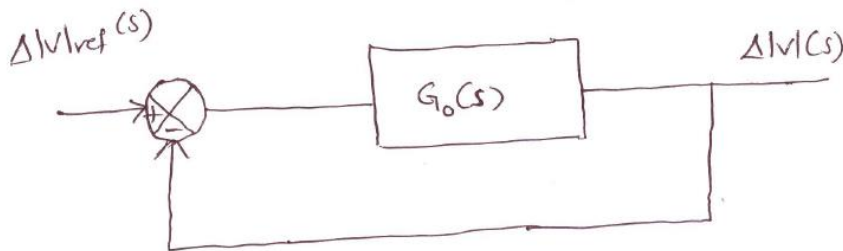


Open loop transfer function:

$$G_o(s) = G_A(s) \cdot G_E(s) \cdot G_F(s) = \frac{K_A \cdot K_e \cdot K_f}{(1+sT_A)(1+sT_e)(1+sT_{do}')$$

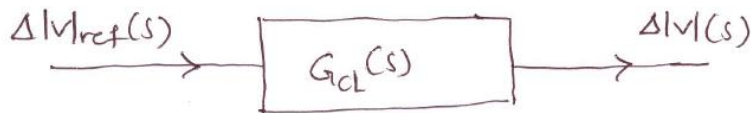
$$\text{Let } k = K_A \cdot K_e \cdot K_f$$

$$G_o(s) = \frac{k}{(1+sT_A)(1+sT_e)(1+sT_{do}')$$



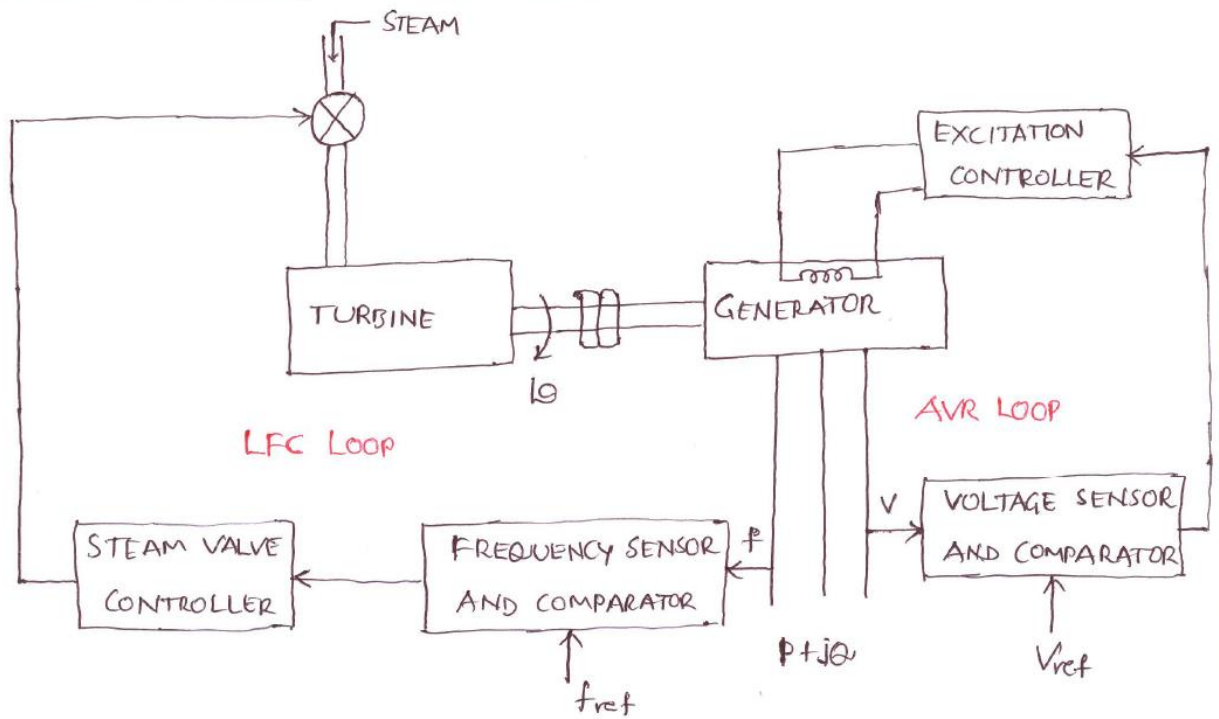
Closed loop transfer function:

$$G_{cl}(s) = \frac{G_o(s)}{1+G_o(s)} = \frac{k}{k + (1+sT_A)(1+sT_e)(1+sT_{do}')$$



3(a). Write notes on basic generator control loops, and cross coupling between loops.

