CMR
INSTITUTE OF
TECHNOLOGY

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Internal Assessment Test I

Sub	ENGINEERING MATHEMATICS IV							Code:	15MAT41
Date:	27 / 3 / 2017	Duration	90 mins	Max Marks	50	Sem	IV	Branch	IS B,CS C

First question is compulsary Answer ANY SIX questions from 2 to 8

		Marks	OBE CO	OBE RBT
1.	Define analytic function. Derive the Cauchy – Riemann equations in Cartesian form.	[08]	CO1	L3
2.	Using modified Euler's method find $y(0.2)$ correct to four decimal places solving the equation $\frac{dy}{dx} = x - y^2, y(0)=1$ taking h=0.1	[07]	CO1	L3
3.	If $\frac{dy}{dx} = 2e^x - y, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040$ and $y(0.3) = 2.090$ find $y(0.4)$ correct to four decimal places by using Adams-Bashforth predictor and corrector method (Apply the corrector formula twice).	[07]	CO1	L3
4.	Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1, y(0) = 1, y'(0) = 0$. Evaluate $y(0.1)$,using Runge-kutta method of order 4.	[07]	CO1	L3
5.	Show that f(z)=z^n, where n is a positive integer is analytic and hence find its derivative.	[07]	CO3	L3
6.	Construct the analytic function whose real part is $y + e^x \cos y$.	[07]	CO3	L3
7.	Verify Cauchy's theorem for the integral of f(z)=z^3 taken along the boundary of the rectangle with vertices -1,1,1+I,-1+i.	[07]	CO3	L3
8.	Evaluate $\int_C \frac{3z^2 + 7z + 1}{z + 1} dz$, in the cases where C is the circle a) $\lvert z\rvert = 1.5$ b) $\lvert z\rvert = 0.5$	[07]	CO3	L3

	Course Outcomes	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Use appropriate single-step and multi-step	3	3	0	0	0	0	0	0	1	0	0	0

	numerical methods to solve first and second order ordinary differential equations.												
CO2:	Apply Frobenius method to find the power series solution of second order differential equations such as Bessel's and Legendre's differential equations.	3	3	0	0	0	0	0	0	1	0	0	0
CO3:	Apply the idea of analyticity and the calculus of residues to evaluate real and complex integrals and analyze conformal transformations.	2	3	0	0	0	0	0	0	1	0	0	0
CO4:	Describe random variables and probability distributions using rigorous statistical methods and translate real-world problems into probability models.	3	3	1	0	0	0	0	0	1	0	0	0
CO5:	Explain and successfully apply parametric testing techniques including single and multi-sample tests for mean and proportion.	3	3	0	0	0	0	0	0	1	0	0	0
CO6:	Estimate the nature and strength of relationship between two variables of interest using joint probability distribution and describe a discrete time Markov chain in terms of a transition matrix.	3	3	0	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

Solution Manual

IAT - 1 Engineering Maths - IV

(1.) A complex valued function $w = f(z)$ is said to be analytic at a point $z = z_0$ if

$$\frac{dw}{dz} = f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z) - f(z)}{\delta z}$$ exists and is unique at z_0 and in the neighbourhood of z_0.

Further $f(z)$ is called analytic in a region if it is analytic at every point of the region. — (2)

let $f(z)$ be analytic at a point $z = x+iy$ and hence by definition of analytic function we have

$$f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z) - f(z)}{\delta z}$$ exists and is unique.

let $f(z) = u(x,y) + iv(x,y)$ also $\delta x, \delta y$ and δz are the increments in x, y and z respectively. — (1)

$$\therefore f'(z) = \lim_{\delta z \to 0} \frac{[u(x+\delta x, y+\delta y) + iv(x+\delta x, y+\delta y)]}{\delta z} - \frac{[u(x,y) + iv(x,y)]}{\delta z}$$ — (1)

$$f'(z) = \lim_{\delta z \to 0} \frac{[u(x+\delta x, y+\delta y) - u(x,y)]}{\delta z} + i \lim_{\delta z \to 0} \frac{[v(x+\delta x, y+\delta y) - v(x,y)]}{\delta z}$$ — (1)

Now $\delta z = (z+\delta z) - z$, $z = x+iy$

$\delta z = [(x+\delta x) + i(y+\delta y)] - [x+iy]$

$\delta z = \delta x + i\delta y$

since δz tends to zero, we have the foll. 2 possibilities

Case I : let $\delta y = 0$, so that $\delta z = \delta x$ and $\delta z \to 0$

$\Rightarrow \delta x \to 0$.

