CMR
INSTITUTE OF
TECHNOLOGY

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Internal Assessment Test I

Sub:	ENGINEERING MATHEMATICS - 4									
Date:	27/03/2017	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	CS-A&B, IS-A

Q.1 is compulsory. Answer any SIX questions from Q.2 to Q.8

	Marks	OBE CO	OBE RBT
1. Find y(20.2) and y(20.4) given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y(20) = 5 taking h = 0.2 by modified Euler's method.	[8]	CO1	L3
2. Apply Adams- Bashforth predictor and corrector formulae to compute y(1.2) given $\frac{dy}{dx} = 3x - 4y^2$ with y(0) =1, y(0.3) =1.3020, y(0.6) = 1.3795, y(0.9) =1.4762.	[7]	CO1	L3
3. Solve $2y'' = 4x + \frac{dy}{dx}$ for x = 1.4 by Milne's method given y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514, $y'(1) = 2$, $y'(1.1) = 2.3178$, $y'(1.2) = 2.6725$ and $y'(1.3) = 3.0657$.	[7]	CO1	L3
4. By Runge-Kutta method solve $y'' = x(y')^2 - y^2$ for x = 0.2 correct to four decimal places given y = 1 and $y' = 0$ when x = 0.	[7]	CO1	L3
5. Find the bilinear transformation that transforms the points z = i, 1, -1 on to the points w = 1, 0 , ∞. Find fixed points also.	[7]	CO3	L3
6. Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is (i) |z| = 3, (ii) |z| = 1/2, (iii) |z| = 3/2.	[7]	CO3	L3
7. Find the analytic function whose real part is $r^2 \cos 2\theta$ and hence find its imaginary part.	[7]	CO3	L3
8. State and prove Cauchy's integral theorem.	[7]	CO3	L3

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Use appropriate single-step and multi-step numerical methods to solve first and second order ordinary differential equations.	3	3	0	0	0	0	0	0	1	0	0	0
CO2:	Apply Frobenius method to find the power series solution of second order differential equations such as Bessel's and Legendre's differential equations.	3	3	0	0	0	0	0	0	1	0	0	0
CO3:	Apply the idea of analyticity and the calculus of residues to evaluate real and complex integrals and analyze conformal transformations.	2	3	0	0	0	0	0	0	1	0	0	0
CO4:	Describe random variables and probability distributions using rigorous statistical methods and translate real-world problems into probability models.	3	3	1	0	0	0	0	0	1	0	0	0
CO5:	Explain and successfully apply parametric testing techniques including single and multi-sample tests for mean and proportion.	3	3	0	0	0	0	0	0	1	0	0	0
CO6:	Estimate the nature and strength of relationship between two variables of interest using joint probability distribution and describe a discrete time Markov chain in terms of a transition matrix.	3	3	0	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

Solution - IAT-1 (M4) CS- A&B, IS-A

(1.) : $f(x,y) = \log_{10}\left(\frac{x}{y}\right) \Rightarrow f(x_0, y_0) = 0.6021$

$x_0 = 20, y_0 = 5, \quad x_1 = x_0 + h = 20.2$

$\& \; y_1 = y(x_1) = y(20.2) = ?$

By Euler's formula - $y_1^{(0)} = y_0 + h f(x_0, y_0) = 5.1204$ — (2)

By modified $y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right] = 5.1198$ — (2)

$y_1^{(2)} = 5.1198 \Rightarrow y(20.2) = 5.1198$ — (1)

Now take $x_0 = 20.2, \quad y_0 = 5.1198 \; \& \; x_1 = 20.4$

— (1)

$f(x_0, y_0) = 0.5961$

$y_1^{(0)} = 5.239$ —(1) , $y_1^{(1)} = 5.2384$

$y_1^{(2)} = 5.2385$ —(1)

(2) Given $y' = 3x - 4y^2$

x	y	$y' = 3x - 4y^2$
$x_0 = 0$	$y_0 = 1$	$y_0' = -4$
$x_1 = 0.3$	$y_1 = 1.3020$	$y_1' = -5.88082$
$x_2 = 0.6$	$y_2 = 1.3795$	$y_2' = -5.81208$
$x_3 = 0.9$	$y_3 = 1.4762$	$y_3' = -6.01667$
$x_4 = 1.2$	$y_4 = ?$	y_4'

—(4)

By Adams - Bashforth predictor formula -

$$y_4^{(p)} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 1.4762 + \frac{0.3}{24}\left[55\times(-6.01667) - 59\times(-5.81208) + 37(-5.88082) - 9\times(-4)\right]$$

$$= -0.643731$$

— (1.5)

