

SCHEME OF EVALUATION

2016-17 EVEN SEMESTER

IAT-1

1.a. :[7]

Max Planck developed a structural model for black body radiation that leads to a theoretical equation for the wavelength distribution that is in complete agreement with the experimental results at all wavelengths. [2]

According to his theory

1. a black body is imagined to be consisting of large number of electrical oscillators.
2. an oscillator emits or absorbs energy in discrete units. It can emit or absorb energy by making a transition from one quantum state to another in the form of discrete energy packets known as photons whose energy is an integral multiple of $h\nu$ where h is the planks constant and ν is the frequency.
3. The Energy density per unit wavelength range per unit volume is given by

$$E_{\lambda} d\lambda = \frac{8\pi hc}{\lambda^5} \left[\frac{1}{e^{\left[\frac{h\nu}{kT}\right]} - 1} \right] d\lambda \quad [1]$$

Where h is Planck's constant, c is velocity of light, T is absolute temperature, λ is the wavelength and k is Boltzmann constant

Deduction of Weins law:[2]

It is applicable at smaller wavelengths.

For smaller wavelengths $e^{\frac{h\nu}{kT}} \gg 1$

$$\therefore e^{\frac{h\nu}{kT}} \gg 1 = e^{\frac{h\nu}{kT}}$$

So Planck's radiation law becomes

$$E_{\lambda} d\lambda = \frac{8\pi hc}{\lambda^5} \left[\frac{1}{e^{\left[\frac{h\nu}{kT}\right]}} \right]$$

Deduction of Rayleigh Jeans Law: :[2]

It is applicable at longer wavelengths.

For longer wavelengths $\frac{h\nu}{kT} \ll 1$

$$\therefore e^{\frac{h\nu}{kT}} = 1 + \frac{h\nu}{kT} + \left(\frac{h\nu}{kT}\right)^2 \frac{1}{2!} + \dots = 1 + \frac{h\nu}{kT}$$

$$E_{\lambda} d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{1 + \frac{h\nu}{kT}} d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

1.b. :[3]

Change in wavelength of x ray after Compton scattering is given by

$$d\lambda = \lambda^1 - \lambda = \frac{h}{m_0 c} (1 - \cos\theta) \quad [1]$$

Wavelength of Incident photon = $E = \frac{hc}{\lambda} = 90000 \times 1.6 \times 10^{-19}$

$$\lambda = 1.37 \times 10^{-11} \text{ m}$$

Here, when $\theta = 60^\circ$,

Wavelength of scattered photon : [1]

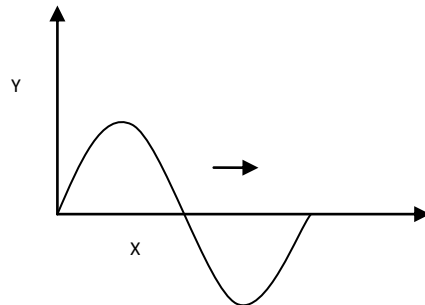
$$\lambda^1 = \lambda + \frac{h}{m_0 c} (1 - \cos 60) = 1.37 \times 10^{-11} + 0.121 \times 10^{-11} = 1.491 \times 10^{-11} \text{ m}$$

Energy of scattered photon = $E = \frac{hc}{\lambda^1} = 13.28 \times 10^{-15} \text{ J} = 83 \text{ keV}$

: [1]

2.a. :[6]

Phase velocity(V_p): It is the speed with which an isolated pulse / constant phase propagates in a medium. : [1]



A single pulse is shown in this diagram .It is represented as

$$Y = A \sin [wt - kx]$$

where Y is the displacement of a particle at a distance ' x ' from the origin at a time ' t ', A is the amplitude , w is the angular velocity and k is the wave number.

Imagine two points A and B at X_1 and X_2 at same phase on the wave. Then

$$(wt_1 - kX_1) = (wt_2 - kX_2)$$

$$\frac{w}{k} = \frac{X_1 - X_2}{t_1 - t_2}$$

$$\therefore v_p = \frac{w}{k}$$

Group Velocity(V_g): It is the velocity with which the resultant envelope of varying amplitude formed by the superposition of two or more waves propagates. : [1]

$$v_g = \lim_{dk \rightarrow 0} \frac{L}{dk} \frac{dw}{dk}$$

Relation between phase velocity and group velocity: **:[4]**

We have phase velocity $v_p = \frac{w}{k}$

$$v_g = \lim_{dk \rightarrow 0} \frac{L}{dk} \frac{dw}{dk} = \frac{d}{dk} (kV_p) = v_p + k \cdot \frac{dV_p}{dk} \dots\dots\dots (1)$$