(1) becomes

$$f'(z) = \lim_{\delta x \to 0} \frac{u(x+\delta y, y) - u(x,y)}{\delta x} + i \lim_{\delta x \to 0} \frac{v(x+\delta x, y) - v(x,y)}{\delta x}$$

$$\therefore f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$ (def^n of partial derivatives) — (2)

— 2

Case II : let $\delta x = 0$ so that $\delta z = i\delta y$ and $\delta z \to 0$

$\Rightarrow i\delta y \to 0$ or $\delta y \to 0$

(1) becomes

$$f'(z) = \lim_{\delta y \to 0} \frac{u(x, y+\delta y) - u(x,y)}{i\delta y} + i \lim_{\delta y \to 0} \frac{v(x, y+\delta y) - v(x,y)}{i\delta y}$$

Now $\frac{1}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$

— 2

$$f'(z) = \lim_{\delta y \to 0} (-i) \left\{ \frac{u(x, y+\delta y) - u(x,y)}{\delta y} \right\} + \lim_{\delta y \to 0} \left\{ \frac{v(x, y+\delta y) - v(x,y)}{\delta y} \right\}$$

$$= -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$$ — (3)

equating the R.H.S of (2) and (3)

we get $\quad \dfrac{\partial u}{\partial x} + i\dfrac{\partial v}{\partial x} = \dfrac{\partial v}{\partial y} - i\dfrac{\partial u}{\partial y}$

equating the real and imaginary parts

we get

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \quad (u_x = v_y,\ v_x = -u_y)$$

(2) <u>I stage</u> : $x_0 = 0,\ y_0 = 1,\ h = 0.1,\ f(x, y) = x - y^2$

$f(x_0, y_0) = -1, \qquad x_1 = x_0 + h = 0.1$ — 1

By Euler's formula

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$

$$= 0.9$$

By modified Euler's formula

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= \underline{0.9145}$$

~~[y(0.1) = 0.9145]~~

$y_1^{(2)} = \underline{0.9132}$ — 3

$y_1^{(3)} = \underline{0.9133}$

$y(0.1) = \underline{0.9133}$

II stage let $x_0 = 0.1$, $y_0 = 0.9133$, $f(x,y) = x - y^2$

$f(x_0, y_0) = 0.7341$

$x_1 = x_0 + h = 0.2$

$y_1^{(0)} = 0.9133 + (0.1)(-0.7341) = 0.8399$

$y_1^{(1)} = 0.8513$

$y_1^{(2)} = 0.8504$ — 3

$y_1^{(3)} = 0.8504$

$y(0.2) = 0.8504$

3) $\frac{dy}{dx} = 2e^x - y$

x	y	$y' = 2e^x - y$
$x_0 = 0$	$y_0 = 2$	$y_0' = 2e^0 - 2 = 0$
$x_1 = 0.1$	$y_1 = 2.010$	$y_1' = 0.2003$
$x_2 = 0.2$	$y_2 = 2.040$	$y_2' = 0.4028$
$x_3 = 0.3$	$y_3 = 2.090$	$y_3' = 0.6097$
$x_4 = 0.4$	$y_4 = ?$	

— 2

$y_4^{(p)} = y_3 + \frac{h}{24}[55y_3' - 59y_2' + 37y_1' - 9y_0']$

$= 2.1616$ — 1

$y_4' = 0.822$ — 1

$y_4^{(c)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$ — 1

$= 2.1615$

$y_4' = 0.82215$ — (1)

substituting again in the corrector formula

we obtain $y_4^{(1)} = 2.1615$

— (1)

$y(0.4) = \underline{2.1615}$

(4) $\dfrac{d^2y}{dx^2} - x^2\dfrac{dy}{dx} - 2xy = 1$

$y = 1, \quad y' = 0 \quad \text{at} \quad x = 0$

put $\dfrac{dy}{dx} = z$ diff w.r.t x

$\dfrac{d^2y}{dx^2} = \dfrac{dz}{dx}$ — (1)

— (1)

$\dfrac{dz}{dx} - x^2 z - 2xy = 1$

$\dfrac{dy}{dx} = z, \quad \dfrac{dz}{dx} = 1 + 2xy + x^2 z$ where $y = 1, z = 0, x = 0$

$f(x, y, z) = z, \quad g(x, y, z) = 1 + 2xy + x^2 z$

— (1)

$\underline{\text{let } h = 0.1}$

$k_1 = h f(x_0, y_0, z_0) = 0$

$l_1 = 0.1$

$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right) = 0.005$

$l_2 = 0.11$

$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right) = 0.0055$

$l_3 = 0.11004$

$k_4 = h f(x_0+h, y_0+k_3, z_0+l_3)$

$= 0.011$

$l_4 = 0.12022$ — (4)

$y(x_0+h) = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$

$y(0.1) = \underline{1.0053}$

(5) $w = f(z) = z^n$

$z = x+iy = re^{i\theta}$ — (1)

$w = z^n = (re^{i\theta})^n = r^n e^{in\theta}$

$w = u+iv = r^n(\cos n\theta + i\sin n\theta)$

$u = r^n \cos n\theta \qquad v = r^n \sin n\theta$ — (2)

$u_r = n r^{n-1}\cos n\theta \qquad v_r = n r^{n-1}\sin n\theta$

$u_\theta = -n r^n \sin n\theta \qquad v_\theta = n r^n \cos n\theta$

$u_r = \frac{1}{r} v_\theta \qquad v_r = -\frac{1}{r} u_\theta$ — (1)

$\therefore$ CR eq^n are satisfied in polar form hence the given function is analytic.

