2002 SCHEME

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CS34

Third Semester B.E. Degree Examination, December 2011

Discrete Mathematical Structures

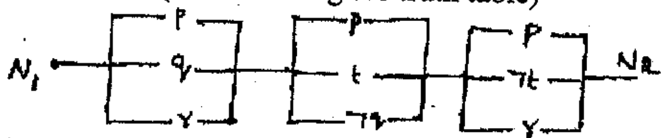
Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. If statement q has the truth value 1, determine all truth value assignments for the primitive statements p,r, and s for which the truth value of the statement.
$(q \to [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \to (\neg r \wedge q)]$ is 1. **(04 Marks)**

b. Simplify the following network. (without using the truth table) **(08 Marks)**



c. Prove that the following argument is valid.

$$\begin{array}{l} (\neg p \vee \neg q) \to (r \wedge s) \\ r \to t \\ \underline{\neg t} \\ \therefore p \end{array}$$

(08 Marks)

2 a. Define an open statement, with an example **(04 Marks)**

b. Find the negation of the statement given with three open statements p (x, y), q (x, y) and r (x, y), $\forall x \, \exists y \, [(p(x, y) \wedge q(x, y)) \to r(x, y)]$. **(06 Marks)**

c. Show that the following argument is valid, by giving proper reasons.

$$\begin{array}{l} \forall x \, [p(x) \to (q(x) \wedge r(x))] \\ \underline{\forall x \, [p(x) \wedge s(x)]} \\ \therefore \forall x \, [r(x) \wedge s(x)] \end{array}$$

(10 Marks)

3 a. If A, B, C are finite sets, prove that, $|A - B - C| = |A| - |A \cap B| - |A \cap C| + A \cap B \cap C$. **(06 Marks)**

b. Let $\cup = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{1, 2, 4, 6, 8\}$
$B = \{2,4,5,9\}$ and $C = \{x \mid x \text{ is +ve integer and } x^2 \leq 6\}$.
Compute the following: i) $A - B$ ii) $B - A$ iii) $A \Delta B$ iv) $B \Delta C$. **(06 Marks)**

c. A survey of 500 television watchers produced the following information: 285 watch football, 195 watch hockey, 115 watch basket ball, 45 watch football and basket ball, 70 watch foot ball and hockey, 50 watch hockey and basket ball and 50 do not watch any of the three kinds of games. i) How many people in the survey watch all three kinds of games? ii) How many people watch exactly one of the sports? **(08 Marks)**

4 a. A box contains six red balls and four green balls. Four balls are selected at random from the box. What is the probability that two of the selected balls will be red and two will be green? **(06 Marks)**

b. Let R be a relation on a set A. Prove the following: i) R is reflexive if and only if $a \in R(a)$ for all $a \in A$. ii) R is symmetric if and only if the following condition holds: $a \in R(b)$ if and only if $b \in R(a)$ iii) R is transitive if and only if the following condition holds: If $b \in R(a)$ and $c \in R(b)$, then $c \in R(a)$. **(08 Marks)**

c. Let $A = \{a, b, c, d\}$. Consider the partition $P = \{\{a, b, c\}, \{d\}\}$ of A. Find the equivalence relation inducing this partition. **(06 Marks)**

5 a. Let $A = \{a, b, c\}$, and R and S be relations on A whose matrices are given below:
$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$; $M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ Find the composite relations. i) SoR ii) RoS iii) RoR iv) SoS and their matrices. **(08 Marks)**

b. Draw the Hasse diagram, representing the positive divisors of 36. **(06 Marks)**

c. Define the following with suitable example: i) Lattices ii) Posets **(06 Marks)**

6 a. Let $A = \{0, \pm1, \pm2, 3\}$. Consider the function, $f: A \rightarrow R$, where 'R' is the set of all real numbers, defined by $f(x) = x^3 - 2x^2 + 3x + 1$ for $x \in A$. Find the range of f. **(04 Marks)**

b. Let $A = \{x \mid x \text{ is real and } x \geq -1\}$ and $B = \{x \mid x \text{ is real and } x \geq 0\}$. Consider the function $f: A \rightarrow B$ defined by $f(a) = \sqrt{a+1}$ for all $a \in A$.
Show that f is invertible and determine f^{-1}. **(08 Marks)**

c. Define the following functions with suitable example:
i) Hashing function ii) Permutation function. **(08 Marks)**

7 a. Define an Abelian group. Prove that a group G is abalian, if and only if $(ab)^2 = a^2 b^2$ for all $a,b \in G$. **(06 Marks)**

b. If $f: G \rightarrow H$, $g: H \rightarrow K$ are homomorphisms, prove that the composite function $g \circ f: G \rightarrow K$, where $(g \circ f) = g(f(x))$ is a homomorphism. **(06 Marks)**

c. Let $G = S_4$ i) For $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, find the subgroup $H = \langle \alpha \rangle$. Define the left caret and determine the left carets of H in G. **(08 Marks)**

8 a. Given a ring $(R, +, \cdot)$, for all $a, b \in R$. Prove that i) $-(-a) = a$ ii) $a(-b) = (-a)b = -(ab)$. **(06 Marks)**

b. Let S and T be subrings of ring R. Prove that $S \cap T$ is a subring of R. **(06 Marks)**

c. Explain the following:
i) Hamming matrices
ii) Parity check and generator matrices. **(08 Marks)**

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