But $\frac{dV_p}{dk} = \frac{dV_p}{d\lambda} \times \frac{d\lambda}{dk}$ and $k = \frac{2\pi}{\lambda}$; $\frac{d\lambda}{dk} = \frac{-2\pi}{k^2}$

Substituting these in equation (1) we get

$$v_g = v_p - \lambda \frac{dV_p}{d\lambda}$$

2.b. :[4]

Debroglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$ **:[1]**

$E = 40\text{eV} = 40 \times 1.6 \times 10^{-19} \text{J}$ **:[2]**

$\lambda = 6.1 \times 10^{-12} \text{m}$ **:[1]**

3.a. :[7]

The position and momentum of a particle cannot be determined accurately and simultaneously. The product of uncertainty in the measurement of position (Δx) and momentum (Δp) is always greater than or equal

to $\frac{h}{2\pi}$.

$$\boxed{(\Delta x) \cdot (\Delta p) \geq \frac{h}{4\pi}}$$

This uncertainty is not due to discrepancy with the apparatus or with the method of measurement, but because of the very wave nature of the object. This uncertainty persists as long as matter possesses wave nature.

Different forms of Heisenberg's Principle:

$$(\Delta x) \cdot (\Delta p) \geq \frac{h}{4\pi}$$

$$(\Delta L) \cdot (\Delta \theta) \geq \frac{h}{4\pi}$$

$$(\Delta E) \cdot (\Delta t) \geq \frac{h}{4\pi}$$

Here ΔL is the uncertainty in angular momentum

$\Delta \theta$ is the uncertainty in angular displacement

ΔE is the uncertainty in the energy

Δt is the uncertainty in the time interval during which the particle exists in the state E: **:[2]**

To Show that electron does not exist inside the nucleus:

:[5]

We know that the diameter of the nucleus is of the order of 10^{-15} m. If the electron is to exist inside the nucleus, then the uncertainty in its position Δx cannot exceed the size of the nucleus

$$\Delta x \leq 10^{-15} \text{m}$$

Now the uncertainty in momentum is

$$\Delta p \geq \frac{h}{4\pi \Delta x}$$

$$\Delta p \geq \frac{6.62 \times 10^{-34}}{4\pi \times 10^{-15}} \text{ :[2]}$$

$$\Delta p \geq 0.5 \times 10^{-19} \text{Ns}$$

Then the momentum of the electron can at least be equal to the uncertainty in momentum.

$$p \geq 0.5 \times 10^{-20} \text{Ns} \text{ :[1]}$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by - for small velocities (non-relativistic) case-

$$E = \frac{p^2}{2m} = \frac{(0.5 \times 10^{-20})^2}{2 \times 9.1 \times 10^{-31}} = 1.37 \times 10^{-11} \text{J} = 85 \text{MeV}$$

For high velocities,

$$E = \left[\frac{1}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \right] m_0 c^2 \dots\dots\dots(1)$$

$$E^2 = \left[\frac{1}{1 - \left(\frac{v^2}{c^2}\right)} \right] m^2_0 c^4$$

Momentum $p = \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right] m_0 v$

$$p^2 = \left[\frac{1}{1 - \left(\frac{v^2}{c^2}\right)} \right] m^2_0 v^2$$

$$p^2 c^2 = \left(\frac{1}{1 - \left(\frac{v^2}{c^2} \right)} \right) m^2 v^2 c^2 \dots\dots\dots(2)$$

From (1) and (2) $E^2 = p^2 c^2 + m^2 c^4$

On substitution **:[2]**

$$E = (9.1 \times 10^{-31})^2 (3 \times 10^8)^2 + (0.5 \times 10^{-19})^2 (3 \times 10^8)^4 = 9.4 \text{ MeV}$$

3.b. :[3]

Given $v = 4.8 \times 10^5 \text{ m}$ accurate to 0.012%

$$\Delta v = 4.8 \times 10^5 \times \frac{0.012}{100} = 57.6 \text{ m/s} \text{ :[1]}$$

From Heisenberg's Principle,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \cdot m \Delta v = \frac{h}{4\pi} \text{ :[2]}$$

$$\Delta x = \frac{h}{4\pi \cdot \Delta v \cdot m} = 1 \times 10^{-6} \text{ m}$$

4.a. :[7]

Time independent Schrödinger equation

A matter wave can be represented in complex form as

$$\Psi = A \sin kx (\cos \omega t + i \sin \omega t) \text{ :[1]}$$

$$\Psi = A \sin kx e^{i\omega t}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = k A \cos kx e^{i\omega t}$$

$$\frac{d^2\Psi}{dx^2} = -k^2 A \sin kx e^{i\omega t} = -k^2 \Psi \dots\dots\dots(1) \text{ :[1]}$$

