Q2. Strain displacement relation.

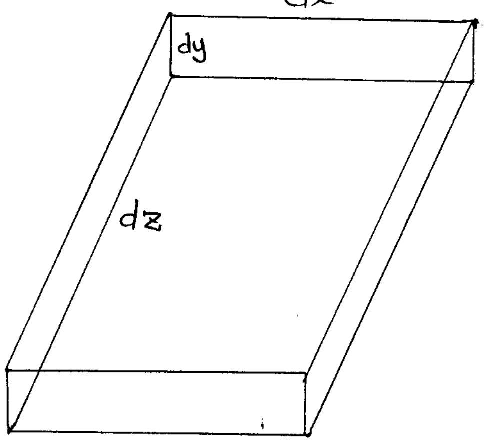

fig-1

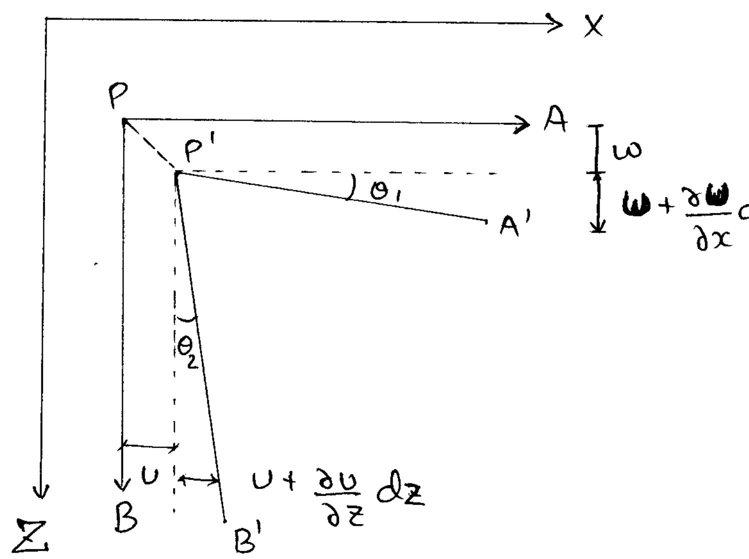

fig 2

Consider a rectangular body of dimension dx, dy & dz along x, y & z axis as shown in fig 1.

After deformation let the displacement be U, V, & ω along x, y & z axis.

So, Displacement in x-direction at point A along x-axis is given as.

$$U + \frac{\partial U}{\partial x} dx$$

$$\varepsilon_x = \frac{\text{change in length}}{\text{Original length}}$$

$$\therefore \quad \varepsilon_x = \frac{\cancel{u} + \frac{\partial u}{\partial x}\cancel{dx} - \cancel{u}}{\cancel{dx}}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad \text{--- (1)}$$

Similarly, $$\varepsilon_y = \frac{\partial v}{\partial y} \quad \text{--- (2)}$$

and $$\varepsilon_z = \frac{\partial w}{\partial z} \quad \text{--- (3)}$$

Above equations are the linear strain displacement equation

After deformation, let the angle of elements PA and PB be θ_1 and θ_2 respectively.

The change in the element is seen after deformation So, the new element after displacement is P'A' and P'B' as shown in the figure 2.

point A is turning towards Z direction and point B is turning towards X-direction.

A in Z direction.

$$= w + \frac{\partial w}{\partial x} dx$$

$$\tan\theta_1 = \frac{\cancel{w} + \frac{\partial w}{\partial x}\cancel{dx} - \cancel{w}}{\cancel{dx}}$$

$$\tan\theta_1 = \frac{\partial w}{\partial x} \quad \Rightarrow \quad \theta_1 = \frac{\partial w}{\partial x}$$

$\therefore$ ~~but~~ $\tan\theta_1 \simeq \theta_1$ ($\because$ Angle is very small)

and B in x- direction,

$$= u + \frac{\partial u}{\partial z} dz$$

$$\tan\theta_2 = \frac{\cancel{u} + \frac{\partial u}{\partial z} \cancel{dz} - \cancel{u}}{\cancel{dz}}$$

$$\tan\theta_2 = \frac{\partial u}{\partial z}$$

$$\theta_2 = \frac{\partial u}{\partial z} \qquad [\because \tan\theta_2 \simeq \theta_2]$$