BASIC GENERATOR CONTROL LOOPS:



SCHEMATIC DIAGRAM OF LOAD FREQUENCY AND EXCITATION
VOLTAGE REGULATOR OF A TURBO-GENERATOR

The two control loops are:

- Control of turbine input also called as:
 - Load frequency control (LFC)
 - Automatic Generation Control (AGC)
 - Automatic Load Frequency Control (ALFC)
 - MW-f control loop
 - Power-frequency control loop

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— Excitation control (or) MVAR-Voltage (Q-V) Control

CROSS-COUPLING BETWEEN CONTROL LOOPS:

- * Active power change is dependent on internal machine angle ' δ ' and is independent of bus voltage. Change in angle ' δ ' is caused by momentary change in generator speed.
- * While bus voltage is dependent on machine excitation and therefore on reactive power generation ' Q ' and is independent of machine angle ' δ '.
- * Therefore, load frequency and excitation voltage controls are non-interactive and can be modelled, analysed independently.
- * Excitation voltage control is fast acting in which the major time constant is that of generator field.
- * Power-frequency control is slow acting with major time constant contributed by the turbine and generator moment of inertia. This time constant is much larger than that of the generator field.
- * Thus the transients in excitation voltage control vanish much faster and do not affect the dynamics of power frequency control.

3(b). Determine the primary ALFC loop parameters for control area having the following data.

Total rated area capacity $P_r = 2000$ MW

Inertia Constant $H = 5.0$ s

Frequency $f_0 = 60$ Hz

Normal operating load = 1000 MW

Assume that the load frequency dependency is linear, meaning that the load would increase 1% for 1% frequency change.

$$\Delta P_D = 1\% \text{ of } 1000 = 10 \text{ MW}$$

$$\Delta f = 1\% \text{ of } 60 = 0.6 \text{ Hz.}$$

$$D = \frac{\Delta P_D}{\Delta f} = \frac{10}{0.6} = 16.67 \text{ MW/Hz} = \frac{16.67}{2000} = 0.00833 \text{ pu MW/Hz.}$$

$$K_p = \frac{1}{D} = 120 \text{ Hz/pu MW} \text{ — Power system gain}$$

$$T_p = \frac{2H}{f_0 D} = 20 \text{ Sec.} \text{ — power system time constant}$$

$$G_p(s) = \frac{K_p}{1+sT_p} = \frac{120}{1+20s}$$

↳ Power System Transfer Functions

4(a). Derive the equations to get the relation between voltage, power and reactive power at a node.

The phase voltage V at a node is a function of P and Q at that node.

$$\text{i.e. } V = f(P, Q)$$

The voltage is also independent of adjacent nodes and assume that these are infinite buses.

The total differential of V_i

$$dV = \left(\frac{\partial V}{\partial P}\right) \cdot dP + \left(\frac{\partial V}{\partial Q}\right) \cdot dQ$$

and using the relation

$$\left(\frac{\partial P}{\partial V}\right) \cdot \left(\frac{\partial V}{\partial P}\right) = 1 \text{ and}$$

$$\left(\frac{\partial Q}{\partial V}\right) \cdot \left(\frac{\partial V}{\partial Q}\right) = 1$$

$$dV = \frac{dP}{\left(\frac{\partial P}{\partial V}\right)} + \frac{dQ}{\left(\frac{\partial Q}{\partial V}\right)}$$

From the above equation it is seen that the change in voltage at a node is defined by two quantities.

