2002 SCHEME

USN | | | | | | | | | | |

CS34

Third Semester B.E. Degree Examination, December 2010
Discrete Mathematical Structures

Time: 3 hrs.

Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Let A be a set such that $|A| = n$. Show that $|P(A)| = 2^n$, where P(A) denotes the power set of A. **(05 Marks)**

b. Let A and B be two arbitrary sets. Prove that $A \cup B = B$ if and only if $A \subseteq B$. **(04 Marks)**

c. If the letters in the acronym WYSIWYG are arranged in a random manner, what is the probability that i) arrangement starts and ends with the same letter? ii) arrangement has no pair of consecutive identical letters? **(07 Marks)**

d. Give a recursive definition for
i) the sequence of Fibonacci numbers. ii) the sequence is integers $\{C_n\}$ where $C_n = 3n + 7$. **(04 Marks)**

2 a. Write the converse, inverse and contrapositive of the implication:
'If today is Labour Day, then tomorrow is Tuesday.' **(03 Marks)**

b. Explain the NAND and NOR logical connectives using truth tables. **(04 Marks)**

c. Is $(p \vee q) \rightarrow [q \rightarrow (p \wedge q)]$ a tautology? **(05 Marks)**

d. If Rochelle gets the supervisor's position and works hard, then she'll get a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore, either Rochelle didn't get the supervisor's position or she did not work hard. Write these arguments in the symbolic form and determine the validity of the argument. If the argument is valid, identify the rule of inference that establishes its validity. **(08 Marks)**

3 a. Consider the following program segment where A is a two dimensional array:

```
for m := 1 to 10 do
    for n := 1 to 20 do
        A[m, n] = m + 3 * n
```

Write the following statements in symbolic form:
i) All entries of A are positive.
ii) All entries of A are positive and less than 70.
iii) The entries in each row of A are sorted into the ascending order.
iv) The entries in each column of A are sorted into the ascending order.
v) Sum of the entries of A exceeds 70. **(06 Marks)**

b. Let p(x), q(x) and r(x) denote the following open statements:
$p(x) := x^2 - 8x + 15 = 0$
$q(x)$: x is odd
$r(x)$: $x > 0$

For the universe of integers, determine the truth or falsity of each of the following statements. If a statement is false, give a counter example.
i) $\forall x\, [p(x) \rightarrow q(x)]$ ii) $\forall x\, [q(x) \rightarrow p(x)]$ iii) $\exists x\, [r(x) \rightarrow q(x)]$
iv) $\forall x\, [\neg q(x) \rightarrow \neg p(x)]$ v) $\exists x\, [p(x) \rightarrow (q(x) \wedge r(x))]$ vi) $\forall x\, [(p(x) \vee (q(x)) \rightarrow r(x))]$ **(06 Marks)**

c. i) If n^2 is odd then prove that n is also odd, where n is an integer. **(04 Marks)**
ii) Let n be a positive integer. Show that for $n > 4$, $n^2 < 2^n$. **(04 Marks)**

4 a. Let A, B and C be subsets of an universal set U. Prove that
$(A \cup B) \times C = (A \times C) \cup (B \times C)$ **(06 Marks)**

b. In the set Z of integers, define a relation R by aRb if and only if a – b is an odd integer. Discuss the various properties of R. **(06 Marks)**

c. i) For A = {1, 2, 3, 4}, let R = {(1, 1), (1, 2), (2, 3), (3, 3), (3, 4), (4,1) }. Draw the directed graph of R. Also, obtain the relation matrix. **(04 Marks)**

ii) Let A = {1, 2, 3, 4}.
Let R = { (1, 1), (1, 2), (2, 3), (2, 4), (3, 4), (4,1), (4, 2) }
S = { (3, 1), (4, 4), (2, 3), (2, 4), (1, 1), (1,4) }
Compute RoS and SoR. **(04 Marks)**

5 a. Let X = { 1, 2, 3, 4, 5, 6, 7 }. Let R = {(x, y) | x – y is divisible by 3 }
Show that R is an equivalence relation on X. Also, determine the partition induced by R. **(07 Marks)**

b. Let X = { 1, 3, 5, 9, 10, 15, 30, 45, 90 } and the relation ≤ be such that x ≤ y if x divides y. Draw the Hasse diagram of (X, ≤). Also, determine whether the poset is linearly ordered. Explain your answer. **(06 Marks)**

c. i) Explain the topological sorting algorithm. **(04 Marks)**

ii) What is a lattice? Give one example. **(03 Marks)**

6 a. Let A = { 1, 2, 3, 4 } and B = {1, 2, 3, 4, 5} be two sets. Determine which of the following relations R are functions from A to B. If R is a function, give its range.

i) R = {(1, 2), (2, 4), (4, 1), (3, 5)}
ii) R = {(1, 5), (2, 3), (3, 5), (4, 5), (2, 4)}
iii) R = {(1, 4), (2, 4), (3, 4), (4, 4)}
iv) R = {(1, 3), (2, 4), (4, 5)}
v) R = {(1, 1), (2, 2), (3, 3), (4, 4)} **(05 Marks)**

b. Let $f : R \to R$ be a function defined by $f(x) = x^2 + x$. Determine whether f is invertible. If so, give the inverse function (here, R stands for the set of real numbers). **(05 Marks)**

c. Is the permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$ even or odd? **(05 Marks)**

d. Write a short note on hashing function and its applications. **(05 Marks)**

7 a. Define the binary operation * on the set of integers Z by x * y = x + y + 1. Show that (Z, *) is an abelian group. **(07 Marks)**

b. Define a cyclic group. Show that every subgroup of a cyclic group is cyclic. **(07 Marks)**

c. Let Z and E denote respectively, the set of integers and the set of even integers. Show that $f : Z \to (E, +)$ by f(a) = 2a is an isomorphism. (Here (Z, +) and (E, +) are groups). **(06 Marks)**

8 a. Let H(x, y) denote the hamming distance between the codewords x and y. Show that

i) H(x, y) ≥ 0 ii) H(x, y) = 0 ; if and only if x = y.
iii) H(x, y) ≤ H(x, z) + H(z, y) iv) H(x, y) = H(y, x) **(06 Marks)**

b. Define a group code. Given the parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

. Determine the group code $e_H : B^2 \to B^5$ **(08 Marks)**

c. Define a ring structure. Give one example. **(06 Marks)**

* * * * *