2002 SCHEME

USN

CS42

Fourth Semester B.E. Degree Examination, December 2011

Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define the following terms: i) Spanning subgraph ii) K-regular graph iii) Complement of G where G is a loop-free undirected graph on n vertices. iv) Euler circuit. **(04 Marks)**

b. Prove that the sum of the degrees of all vertices in a graph is twice the number of edges. Deduce that the number of vertices of odd degree in a graph must be even. **(06 Marks)**

c. Is it possible to have a graph G with 15 edges, 4 vertices each have degree 2 and the remaining vertices each have degree 4? Justify your answer. **(04 Marks)**

d. A graph G is said to be a self dual graph if G is isomorphic to its dual. Prove that the complete graph on 4 vertices is a self dual graph. **(06 Marks)**

2 a. Let G be a directed graph on n vertices. If the associated undirected graph for G is K_n (complete graph on n vertices), then prove that $\sum_{v \in V}[od(v)]^2 = \sum_{v \in V}[id(v)]^2$ where $id(v)$ denotes the indegree of v and od(v) denotes the out degree of v. **(08 Marks)**

b. Let G = (V, E) be a connected planar graph or multigraph with v vertices, e edges and r regions. Prove that r = e - v + 2. Deduce that in a loop-free connected planar graph with v vertices, e > 2 edges and r regions, $3r \leq 2e$ and $e \leq 3v - 6$. **(12 Marks)**

3 a. Let G = (V, E) be a loop-free undirected graph consisting of $n \geq 3$ vertices. If $\deg(x)+\deg(y) \geq n$ for all non adjacent x, y $\in$ V, prove that G contains a Hamilton cycle. **(08 Marks)**

b. If T = (V, E) is a tree where $|V| \geq 2$, prove that T has at least two pendant vertices. **(06 Marks)**

c. Construct an optimal prefix code for the symbols a, 0, q, u, y, z which occur in a given sample with frequencies 20, 28, 4, 17, 12, 7 respectively. **(06 Marks)**

4 a. Define the following : i) Transport network ii) Matching iii) Complete matching iv) Maximal matching **(04 Marks)**

b. With usual notations, prove that in bipartite graph G = (V, E) where V is partitioned as $X \cup Y$, the maximum number of vertices in X which can be matched with those in Y is $|X| - \delta(G)$. **(08 Marks)**

c. Using Kruskal's algorithm, obtain a minimum cost spanning tree for the graph given below: **(08 Marks)**

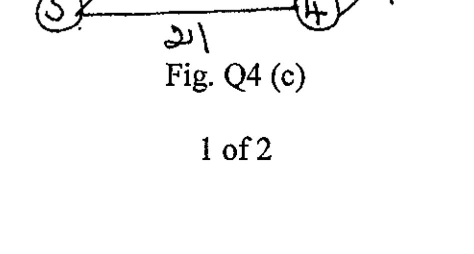

Fig. Q4 (c)

PART – B

5 a. Verify that for each integer $n \geq 1$,
$$\binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1}\binom{2n}{n}$$ **(06 Marks)**

b. Define S(m, n), sterling number of the second kind. If m and n are positive integers with $1 < n \leq m$, then prove that $S(m+1, n) = S(m, n-1) + nS(m, n)$. **(08 Marks)**

c. Show that if any 14 integers are selected from the set S = {1, 2, 3….25}, there are at least two whose sum is 26. **(06 Marks)**

6 a. Determine the number of positive integers n $(1 \leq n \leq 2000)$, which are not divisible by 2, 3 or 5. **(07 Marks)**

b. Prove that the number of partitions of a positive integer n into distinct summands is the same as the number of partitions of n into odd summands. **(07 Marks)**

c. If a set X has 2n+1 elements, find the member of subsets of X with utmost n elements. **(06 Marks)**

7 a. In $a_n (n \geq 0)$ is a solution of the recurrence relation $a_{n+1} - da_n = 0$, where $a_3 = 49$, $a_5 = 81$, determine d. **(04 Marks)**

b. For $n \geq 0$, let S = {1, 2, 3,….n} where n = 0, take $S = \phi$. Let a_n denote the member of subsets of S which contain no consecutive integers. Find a recurrence relation for a_n and solve. **(08 Marks)**

c. Solve the recurrence relation $a_r + 5a_{r-1} + 6a_{r-2} = 42 \times 4^r$ with the boundary conditions $a_2 = 278$, $a_3 = 962$. **(08 Marks)**

8 a. Obtain the exponential generating function for each of the following sequences:

i) $1, 2, 2^2, 2^3, 2^4 \ldots\ldots$

ii) $a, a^3, a^5, a^7, \ldots\ldots$ where a is a real number. **(04 Marks)**

b. Let D_n denote the number of derangements of 1, 2, 3….n. Obtain an expression for D_n. Hence determine D_4 and D_5. **(08 Marks)**

c. Consider the members 1, 2, 3 and 4. Suppose forbidden positions are 1 in the second position, 2 in the third position, 3 in the first or fourth position, 4 in the fourth position. Using the method of rook polynomials, determine the following:
Number of permutations of 1, 2, 3, 4 in which

i) None of the integers is in a forbidden position.
ii) Exactly one integer is in a forbidden position.
iii) Exactly two integers are in forbidden positions.
iv) Exactly three integers are in forbidden positions.
v) Exactly four integers are in forbidden positions. **(08 Marks)**

* * * * *