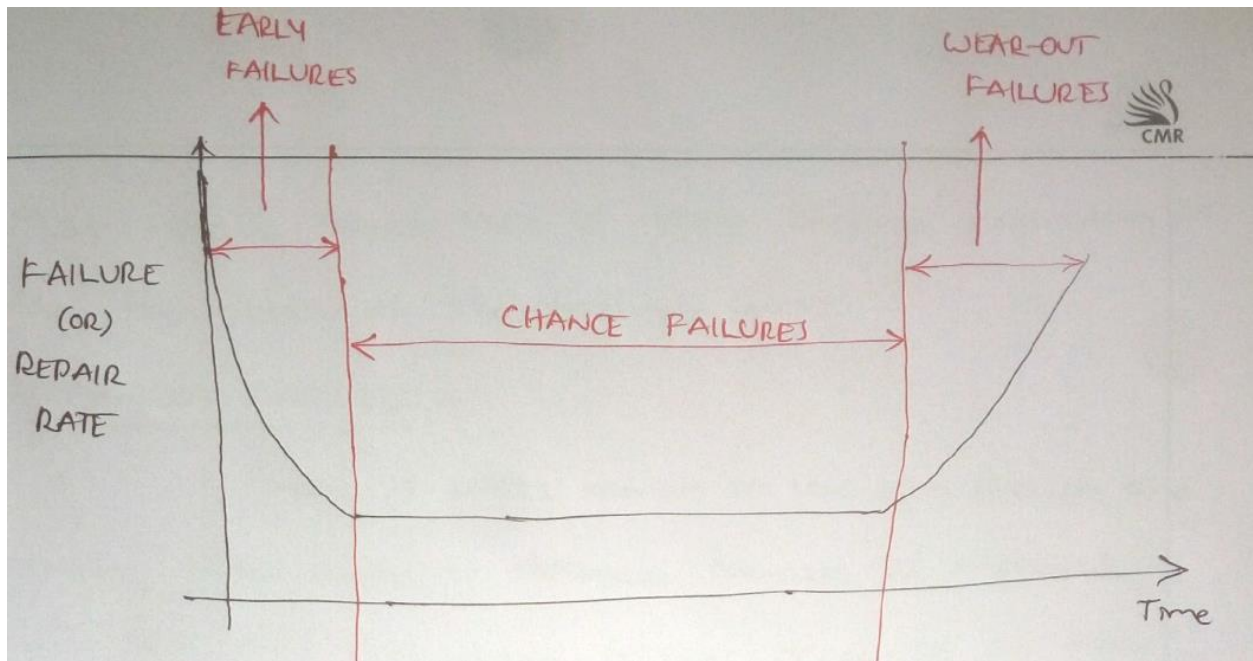
$$\left(\frac{\partial P}{\partial V}\right) \text{ and } \left(\frac{\partial Q}{\partial V}\right)$$

Normally $\left(\frac{\partial Q}{\partial V}\right)$ is the quantity of greater interest and can be experimentally determined using Network Analysis by injecting known quantity of VARs at the node in question and measuring the difference in voltage produced.

4(b). Explain the three modes of failures of a system.

- Early Failures
- Chance Failures
- Wear-out Failures.

A Plot of the failure rate (hazard rate) over time for most products yields a curve that looks like a drawing of a "bathtub".



EARLY FAILURES:-

The initial region that begins at time zero when a customer first begins to use the product is characterized by a high but rapidly decreasing failure rate. This region is known as "Early Failure Period" (Infant Mortality Period). This decreasing failure rate typically lasts several weeks to a few months.

CHANCE FAILURES:-

Next, the failure rate levels off and remains roughly constant for the majority of the useful life of the product. This long period of a level failure rate is known as the Intrinsic (chance or stable) failure period.

Most systems spend most of their lifetimes operating in this flat portion of the 'bathtub curve'.

WEAR-OUT FAILURES:-

Finally if units remain in use long enough, the failure rate begins to ~~decrease~~ increase as materials wear out and degradation failures occur at an ever increasing rate. This is the "wearout failure period."

5. Explain how mathematical model of speed governor system is developed for Automatic Generation Control

(Automatic Load Frequency Control).

SPEED GOVERNING SYSTEM:

Figure shows the schematic diagram of a speed governing system which controls the real power flow in the power system. The speed governing system consists of the following parts:

1. Speed Governor:

This is a fly-ball type of speed governor and constitutes the heart of the system as it senses the change in speed or frequency. With increase in speed the fly-balls move outwards and the point B on linkage mechanism moves downwards and vice-versa.

2. Linkage Mechanism:

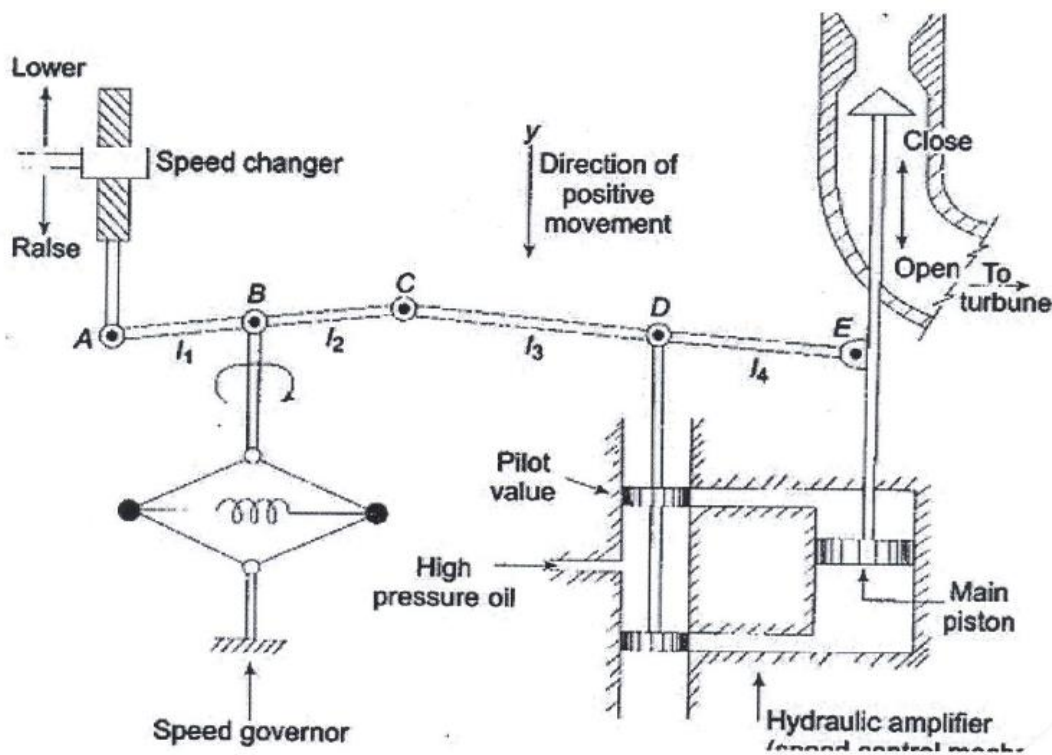
ABC and CDE are the rigid links pivoted at B and D respectively. The mechanism provides a movement to the control valve in the proportion to change in speed. Link 4 (l_4) provides a feedback from the steam valve movement.