From de Broglie's relation

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$$

$$k^2 = 4\pi^2 \frac{p^2}{h^2} \dots\dots\dots(2) \text{ :[1]}$$

Total energy of a particle $E = \text{Kinetic energy} + \text{Potential Energy}$

$$E = \frac{1}{2} m v^2 + V \text{ :[1]}$$

$$E = \frac{p^2}{2m} + V$$

:[1]

$$p^2 = (E - V) 2m$$

Substituting in (2)

$$k^2 = \frac{4\pi^2 (E - V) 2m}{h^2}$$

\therefore From (1)

$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m (E - V) \Psi}{h^2} = 0}$$

:[2]

(For one dimension)

4.b. :[3]

Energy of a particle in an infinite potential well $E = \frac{n^2 h^2}{8ma^2} \text{ :[1]}$

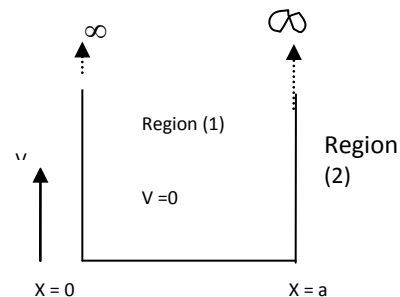
Lowest energy corresponds to $n = 1 \text{ :[1]}$

$$E = \frac{1^2 h^2}{8 \cdot (1.67 \times 10^{-27}) \cdot (10^{-14})^2} = 3.27 \times 10^{-13} \text{ J} = 2 \text{ MeV} \text{ :[1]}$$

5.a. :[7]

Particle in an infinite potential well problem:

Consider a particle of mass m moving along X -axis in the region from $X=0$ to $X=a$ in a one dimensional potential well as shown in the diagram. The potential energy is zero inside the region and infinite outside the region.



Applying Schrodinger's equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\Psi}{h^2} = 0 \quad \therefore V = 0 \quad \text{: [1]}$$

$$\text{But } k^2 = \frac{8\Pi^2 mE}{h^2}$$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0 \quad \text{: [1]}$$

The general solution to this expression is given by

$$\Psi = C \cos kx + D \sin kx$$

$$\text{At } x=0, \Psi = 0 \quad \therefore C = 0$$

$$\text{At } x=a, \Psi = 0 \quad D \sin ka = 0 \Rightarrow ka = n\Pi \text{ where } n = 1, 2, 3, \dots$$

$$\Psi = D \sin\left(n \frac{\Pi}{a}\right)x$$

$$E = \frac{n^2 h^2}{8ma^2} \quad \text{: [2]}$$

To evaluate the constant D:

Normalisation : For one dimension

$$\int_0^a \Psi^2 dx = 1 \quad \text{: [1]}$$

$$\int_0^a D^2 \sin^2\left(\frac{n\Pi}{a}\right)x dx = 1$$

$$\text{But } \cos 2\theta = 1 - 2\sin^2 \theta$$

$$\int_0^a D^2 \frac{1}{2} (1 - \cos 2\left(\frac{n\Pi}{a}\right)x) dx = 1$$

$$\int_0^a \frac{D^2}{2} dx - \int_0^a \frac{1}{2} \cos 2\left(\frac{n\Pi}{a}\right)x dx = 1$$

$$\frac{D^2 a}{2} - \left[\sin 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}\right]_0^a = 1$$

$$D^2 \frac{a}{2} - 0 = 1$$

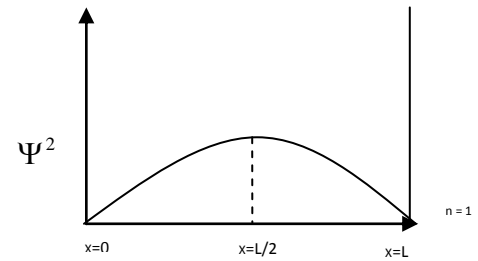
$$D = \sqrt{\frac{2}{a}}$$

$$\therefore \Psi_n = \sqrt{\frac{2}{a}} \sin\left(n \frac{\Pi}{a}\right)x \quad \text{: [2]}$$

For n = 1, First state

$$\therefore \Psi_1 = \sqrt{\frac{2}{a}} \sin\left(1 \cdot \frac{\Pi}{a}\right)x$$

The graph of Ψ^2 versus x is shown below.

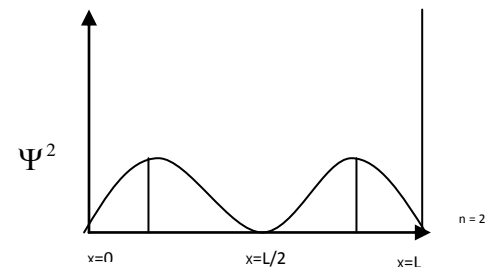


It is seen from the graph that probability density is maximum at the centre for the particle in the first state.