By Adams Bashforth Corrector formula

First $y_4' = 3x_4 - 4(y_4^{(p)})^2 = 3(1.2) - 4(-0.643731)^2$
$= 1.942442$ — (1)

$$y_4^{(c)} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$$= 1.4762 + \frac{0.3}{24}\left[9 \times 1.942442 + 19 \times (-6.01667) - 5(-5.81208) - 5.88082\right]$$

$$= 0.55551$$ — (1)

(3) Given diff. eq^n. $y'' = 2x + \frac{1}{2}y'$

Let $y' = z$ then $z' = 2x + \frac{1}{2}z$

x	y	$y' = z$	z'
$x_0 = 1$	$y_0 = 2$	$y_0' = 2$	$z_0' = 3$
$x_1 = 1.1$	$y_1 = 2.2156$	$y_1' = 2.3178$	$z_1' = 3.3589$
$x_2 = 1.2$	$y_2 = 2.4649$	$y_2' = 2.6725$	$z_2' = 3.73625$
$x_3 = 1.3$	$y_3 = 2.7514$	$y_3' = 3.0657$	$z_3' = 4.13285$
$x_4 =$			

— (2)

By predictor formula

$$y_4^{(p)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3) = 3.0793$$

$$\& \quad z_4^{(p)} = z_0 + \frac{4h}{3}\left[2z_1' - z_2' + 2z_3'\right] = 3.4996$$

— (2.5)

By corrector formula -

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right] = y_2 + \frac{h}{3}\left[z_2 + 4z_3 + z_4\right]$$

$$= 3.0794$$

$$\& \quad z_4^{(c)} = z_2 + \frac{h}{3}\left[z_2' + 4z_3' + z_4'\right] = 3.4997$$ — (2.5)

$$\hookrightarrow z_4' = 2x_4 + \frac{z_4^{(p)}}{2} = 4.5498$$

④ Given $y'' = x(y')^2 - y^2$; $y = 1$ & $y' = 0$ when $x = 0$

Let $\frac{dy}{dx} = z$; $y(0) = 1$ & $\frac{dz}{dx} = xz^2 - y^2$; $z(0) = 0$

i.e. $y_0 = 1$, $z_0 = 0$

$= f(x_0, y_0, z)$ — (1) $\qquad g(x, y, z)$ — (1)

$$k_1 = hf(x_0, y_0, z_0) = 0$$

$$l_1 = hg(x_0, y_0, z_0) = -0.2$$ — (1)

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right) = 0.2 f(0.1, 1, -0.1) = -.02$$

$$l_2 = (0.2)\, g(0.1, 1, -0.1) = -0.1998$$ — (1)

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right) = (0.2) f(0.1, 0.99, -0.0999)$$

$$= -0.01998$$

$$l_3 = -0.1958$$ — (1)

$$k_4 = hf(x_0 + h, y_0 + k_3, z_0 + l_3) = 0.2 f(0.2, 0.98002, -0.1958)$$

$$= -0.03916$$

$$l_4 = -0.19055$$ — (1)

Now $y_1 = y(0.2) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 0.9801$ —— (1)

5. $z_1 = i, \quad z_2 = 1, \quad z_3 = -1, \quad z_4 = z$

$w_1 = 1, \quad w_2 = 0, \quad w_3 = \infty, \quad w_4 = w$

By cross ratio, we have

$$\frac{(z_1 - z_2)(z_3 - z_4)}{(z_2 - z_3)(z_4 - z_1)} = \frac{(w_1 - w_2)(w_3 - w_4)}{(w_2 - w_3)(w_4 - w_1)}$$ —— (1)

$$\frac{(1-0)(w_3 - w)}{(0 - w_3)(w-1)} = \frac{(i-1)(-1-z)}{(1+1)(z-i)}$$ —— (1)

$$\frac{1}{(w-1)} \lim_{w_3 \to \infty} \frac{\cancel{w_3}(1 - w/w_3)}{\cancel{w_3}\left(\frac{0}{w_3} - 1\right)} = \frac{-i - iz + 1 + z}{z - i + z - i}$$ —— (1)

$$\frac{-1}{(w-1)} = \frac{z + 1 - i - iz}{2z - 2i}$$ —— (1)

$$-2z + 2i = wz + w - wi - wzi - z - 1 + i + iz$$

$$\Rightarrow w = \frac{(z-1)(i+1)}{(z+1)(i-1)} = \frac{(i+1)z - (i+1)}{(i-1)z + (i-1)}$$