3. Hydraulic Amplifier:

This consists of the main piston and pilot valve. Low power level pilot valve movement is converted into high power level piston valve movement which is necessary to open or close the steam valve against high pressure steam.

4. Speed Changer:

The speed changer provides a steady state power output setting for the turbine. The downward movement of the speed changer opens for the upper pilot valve so that more steam is admitted to the turbine under steady condition. The reverse happens when speed changer moves upward.



MODEL OF SPEED GOVERNING SYSTEM :

We consider the steady state condition by assuming that the linkage mechanism is stationary, pilot valve closed, steam valve opened by a definite magnitude, the turbine output balances the generator output and the turbine or generator is running at a particular speed

- Two factors contribute to the movement of C
- Increase in frequency causes B to move by Δx_B , downward
 - The lowering of speed changer by an amount Δx_A lifts the point C upwards

\therefore Movement or change at C, $\Delta x_C = K_1 \Delta f - K_2 \Delta P_C$ — (1)

SPEED CHANGER 'RAISE' CASE

A	↓	Δx_A	+ve ←
B	—	Δx_B	0
C	↑	Δx_C	-ve ←
D	↑	Δx_D	-ve ←
E	↓	Δx_E	+ve ←

SPEED CHANGER 'LOWER' CASE. (3)

A	↑	Δx_A	-ve ←
B	—	Δx_B	0
C	↓	Δx_C	+ve ←
D	↓	Δx_D	+ve ←
E	↑	Δx_E	-ve ←

TURBINE @ HIGHER SPEED ($\omega \uparrow$)

A	—	Δx_A	0
B	↓	Δx_B	+ve ←
C	↓	Δx_C	+ve ←
D	↓	Δx_D	+ve ←
E	↑	Δx_E	-ve ←

TURBINE @ LOWER SPEED ($\omega \downarrow$)

A	—	Δx_A	0
B	↑	Δx_B	-ve ←
C	↑	Δx_C	-ve ←
D	↑	Δx_D	-ve ←
E	↓	Δx_E	+ve ←

The movement of D is contributed by the movement of C and E
Therefore, $\therefore \Delta x_D = k_3 \Delta x_C + k_4 \Delta x_E$ — (2)

Assuming that oil flow into hydraulic cylinder is proportional to position Δx_D of the pilot valve, then

$$\Delta x_E = k_5 \int_0^t \Delta x_D \cdot dt \text{ — (3)}$$

where k_1, k_2, k_3, k_4 depend upon the length of linkage arms and k_5 depends upon the fluid pressure and the geometry of the cylinder.

Laplace transform of eq(1), eq(2), eq(3) \rightarrow

$$\Delta X_c(s) = K_1 \Delta F(s) - K_2 \Delta P_c(s) \text{ --- (4)}$$

$$\Delta X_D(s) = K_3 \Delta X_c(s) + K_4 \Delta X_E(s) \text{ --- (5)}$$

$$\Delta X_E(s) = \frac{-K_5}{s} \cdot \Delta X_D(s)$$

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$$s \cdot \Delta X_E(s) = -K_5 \cdot \Delta X_D(s) \text{ --- (6)}$$

(4) in (5) \rightarrow

$$\Delta X_D(s) = K_3 K_1 \Delta F(s) - K_3 K_2 \Delta P_c(s) + K_4 \Delta X_E(s) \text{ --- (7)}$$

