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Internal Assessment Test 1 – November 2017

Sub:	Discrete Mathematical Structures							Code:	16MCA15
Date:	09/11/17	Duration:	90 mins	Max Marks:	50	Sem:	1	Branch:	MCA

Q1. This question is compulsory.

Discuss different types of logical connectives with example and truth table. [10]

Answer any five of Q2 to Q8. [5 x 8]

Q2. Define Tautology, Contradiction and prove that for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to [p \to r]$ is a tautology. [8]

Q3. Prove the result using laws of logic ,

$\neg[\{(p \vee q) \wedge r\} \to \neg q] \Leftrightarrow (q \wedge r)$. [8]

Q4. (a) Indicate how many rows are needed in the truth table for the compound proposition $(p \vee \neg q) \leftrightarrow \{(\neg r \wedge s) \to t\}$. Find the truth table of this proposition if p and r are true and q, s, t are false. [5]

(b) Identify the bound variables and free variables in each of the following statements: [3]

i) $\forall y \exists z[\cos(x+y) = \sin(z-x)]$

ii) $\exists x \exists y[x^2 - y^2 = z]$

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Q5. Negate and simplify each of the following

i) $p \to (\neg q \wedge r)$ ii) $(p \vee q) \wedge \neg(\neg p \wedge q)$ iii) $q \to \neg[(p \vee q) \wedge r]$ [2+3+3]

Q6. Test the validity of the argument. If I study then I will not fail in the examination. If I do not watch TV in the evenings, I will study. I failed in the examination. Therefore I must have watched TV in the evening. [8]

Q7. For the universe of all integers , let p(x), q(x), r(x), s(x), t(x) be the following open statements: p(x) : x > 0, q(x): x is even, r(x): x is a perfect square, s(x): x is divided by 4, t(x) = x is divided by 5. Write the following statements in symbolic form and determine whether they are true or false. [2+3+3]

i) If x is even then x is not divisible by 5

ii) If x is a perfect square then it is positive

iii) If x is even and perfect square then x is divisible by 4.

Q8. Write down the negation of the proposition : " if x is not a real number, then it is not a rational number and not an irrational number". [8]

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DMS- Solution - IAT-1 (MCA-I sem)

Nov. 2017

Ans 1

1) Conjunction - Let p and q be two propositions. The conjunction of p & q, denoted by $p \wedge q$ ("p and q")

e.g. Today is Friday and it is raining

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

— ($1\frac{1}{2}$)

2) Disjunction - ("p or q") and denoted by $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The above example is true on any day that is either Friday or rainy day (including rainy Friday) — ($1\frac{1}{2}$)

3) Negation -

Let p be a proposition.
"It is not the case that p"
and denoted by $\neg p$ or $\sim p$
or $\bar{p}$. e.g. Delhi is capital of Karnataka

$\neg p$: Delhi is not capital of Karnataka.

p	$\neg p$
T	F
F	T

— (1)

4) Exclusive disjunction

It is denoted by $\oplus$ or $\underline{\vee}$
i.e. $p \oplus q$ or $p \underline{\vee} q$
and it is true when exactly one of p & q is true and false otherwise

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

— (2)

e.g. Either Anil will be in class room or he'll be out of the class.

5) Conditional – "If p then q" & is denoted by $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

— (2)

e.g. If 2 is an even no. then 2+9 is an even no.

6) Biconditional – "p iff q" and is denoted by $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

— (2)

x is a real no. iff $x^2 > 0$.

Ans-2 Tautology :- If a compound proposition is true for all possible truth values of its components is called Tautology. — (2)

And if it is false in all the cases then it is called contradiction. — (2)

p	q	r	① $p \to q$	② $q \to r$	③ $p \to r$	④ ① ∧ ②	④ → ③
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

— (4)

∴ It is a tautology.

③ $\neg[\{(p \vee q) \wedge r\} \to \neg q]$

$\equiv \neg[\neg\{(p \vee q) \wedge r\} \vee \neg q]$ $(\because p \to q \equiv \neg p \vee q)$ — (1)

$\equiv \{(p \vee q) \wedge r\} \wedge q$ (DeMorgan's and double negation) — (2)

$\equiv (p \vee q) \wedge (r \wedge q)$ (associative) — (1)

$\equiv [(p \vee q) \wedge q] \wedge r$ (commu. & asso.) — (2)

$\equiv q \wedge r$ (absorption) — (2)

4. (a) $\because$ The compound proposition is containing 5 atomic propositions.

