2002 SCHEME

USN | | | | | | | | | | | **CS42**

Fourth Semester B.E. Degree Examination, June/July 2011
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Illustrate with example for each of the following terms :
i) Degree of a vertex ii) Walk iii) Path. **(06 Marks)**

b. Prove that the number of odd degree vertices in a graph is always even. Illustrate with two examples. **(07 Marks)**

c. What do you understand by isomorphic graphs? Give the properties to be fulfilled with an example. **(07 Marks)**

2 a. Write two Kuratowski's graphs and list the characteristics of each graph. **(06 Marks)**

b. For the following graph detect its planarity. Identify clearly the steps followed during detection. [Refer Fig.Q2(b)]. **(07 Marks)**

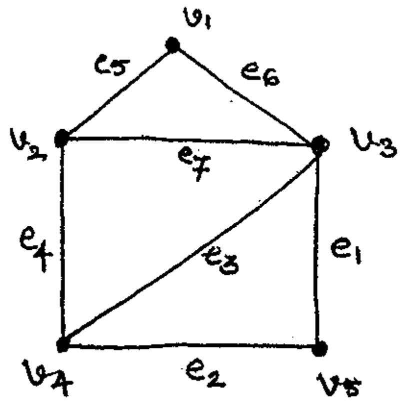

Fig.Q2(b)

c. What is geometric dual of a graph? List the observations between a planar graph and its dual. **(07 Marks)**

3 a. Prove that a tree with n vertices has (n - 1) edges. **(06 Marks)**

b. Define height of a tree. Derive expressions for maximum and minimum possible heights of an n-vertex binary tree. **(07 Marks)**

c. Construct a tree for the month of a year, considered from December to January. Obtain its inorder traversal and comment on the result. **(07 Marks)**

4 a. Define the following with examples: i) Vertex connectivity ii) Edge connectivity.
Show that for a graph the edge connectivity cannot exceed the degree of a vertex with the smallest degree. **(06 Marks)**

b. What is a minimal spanning tree? Obtain minimal spanning tree for the graph given in Fig.Q3(b), using the Kruskal's algorithm. **(10 Marks)**

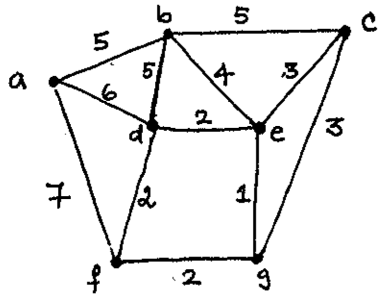

Fig.Q3(b)

c. What is a transport network? List the conditions to be satisfied. **(04 Marks)**

PART – A

5 a. How many permutations are there for the eight letters a, c, f, g, i, t, w, x ? How many start with the letter t and end with letter c. **(06 Marks)**

b. Find the number of arrangements of the letters in BASSAHASSEE. How many of these arrangements have no adjacent As. **(06 Marks)**

c. What is a binomial coefficient? Obtain the coefficients of the following terms:
i) x^5y^2 in $(x + y)^7$ ii) a^5b^2 in $(2a - 3b)^7$ iii) $x^2y^2z^3$ in $(x + y + z)^7$ **(08 Marks)**

6 a. State the principle of inclusion and exclusion for two and three variables. Derive the expressions. **(07 Marks)**

b. Determine the number of positive integers n, where $1 \le n \le 100$ and 2 is not divisible by 2, 3 or 5. **(10 Marks)**

c. Briefly explain the use of Venn diagrams in the application of the principle of inclusion and exclusion. **(03 Marks)**

7 a. Define generating function and give two examples. **(06 Marks)**

b. Obtain the generating sequence for the Maclaurin series expansion for $(1 + x)^n$. **(07 Marks)**

c. Find the generating function for the number of partitions of a positive integer n into distinct summands. **(07 Marks)**

8 a. Solve the recurrence relation $a_{n+1} = 3a_n$, where $n \ge 0$ and $a_0 = 5$. **(06 Marks)**

b. Distinguish between the homogeneous and non-homogeneous recurrence relations and give the corresponding characteristic equations. **(06 Marks)**

c. Solve the relation $a_{n+2} - 5a_{n+1} + 6a_n = 2$ where $n \ge 0$, $a_0 = 3$ and $a_1 = 7$. **(08 Marks)**

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