(7) in (6) \rightarrow

$$s \cdot \Delta X_E(s) = -K_3 K_1 K_5 \Delta F(s) + K_3 K_2 K_5 \Delta P_c(s) - K_4 K_5 \Delta X_E(s) \left[\div K_4 K_5 \right]$$

$$\frac{s \cdot \Delta X_E(s)}{K_4 \cdot K_5} = \frac{-K_3 K_1 K_5}{K_4 \cdot K_5} \cdot \frac{K_2}{K_2} \Delta F(s) + \frac{K_3 K_2 K_5}{K_4 \cdot K_5} \Delta P_c(s) - \Delta X_E(s)$$

$$\Delta X_E(s) \left[1 + s \cdot \frac{1}{K_4 \cdot K_5} \right] = - \left(\frac{K_2 K_3}{K_4} \right) \left(\frac{K_1}{K_2} \right) \Delta F(s) + \left(\frac{K_2 K_3}{K_4} \right) \Delta P_c(s)$$

Define ; $K_G = \text{Governor gain constant} = \frac{K_2 K_3}{K_4}$

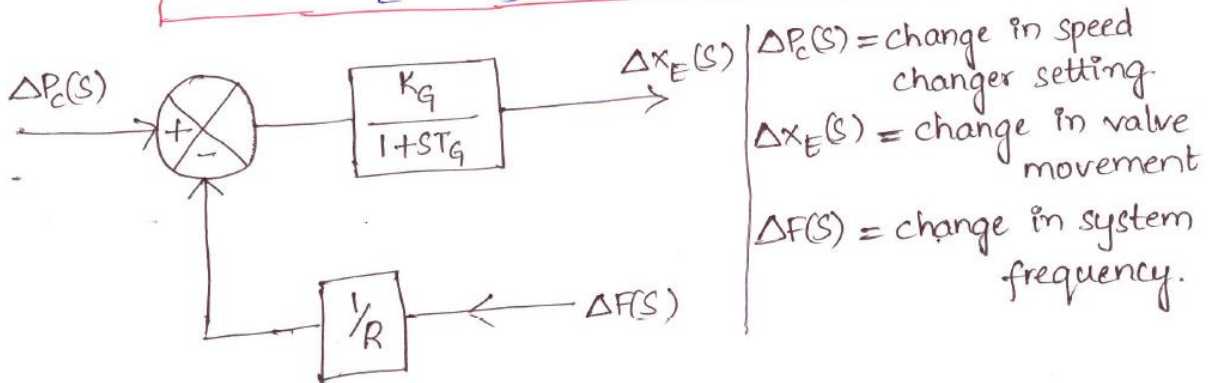
$T_G = \text{Governor time constant} = \frac{1}{K_4 K_5}$

$R = K_2 / K_1 = \text{Regulation of governor.}$

Then

$$\Delta X_E(s) [1 + sT_G] = -K_G \cdot \left(\frac{1}{R}\right) \Delta F(s) + K_G \Delta P_C(s)$$

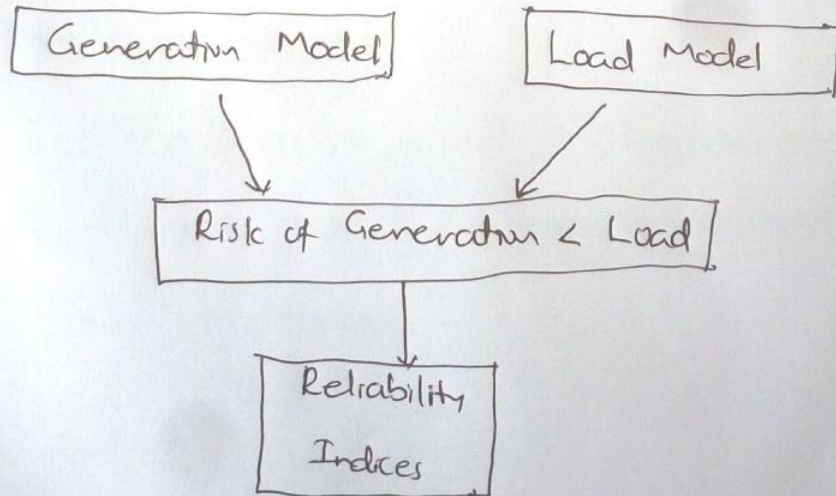
$$\Delta X_E(s) = \left[\frac{K_G}{1 + sT_G} \right] \left[\Delta P_C(s) - \frac{1}{R} \cdot \Delta F(s) \right]$$