$\therefore$ The no. of rows are needed in Truth Table

$= 2^5 = 32$ — (2)

Given p & r are T & q, s, t are F

p	q	r	s	t	$\neg q$	$p \vee \neg q$ ①	$\neg r$	$\neg r \wedge s$	$(\neg r \wedge s) \to t$ ②	① $\leftrightarrow$ ②
T	F	T	F	F	T	T	F	F	T	T

$\therefore (p \vee \neg q) \longleftrightarrow \{(\neg r \wedge s) \to t\}$ is T. — (3)

(b) (i) $\forall y \exists z [\cos(x+y) = \sin(z-x)]$

Here x is free variable & y and z are bound variables — $(1\frac{1}{2})$

(ii) $\exists x \exists y [x^2 - y^2 = z]$

Here z is free variable & x and y are bound variables

— $(1\frac{1}{2})$

5. Negate and Simplify

i) $p \to (\neg q \wedge r)$

$\equiv \neg p \vee (\neg q \vee r) \quad (\because p \to q \equiv \neg p \vee q)$ —— (1)

Negation : $\neg[\neg p \vee (\neg q \vee r)]$

$\equiv p \wedge \neg(\neg q \vee r)$ (De Morgan's & double negation)

$\equiv p \wedge (q \wedge \neg r)$ (") —— (1)

(ii) $(p \vee q) \wedge \neg(\neg p \wedge q)$

Neg: $\neg[(p \vee q) \wedge \neg(\neg p \wedge q)]$

$\equiv \neg(p \vee q) \vee (\neg p \wedge q)$ (De Morgan's & double neg.)

$\equiv (\neg p \wedge \neg q) \vee (\neg p \wedge q)$ (") —— (1)

$\equiv \neg p \wedge (\neg q \vee q)$ (Distributive)

$\equiv \neg p \wedge T_0$ (Inverse)

$\equiv \neg p$ (Identity) —— (2)

(iii) $q \to \neg[(p \vee q) \wedge r]$

$\equiv \neg q \vee \neg[(p \vee q) \wedge r] \quad (\because p \to q \equiv \neg p \vee q)$ —— (1)

Negation : $\neg[\neg q \vee \neg\{(p \vee q) \wedge r\}]$

$\equiv q \wedge \{(p \vee q) \wedge r\}$ (De Morgan's & double negation)

$= (q \wedge (p \vee q)) \wedge r$ (assoc.)

$\equiv q \wedge r$ (Absorption) —— (2)

6. p: I study

q: I'll fail in exam

r: I watch TV in the evening —— (2)

Given argument in symbolic form:

$$\begin{array}{l} p \rightarrow \neg q \\ \neg r \rightarrow p \\ \quad q \\ \hline \therefore r \end{array}$$ —— (2)

Now only take premises

$$\begin{array}{l} p \rightarrow \neg q \\ \neg r \rightarrow p \\ \quad q \\ \hline \end{array}$$

$\Rightarrow$ $\neg r \rightarrow \neg q$ (Syllogism in I & II) —— (1)

$$\begin{array}{l} \quad q \\ \hline \neg(\neg r) \end{array}$$ (Modus Tollens) —— (1)

$$\begin{array}{l} \hline \therefore r \end{array}$$ (Double negation) —— (1)

$\therefore$ Argument is valid. —— (1)

7. Given $p(x): x > 0$; $q(x)$: x is even ; $r(x)$: x is a perfect square ; $s(x)$: x is divided by 4; $t(x)$: x is divided by 5.

(i) If x is even then x is not divisible by 5.

in symbolic form:

$q(x) \rightarrow \neg t(x)$ (It is F for $x = 10, 20, 30 \ldots$)

—— (3)

(ii) If x is a perfect square then it is positive.

In symbolic form: $r(x) \rightarrow p(x)$

(F), ~~for x^2=~~ —— (2)

(iii) If x is even and perfect square then x is divisible by 4.

$q(x) \wedge r(x) \rightarrow s(x)$ (T) —— (3)

8 "If x is not a real number, then it is not a rational number and not an irrational no."

p: x is a real no.

q: x is a rational no. —— (1)

"In symbolic form"

$\neg p \rightarrow (\neg q \wedge q)$ —— (1)

$\equiv \neg(\neg p) \vee (\neg q \wedge q)$ $(\because p \rightarrow q \equiv \neg p \vee q)$ —— (1)

Negation $\neg[\neg(\neg p) \vee (\neg q \wedge q)]$ —— (1)

$\equiv \neg p \wedge \neg(\neg q \wedge q)$ (By DeMorgan's & double negation)

$\equiv \neg p \wedge (q \vee \neg q)$ (") —— (2)

" x is not a real no. and it is a rational no. or not a rational no. —— (2)