For n = 2, Second state

$$\therefore \Psi_2 = \sqrt{\frac{2}{a}} \sin\left(2 \cdot \frac{\Pi}{a}\right)x$$

The graph of Ψ^2 versus x is shown below.



It is seen from the graph that probability density is maximum at $x = L/4$ and $x = 3L/4$ for the particle in the second state.

5.b: [3]

A wave function (ψ) is a matter wave associated with a particle. It is obtained as a solution to Schrodinger equation.

Significance of Ψ^2 : According to Max Born, $|\Psi^2|$ represents probability of finding a particle per unit volume. **: [1]**

A well behaved wave function has following properties. **: [2]**

1. Single valued
2. Continuous
3. Finite
4. Normalisable.

6.a. :[6]

Failures of Classical free electron theory:

1. Prediction of low specific heats for metals: :[2]

Classical free electron theory assumes that conduction electrons are classical particles similar to gas molecules. Hence, they are free to absorb energy in a continuous manner. Hence metals possessing more electrons must have higher heat content. This resulted in high specific heat given by the expression

$$C_v = C_v = \frac{3}{2} R.$$

This was contradicted by experimental results which showed low specific heat for metals given by $C_v = 10^{-4} R$.

2. Temperature dependence of electrical conductivity: :[2]

From the assumption of kinetic theory of gases

$$\frac{3}{2} kT = \frac{1}{2} mv^2$$

$$\therefore v \propto \sqrt{T}$$

Also mean collision time τ is inversely proportional to velocity,

$$\tau \propto \frac{1}{v}$$

$$\tau \propto \frac{1}{\sqrt{T}}$$

$$\therefore \sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

However experimental studies show that $\sigma \propto \frac{1}{T}$

3. Dependence of electrical conductivity on electron concentration: :[2]

As per free electron theory, $\sigma \propto n$

The electrical conductivity of Zinc and Cadmium are 1.09×10^7 /ohm m and $.15 \times 10^7$ /ohm m respectively which are very much less than that for Copper and Silver for which the values are 5.88×10^7 /ohm m and 6.2×10^7 /ohm m. On the contrary, the electron concentration for zinc and cadmium are $13.1 \times 10^{28} /m^3$ and $9.28 \times 10^{28} /m^3$ which are much higher than that for Copper and Silver which are $8.45 \times 10^{28} /m^3$ and $5.85 \times 10^{28} /m^3$.

These examples indicate that $\sigma \propto n$ does not hold good.

4. Mean free path, mean collision time found from classical theory are incorrect.

6.b. :[4]

Case 1:

For $E < E_F$, at $T = 0$,

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1} = 1 \quad :[2]$$

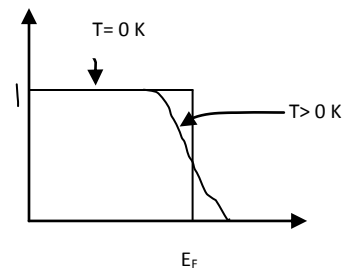
This shows that energy levels below Fermi energy are completely occupied:

Case 2:

For $E > E_F$, at $T=0$

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1} = 0 \quad :[2]$$

This shows that energy levels above Fermi energy are empty:



7.a. :[6]

Expression for Electrical conductivity:

Imagine a conductor across which an electric field E is applied. The equation of motion for an electron moving under the influence of external field is given by $F = dp/dt = eE$

Let the wave number change from k_1 to k_2 in time interval τ_f in the presence of electric field.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{2\pi p}{h}$$

$$p = \frac{hk}{2\pi}$$

$$\frac{dp}{dt} = \frac{h}{2\pi} \left(\frac{dk}{dt} \right) \quad :[2]$$

$$dk = \frac{2\pi}{h} eEdt$$

On integration $k_2 - k_1 = \Delta k = \frac{2\pi \cdot eE \cdot \tau_F}{h} \dots\dots(1)$

From quantum theory, conductivity $J = \Delta k \cdot ne \cdot \frac{h}{2\pi \cdot m} \dots\dots\dots(2)$

Substituting (1) in (2)

$$\text{We get } J = \frac{ne^2\tau_F}{m^*} E \dots(3) \text{ :}[2]$$

Since from Ohm's, $J = \sigma E$, conductivity σ can be written as

$$\sigma = \frac{ne^2}{m^*} \frac{\lambda}{v_F} \text{ where } \tau = \frac{\lambda}{v_F} \text{ :}[2]$$

7.b. :[4]

Fermi probability factor

$$f(E) = \frac{1}{e^{\frac{E-E_F}{kT}} + 1} \text{ :}[1]$$

For $E - E_F = 0.02 \text{ eV} = 0.02 \times 1.6 \times 10^{-19} \text{ J}$; $T = 200 \text{ K}$:[2]

$$f(E) = 0.24$$

$$f(E) = 24\% \text{ :}[1]$$