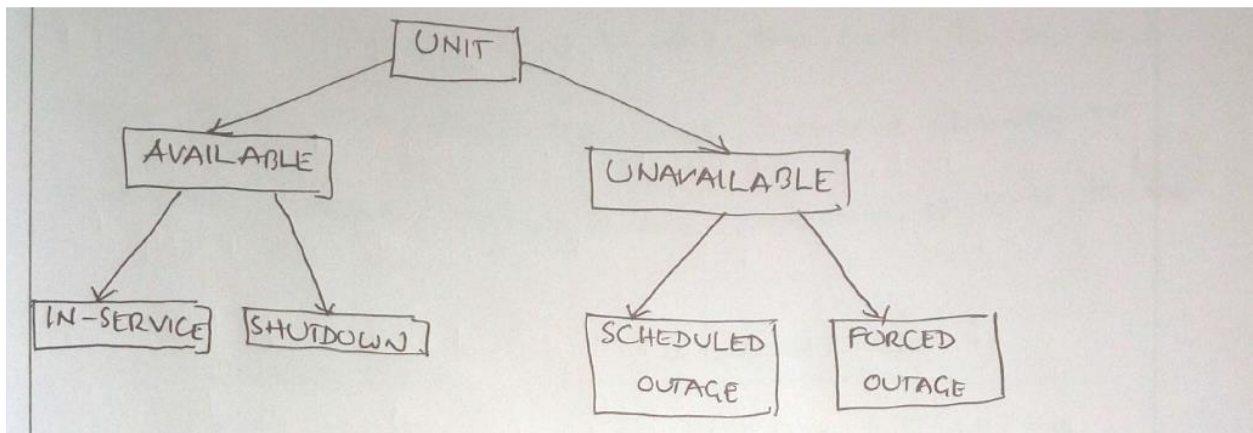
BLOCK DIAGRAM OF SPEED GOVERNOR MODEL.

6. Briefly explain the two state generator model. With usual notations derive the expression for availability and unavailability in terms of failure and repair rate.

The basic elements used to evaluate generation adequacy are shown in figure below. The system is assumed to operate successfully as long as there is sufficient generation capacity to supply the load. First, mathematical representations of generation and load are combined to model the risk of supply shortages in the system. Secondly, probabilistic estimates of shortage risk are used as indices of bulk power reliability for the considered configuration.



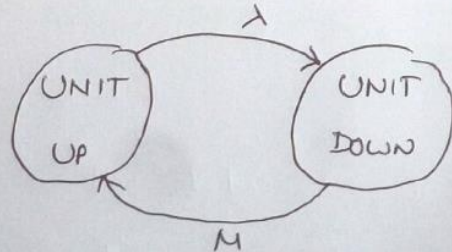
The status of a generating unit is conveniently described as residing in one of several possible states listed below.



Forced Outage — An outage that results ~~when~~ from emergency conditions, requiring that the component be taken out of service, immediately.

Scheduled Outage — An outage that results when a component is deliberately taken out of service, usually for purpose of preventive maintenance (or) repair.

The operating life of a generation unit can be represented by a simple two-state model in a "service-repair" process as shown in figure.



Where λ and μ are the unit failure and repair rate respectively. The most important quantity for generation reliability analysis is the probability of unit failure.

$$\therefore \text{UNAVAILABILITY (U)} = \frac{\Sigma (\text{DOWN TIME})}{\Sigma (\text{DOWN TIME}) + \Sigma (\text{UP TIME})}$$

$$\text{i.e. } U = \frac{\text{MTTR}}{\text{MTTR} + \text{MTTF}}$$

But $MTTR = 1/\mu$ and $MTTF = 1/\lambda$

$$U = \frac{1/\mu}{(1/\mu) + (1/\lambda)}$$

$$U = \frac{\lambda}{\lambda + \mu}$$

→ Expression for UNAVAILABILITY.

Also $AVAILABILITY (A) = \frac{\sum (UP TIME)}{\sum (DOWN TIME) + \sum (UP TIME)}$

i.e. $A = \frac{MTTF}{MTTR + MTTF}$

But $MTTR = 1/\mu$ and $MTTF = 1/\lambda$

$$A = \frac{(1/\lambda)}{(1/\lambda) + (1/\mu)}$$

$$A = \frac{\mu}{\lambda + \mu}$$

→ Expression for availability

It is clear from above discussion that;

