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Internal Assessment Test II – May 2019

Sub:	Discrete Mathematics and Structures					Sub Code:	18MCA23	
Date:	14^{th} May 2019	Duration:	90 mins	Max Marks:	50	Sem / Sec:	II MCA	OBE

Answer all questions	MARKS	CO	RBT
1.Define exponential distribution. In a certain town, the duration of a shower is exponentially distributed with mean equal to 5 minutes. What is the probability that a shower will last for i) less than 10 minutes ii) 10 minutes or more?.	[12]	CO3	L3
(OR)			
2.Write the formula for mean and variance of exponential distribution. The length of a telephone conversation has an exponential distribution with a mean of 3 minutes. Find the probability that a call i) ends less than 3 minutes ii) takes between 3 and 5 minutes.	[12]	CO3	L3
3.a) Verify Identity, Inverse and domination laws using truth table. b) Prove that for any propositions p,q,r the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology.	[10]	CO1	L3
(OR)			
4.a)Prove the law for negation of a conditional. b)Rewrite the following conditional in the form of disjunction and hence write its negation "If I am awake, then I will work on the computer or read a novel"	[10]	CO1	L3

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5. a)Define tautology with an example. b)State and prove distributive laws using truth table. [15] CO3 L3
c) Prove the logical equivalence $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$ using the laws of logic.

(OR)

6.. a)Define biconditional with an example. b) State and prove DeMorgan's laws using truth table. c) Prove the logical equivalence $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$ using the laws of logic. [15] CO3 L3

7. Define normal distribution. The weekly wages of workers in a company are normally distributed with mean of Rs. 700 and standard deviation of Rs.50. Find the probability that the weekly wage of a randomly chosen worker is i) between Rs.650 and 750 ii) more than Rs.750 given A(1)=0.3413. [13] CO1 L3

(OR)

8.Write the formula for mean and variance of normal distribution The mean weight of 500 students of a certain school is 50 Kgs and the standard deviation is Rs.6 Kgs. Assuming that the weights are normally distributed, find the expected number of students, weighing i)between 40 and 50 Kgs ii) more than 60 Kgs, given that A(1.6667)=0.45 [13] CO1 L3

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(i)

1.a) Let α be a real constant > 0. Then the continuous probability function for which

$$\in(\alpha, x) = \begin{cases} \alpha e^{-\alpha x} & 0 \le x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

is the P.D.F called as the exponential distribution. x is the exponential variate. It is found that

$\in(\alpha, x) \geq 0$ and $\int_{-\infty}^{\infty} \in(\alpha, x)\, dx = 1$ 2

b) $\text{Mean} = \frac{1}{\alpha} = 5 \Rightarrow \alpha = \frac{1}{5}$

$$\in(\alpha, x) = \in\left(\tfrac{1}{5}, x\right) = \frac{1}{5} e^{-\frac{1}{5}x}, \; x > 0$$ 2

$$P(x < 10) = \int_0^{10} \in\left(\tfrac{1}{5}, x\right) dx$$

$$= \frac{1}{5} \int_0^{10} e^{-\frac{1}{5}x}\, dx$$

$$= \frac{1}{5} \left(\frac{e^{-x/5}}{-1/5} \right)_0^{10}$$

$$= -\left(e^{-2} - e^{0}\right) = 1 - e^{-2}$$

$$= 0.8647$$ 4

$$P(x \geq 10) = 1 - P(x < 10)$$

$$= 1 - 0.8647 = 0.1353 \quad 4 \qquad 12$$

2\. a) $\mu = \frac{1}{\alpha} \quad \sigma^2 = \frac{1}{\alpha^2}$ 2

$\frac{1}{\alpha} = 3 \quad \alpha = \frac{1}{3}$

b)

$$E\left(\frac{1}{3}, x\right) = \frac{1}{3} e^{-x/3}, \quad x > 0 \qquad 2$$

$$P(x < 3) = \int_0^3 \frac{1}{3} e^{-x/3} dx$$

$$= \frac{1}{3}\left(\frac{e^{-x/3}}{-1/3}\right)_0^3$$

$$= -\left(e^{-1} - e^{0}\right) = 1 - e^{-1}$$

$$= 0.6321$$

$$P(3 < x < 5) = \frac{1}{3}\int_3^5 e^{-x/3} dx \qquad 4$$

$$= \frac{1}{3}\left(\frac{e^{-x/3}}{-1/3}\right)_3^5$$

$$= -\left(e^{-5/3} - e^{-1}\right) = 0.149$$

4

12

3a) Identity Laws

For any proposition p,

a) $p \vee F_0 \equiv p$ b) $p \wedge T_0 \equiv p$

p	F_0	T_0	$p \vee F_0$	$p \wedge T_0$
0	0	1	0	0
1	0	1	1	1

2

Inverse Laws For any proposition p,

a) $p \vee \sim p \equiv T_0$ b) $p \wedge \sim p \equiv F_0$

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
0	1	1	0
1	0	1	0

2

Domination Laws

For any proposition p

a) $p \vee T_0 \equiv T_0$ b) $p \wedge F_0 \equiv F_0$

p	T_0	F_0	$p \vee T_0$	$p \wedge F_0$
0	1	0	1	0
1	1	0	1	0

2

6

b)

p	q	r	$p \to q$	$q \to r$	$\overset{B}{p \to r}$	$\overset{A}{p \to q \wedge q \to r}$	$A \to B$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	1	0	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	1	0	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