For fixed pts we have $w = z$ —— (1)

$$\therefore w = \frac{(z-1)(i+1)}{(z+1)(i-1)} = z$$

$$zi + z - i - 1 = z(zi - z + i - 1)$$

$$\cancel{zi} + z - i - 1 = z^2 i - z^2 + \cancel{zi} - z$$

$$z^2(1-i) + 2z - (i+1) = 0$$

Here $a = 1 - i, \quad b = 2, \quad c = -(i+1)$ —— (1)

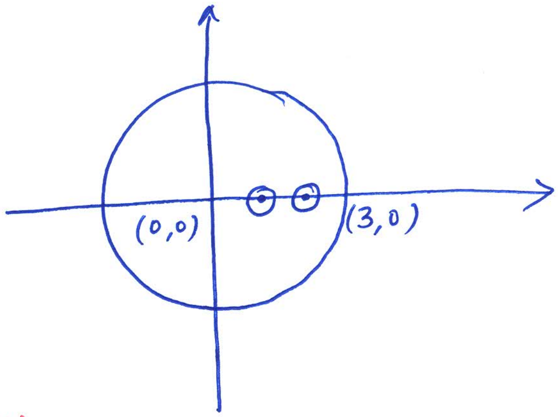
$$z = \frac{-2 \pm \sqrt{4 + 4(1-i)(1+i)}}{2(1-i)}$$

$$= \frac{-2 \pm \sqrt{4+8}}{2(1-i)} = \frac{-2 \pm 2\sqrt{3}}{2(1-i)}$$

$$z = \frac{-1 \pm 1\sqrt{3}}{1-i}$$ —— (1)

(6.) $\displaystyle\oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)}\, dz$

(i) $C: |z| = 3$



Let $\displaystyle\frac{1}{(z-1)^2(z-2)} = \frac{A}{(z-1)} + \frac{B}{(z-1)^2} + \frac{C}{(z-2)}$

$A = -1,\ B = -1,\ C = 1$ — (1)

$$\therefore \oint \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)^2(z-2)} = \oint_C \frac{-(\sin\pi z^2 + \cos\pi z^2)}{(z-1)}\,dz - \oint_C \frac{+(\sin\pi z^2 + \cos\pi z^2)}{(z-1)^2}\,dz$$

$$+ \int \frac{\sin\pi z^2 + \cos\pi z^2}{(z-2)}\,dz$$ — (1)

$= -2\pi i(-1) + 4\pi^2 i + 2\pi i = 4\pi i(\pi + 1)$ — (2)

(By Cauchy's integral theorem & formula)

(ii) $C: |z| = \frac{1}{2}$ $\because$ both points are outside

$=$ Ans $- 0$ — (1)

(iii) $C: |z| = 3/2$ only $z = 1$ lies inside the C

$\therefore$ Consider $f(z) = \dfrac{\sin\pi z^2 + \cos\pi z^2}{(z-2)}$.

$\therefore \displaystyle\oint_C \frac{f(z)}{(z-1)^2}\,dz = 2\pi i(2\pi + 1)$ — (2)

(7) Given $u = r^2 \cos 2\theta$

$u_r = 2r\cos 2\theta, \quad u_\theta = -2r^2 \sin 2\theta$ — (1)

Let $f'(z) = e^{-i\theta}(u_r + i v_r)$

by C-R eq^n – $v_r = -\frac{1}{r} u_\theta \Rightarrow v_r = 2r\sin 2\theta$ — (1)

$\therefore f'(z) = e^{-i\theta} \cdot (2r\cos 2\theta + i2r\sin 2\theta)$

$= 2re^{-i\theta} \cdot e^{i2\theta} = 2re^{i\theta}$ — (1)

Put $r = z$ & $\theta = 0$ — (1)

$f'(z) = 2z$

Integrating $f(z) = z^2 + C$ — (1)

Let $f(z) = z^2 = (re^{i\theta})^2$

$= r^2 e^{i2\theta}$

$= r^2[\cos 2\theta + i\sin 2\theta]$

$= \underbrace{r^2 \cos 2\theta}_{u} + \underbrace{i r^2 \sin 2\theta}_{v}$ — (1)

$\therefore$ imaginary part $v = r^2 \sin 2\theta$ — (1)

(8) Theorem Statement — (2)

$\oint f(z)dz = \oint (udx - vdy) + i\oint_c (udy + vdx) = I_1 + I_2$ — (1)

By Green's theorem $I_1 = 0$ & $I_2 = 0$

$\therefore \oint_c f(z)dz = 0$ — (2) — (2)