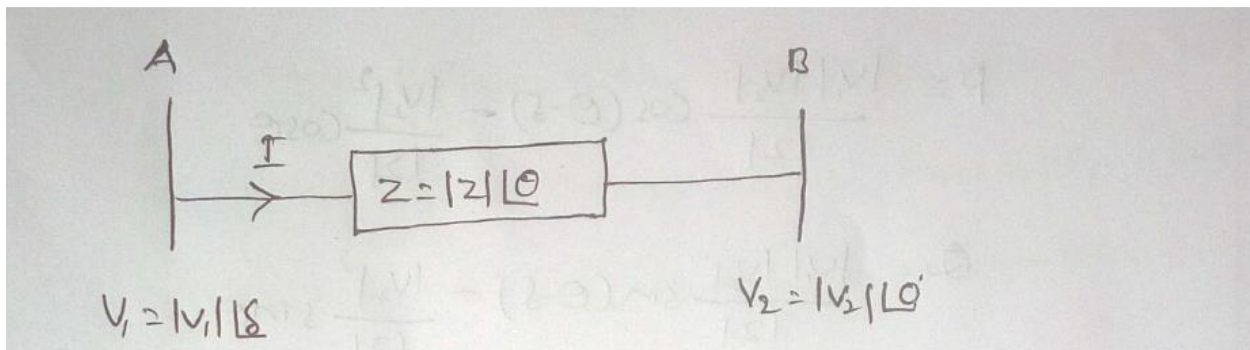
$$\boxed{A+U=1}$$

The unit ~~unavailable~~ unavailability is commonly referred as:

$$\text{Forced Outage Rate (FOR)} = \frac{\text{Forced outage hours}}{\text{In service hours} + \text{forced outage hours}}$$

i.e. $\boxed{\text{UNAVAILABILITY} = \text{FOR}}$

7. Show that the real power flow between two nodes is determined by the transmission angle δ and the reactive power flow is determined by the scalar voltage difference between two nodes.



$$I = \frac{V_1 - V_2}{Z} = \frac{(|V_1| \angle \delta) - (|V_2| \angle 0)}{|Z| \angle \theta}$$

$$I = \frac{|V_1|}{|Z|} \frac{[\delta - 0]}{\angle \theta} - \frac{|V_2|}{|Z|} \frac{[0]}{\angle \theta}$$

$$I^* = \frac{|V_1|}{|Z|} \frac{[0 - \delta]}{\angle \theta} - \frac{|V_2|}{|Z|} \frac{[0]}{\angle \theta}$$

If $I = r \angle \theta$
 $I^* = r \angle -\theta$

Complex power $S = V_2 I^* = (|V_2| \angle 0) \left[\frac{|V_1|}{12} \angle \theta - \frac{|V_2|}{12} \angle 0 \right]$

$$S = \frac{|V_1| |V_2| \angle \theta}{12} - \frac{|V_2|^2 \angle 0}{12}$$

$V \angle \theta = V \cos \theta + j V \sin \theta$

$$S = \left[\frac{|V_1| |V_2|}{12} \cos(\theta - 0) + j \frac{|V_1| |V_2|}{12} \sin(\theta - 0) \right] - \left[\frac{|V_2|^2}{12} \cos 0 + j \frac{|V_2|^2}{12} \sin 0 \right]$$

$$S = P + jQ$$

$$P = \frac{|V_1| |V_2|}{12} \cos(\theta - 0) - \frac{|V_2|^2}{12} \cos 0$$

$$Q = \frac{|V_1| |V_2|}{12} \sin(\theta - 0) - \frac{|V_2|^2}{12} \sin 0$$

\therefore For transmission line; $R \ll X_L$; $\tan^{-1}\left(\frac{X_L}{R}\right) = \theta \approx 90^\circ$

$$\therefore P = \frac{|V_1| |V_2|}{12} \sin 90$$

$$Q = \frac{|V_1| |V_2|}{12} \cos 90 - \frac{|V_2|^2}{12}$$

↳ Thus Reactive power (Q) flow between two nodes is determined by scalar voltage difference between two nodes.

↳ Real power flow between two nodes is determined by transmission angle (δ).

8. Obtain the expressions for steady-state reliability and general reliability expression.

Let $N_0 =$ No. of identical items to be tested.

Also test initiates at time $t=0$

After an interval of time when $t > 0$, let us assume that:

$N_s(t) =$ No. of items survival at time t and

$N_f(t) =$ No. of items which have failed in time interval $(0-t)$

At time t , the reliability $R(t)$ is given by,

$$R(t) = \frac{N_s(t)}{N_0} = \frac{N_0 - N_f(t)}{N_0} = 1 - \frac{N_f(t)}{N_0}$$

Thus $\frac{dR(t)}{dt} = -\frac{1}{N_0} \frac{dN_f(t)}{dt}$, As $dt \rightarrow 0$

①

$\frac{1}{N_0} \cdot \frac{dN_f(t)}{dt}$ is the instantaneous failure density function

which is expressed by $f(t)$, and

$$f(t) = \frac{-dR(t)}{dt} \quad \text{--- (2)}$$

The hazard (failure) rate, $\lambda(t)$ is defined as the percentage of those remaining equipment that will fail in the next interval of time is given by

$$\lambda(t) = \frac{d}{dt} N_f(t) / N_s(t)$$

$$\lambda(t) = \frac{N_0}{N_0} \cdot \frac{1}{N_s(t)} \cdot \frac{dN_f(t)}{dt} = \frac{f(t)}{R(t)}$$