4M

$(p \to q) \wedge (q \to r) \equiv p \to r$
is a tautology

10

4 a) $p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv \sim(\sim p \vee q)$ De Morgan

$\equiv \sim(\sim p) \wedge \sim q$

$\equiv p \wedge \sim q$

$\sim(p \to q) \equiv p \wedge \sim q$ (Negation of a Conditional) 3

b) p: I am awake

q: work on the computer

r: read a novel

$p \to (q \vee r)$

$p \to (q \vee r) \equiv \sim p \vee q \vee r$

I am not awake or I work on the Computer or read a novel 4

$\sim(p \to (q \vee r)) \equiv p \wedge \sim(q \vee r)$ De Morgan

$\equiv p \wedge \sim q \wedge \sim r$

I am awake and I am not working on the Computer and I am not reading a novel 3

10

5 a) A compound proposition which is true for all possible truthvalues of its components is called a tautology.

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ 2

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

for any 3 propositions p, q, r

p	q	r	$q \wedge r$	$q \vee r$	$p \wedge q$	$p \vee q$	$p \wedge r$	$p \vee r$
0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	1
0	1	0	0	1	0	1	0	0
0	1	1	1	1	0	1	0	p
1	0	0	0	0	0	1	0	1
1	0	1	0	1	0	1	1	1
1	1	0	0	1	1	1	0	1
1	1	1	1	1	1	1	1	1

$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
0	0	0	0
0	0	0	0
0	0	0	0
1	1	0	0
1	1	0	0
1	1	1	1
1	1	1	1
1	1	1	1

7

$$[(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)] \equiv p \wedge q$$

$(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r) \equiv \sim(\sim p \vee \sim q) \vee (p \wedge q \wedge r)$

De Morgan

$\equiv \sim(\sim p) \wedge \sim(\sim q) \vee (p \wedge q \wedge r)$

Double negation

$\equiv (p \wedge q) \vee (p \wedge q \wedge r)$

Absorption

$\equiv p \wedge q$

6 15

6. a) Let p and q be two propositions. The conjunction of the conditionals $p \rightarrow q$ and $q \rightarrow p$ is called as the biconditional of p and q, denoted by $p \leftrightarrow q$.

2

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

eg p : 2 is a prime no.

q : 3 is a prime no.

$p \leftrightarrow q$ is true

b) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

for any 2 propositions p, q

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$p \vee q$	$\sim(p \vee q)$
0	0	1	1	0	1	0	1
0	1	1	0	0	1	1	0
1	0	0	1	0	1	1	0
1	1	0	0	1	0	1	0

$\sim(p \vee q)$	$\sim p \vee \sim q$	$\sim p \wedge \sim q$
1	1	1
0	1	0
0	1	0
0	0	0

c) $(p \longrightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] \equiv \sim(q \vee p)$

Consider $(p \longrightarrow q) \wedge [\sim q \wedge (r \vee \sim q)]$

$\equiv p \longrightarrow q \wedge [\sim q \wedge (\sim q \vee r)]$ Commutative

$\equiv p \longrightarrow q \wedge \sim q$ absorption

$\equiv \sim[(p \longrightarrow q) \longrightarrow q]$

$\equiv \sim[\sim(p \longrightarrow q) \vee q]$

$\equiv \sim [(p \wedge \sim q) \vee q]$

$\equiv \sim [q \vee (p \wedge \sim q)]$ Commutative

$\equiv \sim [(q \vee p) \wedge (q \vee \sim q)]$ disterbution

$\equiv \sim [q \vee p \wedge T_0]$ tautology

$\equiv \sim (q \vee p)$ identity law

15 6

7. Let μ and σ be two real constants such that $-\infty < \mu < \infty$ and $\sigma > 0$. Then the continuous probability distribution for which $N(\mu, \sigma, x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

the P.D.F is called a Normal Distribution

2

x is the normal variate

x is the weekly wage, $\mu = 700$ $\sigma = 50$ If x is the weekly wage,

SNV $Z = \frac{x-\mu}{\sigma} = \frac{x-700}{50}$

at $x = 650$ $z = -1$

at $x = 750$ $z = 1$

$P(650 < x < 750)$

$= P(-1 < Z < +1) = 2P(0 < Z < 1)$

$= 2A(1) = 2(0.3413)$

$= 0.6826$

5

$P(x > 750) = P(Z > 1) = P(Z \geq 0) - P(0 < Z < 1)$

$= 0.5 - 0.3413 = 0.1587$

5

12

8. Mean of Normal Distribution and variance equals the mean and variance of the given distribution. 2

$$Z = \frac{x - \mu}{\sigma} = \frac{x - 50}{6}$$

at $x = 50$ $Z = 0$

at $x = 40$ $Z = -1.6667$

$P(40 < x < 50) = \cancel{P(0 < Z <} \; P(-1.6667 < Z < 0)$

$= P(0 < Z < 1.6667)$

$= A(1.6667) = 0.4525$

For 500 students, estimated no. who weigh between 40 & 50 is 0.4525×500

$= 226$ 5

For $x = 60$ $\quad z = 10/6 = 1.6667$

$$P(x > 60) = P(z > 1.6667)$$
$$= P(z \geq 0) - P(0 \leq z \leq 1.6667)$$
$$= 0.5 - A(1.6667)$$
$$= 0.5 - 0.4525 = 0.0475$$

Out of 500 students, estimated no.

$$0.0475 \times 500 = 24$$

5

12