$f'(z) = e^{-i\theta}(u_r + i v_r)$ — (3)

$= e^{-i\theta}(n r^{n-1}\cos n\theta + i n r^{n-1}\sin n\theta)$

$= n r^{n-1} e^{-i\theta}(e^{in\theta})$

$= n r^{n-1} e^{i\theta(n-1)}$

$= n(re^{i\theta})^{n-1}$

$\underline{f'(z) = n z^{n-1}} \qquad \because z = re^{i\theta}$

6) $u = y + e^x \cos y$.

$u_x = e^x \cos y$

$u_y = 1 - e^x \sin y$ $\Big\}$ ②

$f'(z) = u_x + i v_x$

$= u_x - i u_y$ (using CR equations $v_x = -u_y$)

$\therefore f'(z) = e^x \cos y - i(1 - e^x \sin y)$ — ②

Put $x = z$, $y = 0$ we get

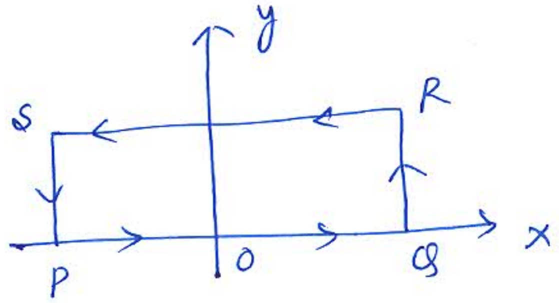
$f'(z) = e^z - i$

Integrating w.r.t z we get — ③

$f(z) = \int e^z dz - i \int dz + c$

$f(z) = e^z - iz + c$

⑦ $f(z) = z^3$



Here the vertices P, Q, R, S are the points $-1, 1, 1+i$, $-1+i$ respectively.

Since $f(z) = z^3$ is analytic everywhere in the complex plane, it is analytic on and within C. — ①

By Cauchy's theorem

$$\int_C f(z) dz = \int_C z^3 dz = 0$$

$$\int_C z^3 dz = \int_{PQ} z^3 dz + \int_{QR} z^3 dz + \int_{RS} z^3 dz + \int_{SP} z^3 dz$$

on PQ, $y=0, \quad -1 \le x \le 1$ $\quad - 1\frac{1}{2}$

$\therefore z = x, \quad dz = dx$

$$\int_{PQ} z^3 dz = \int_{-1}^{1} x^3 dx = 0 \qquad - (1)$$

on QR, $x = 1, \quad 0 \le y \le 1$ $\quad - 1\frac{1}{2}$

$z = (1+iy) \qquad dz = i\,dy$

$$\int_{QR} z^3 dz = \int_0^1 (1+iy)^3 (i\,dy)$$

$$= \frac{1}{4}\left[(1+i)^4 - 1\right] \qquad - (2)$$

$- 1\frac{1}{2}$

on RS, $y = 1 \quad 1 \le x \le -1$

$z = x + i \qquad dz = dx$

$$\int_{RS} z^3 dz = \int_1^{-1} (x+i)^3 dx$$

$$= \frac{1}{4}\left\{(1-i)^4 - (1+i)^4\right\} \qquad - (3)$$

on SP, $x = -1, \quad 1 \le y \le 0$ $\quad - 1\frac{1}{2}$

$z = -1 + iy \qquad dz = i\,dy$

$$\int_{SP} z^3 dz = \int_1^0 (-1+iy)^3 (i\,dy)$$

$$= \frac{1}{4}\left\{1 - (1-i)^4\right\} \qquad - (4)$$

on Adding (1), (2), (3), (4)

$$\int_C z^3 dz = 0$$

(8) $\int_C \frac{3z^2+7z+1}{z+1}\,dz$

$f(z) = 3z^2+7z+1$

(a) $|z| = 1.5$

$z=-1$, lies inside the given circle $C: |z|=1.5$

$\int \frac{f(z)}{z-a}\,dz = 2\pi i f(a)$

$f(z) = 3z^2+7z+1$ — 4

$f(-1) = 3(-1)^2+7(-1)+1 = 3-7+1 = -3$

$= 2\pi i\, f(-1)$

$= 2\pi i(-3) = -6\pi i$

(b) $|z|=0.5$

$f(z)$ is analytic everywhere except at $z=-1$. also $z=-1$ lies out side the given circle $C: |z|=\frac{1}{2}$

$\therefore$ $f(z)$ is analytic on and within C

$\therefore$ By cauchy's theorem $\int_C f(z)\,dz=0$

hence $\int_C \frac{3z^2+7z+1}{z+1}\,dz = 0$. — 3