But from (2); $f(t) = \frac{-dR(t)}{dt}$,

$$\lambda(t) = \frac{-dR(t)}{dt \cdot R(t)} \quad \text{--- (3)}$$

From (2) $-dR(t) = f(t) \cdot dt$.

$$\int_0^{R(t)} -dR(t) = \int_0^t f(t) \cdot dt$$

$$1 - R(t) = \int_0^t f(t) \cdot dt = \text{Failure probability function.}$$

From (3); $-\frac{dR(t)}{R(t)} = \lambda(t) \cdot dt$

$$\int_0^{R(t)} \frac{dR(t)}{R(t)} = -\int_0^t \lambda(t) \cdot dt$$

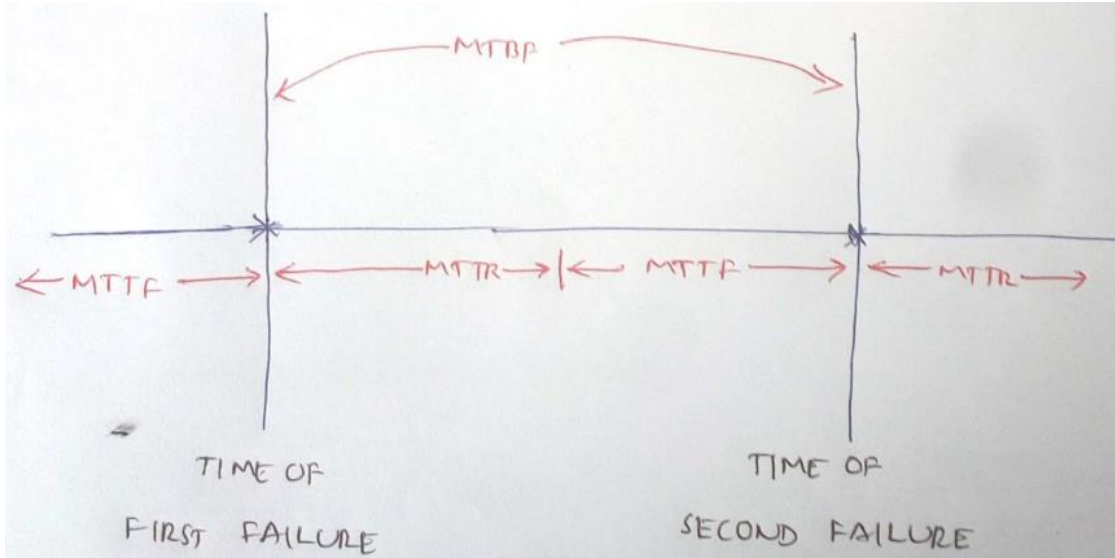
$$\log R(t) = -\int_0^t \lambda(t) \cdot dt$$

$$R(t) = e^{-\int_0^t \lambda(t) \cdot dt}$$

→ General expression for reliability.

From the above expression it is clear that reliability and hazard (or) failure rates are the function of time.

STEADY STATE EXPRESSION FOR RELIABILITY:—



MTTF — Mean Time To Failure

MTTR — Mean Time To Repair

MTBF — Mean Time Between Failures

$$\lambda = \text{Failure (Hazard Rate)} = \frac{1}{\text{MTTF}}$$

$$\mu = \text{Repair rate} = \frac{1}{\text{MTTR}}$$

∴ Reliability (Availability);

$$R = A = \frac{\sum(\text{Up Time})}{\sum(\text{Up Time}) + \sum(\text{Down Time})} = \frac{MTTF}{MTTF + MTTR}$$

$$R = \frac{(1/\lambda)}{(1/\lambda) + (1/\mu)} = \frac{\mu}{\lambda + \mu}$$

$$R = \frac{\mu}{\lambda + \mu}$$

→ STEADY STATE EXPRESSION FOR RELIABILITY.

and Unreliability (Unavailability);

$$Q = U = \frac{\sum(\text{Down Time})}{\sum(\text{Down Time}) + \sum(\text{Up Time})} = \frac{MTTR}{MTTR + MTTF}$$

$$Q = \frac{(1/\mu)}{(1/\lambda) + (1/\mu)} = \frac{\lambda}{\lambda + \mu}$$

$$Q = \frac{\lambda}{\lambda + \mu} \rightarrow \text{STEADY STATE EXPRESSION FOR UNRELIABILITY.}$$