Addition of θ_1 and θ_2 gives strain displacement relation

i.e,

$$\cancel{\gamma_{xy}}\ \gamma_{zx} = \theta_1 + \theta_2$$

$$\boxed{\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}}$$

Similarly, $\boxed{\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}}$

$$\boxed{\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}}$$

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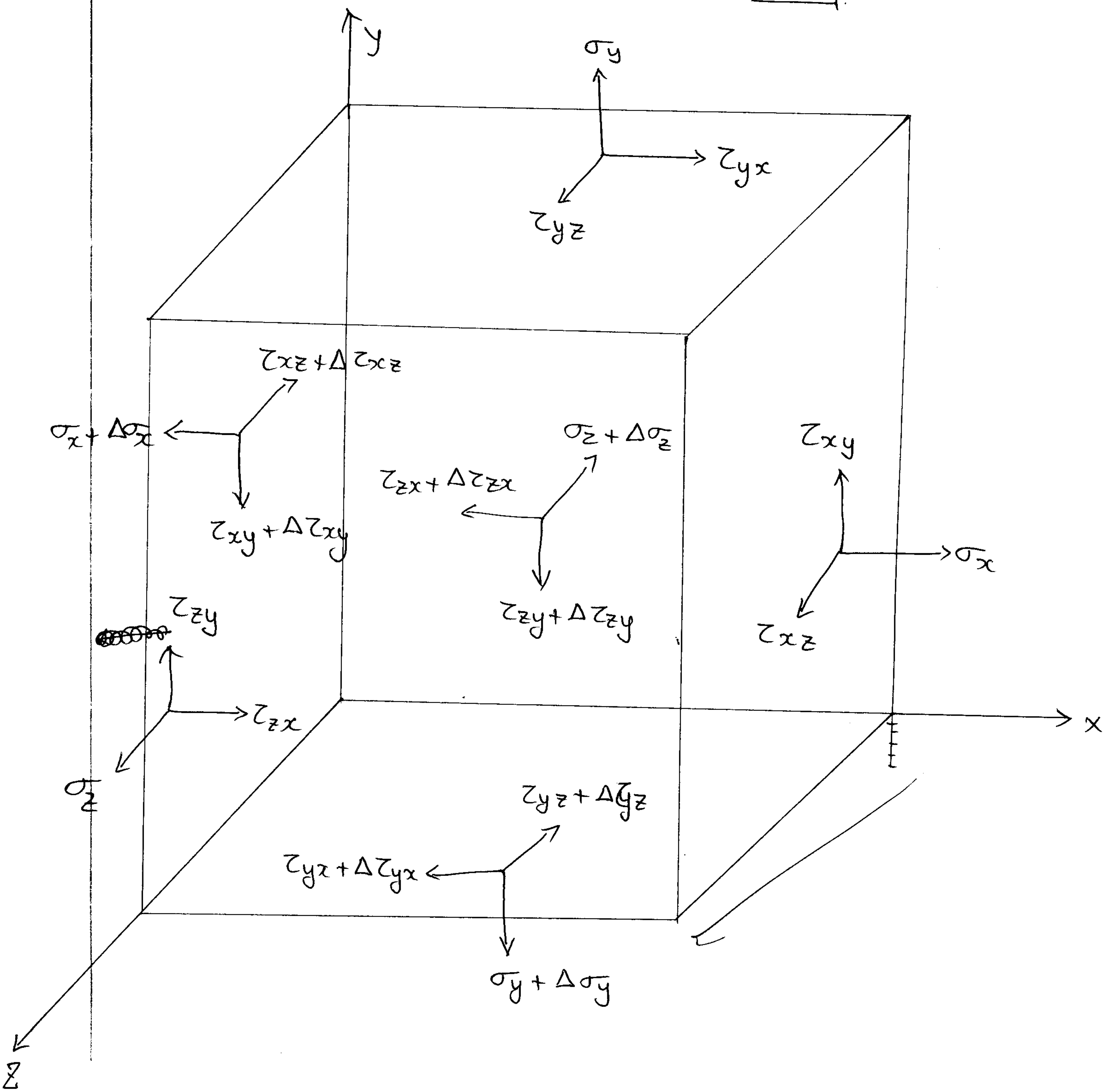
Q 1. Differential equation of equilibrium in Cartesian co-ordinates.



consider a rectangular element of sides Δx, Δy & Δz. let the body forces be x, y & z in x, y & z ~~axis~~ direction respectively. Considering the stress component σ_x, σ_y, & σ_z, τ_{xy}, τ_{xz}, τ_{yz} as linearly varying along its length.

The stress component on each faces are :-

~~for~~ face 1: σ_x, τ_{xy}, τ_{xz}

face 2: $\sigma_x + \frac{\partial \sigma_x}{\partial x}\Delta x$, $\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x}\Delta x$,

$\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x}\Delta x$.

face 3: σ_y, τ_{yx}, τ_{yz}

face 4: $\sigma_y + \frac{\partial \sigma_y}{\partial y}\Delta y$, $\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\Delta y$, $\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y}\Delta y$

face 5: σ_z, τ_{zx}, τ_{zy}

face 6: $\sigma_z + \frac{\partial \sigma_z}{\partial z}\Delta z$, $\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\Delta z$, $\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z}\Delta z$

For equilibrium

$$\Sigma f(x) = 0$$

P.T.O

$$\therefore \quad -\cancel{\sigma_x}\,\Delta z\,\Delta y + \left(\cancel{\sigma_x} + \frac{\partial \sigma_x}{\partial x}\Delta x\right)\Delta z\,\Delta y$$

$$-\cancel{\tau_{yx}}\,\Delta x\,\Delta z + \left(\cancel{\tau_{yx}} + \frac{\partial \tau_{yx}}{\partial y}\Delta y\right)\Delta z\,\Delta x$$

$$-\cancel{\tau_{zx}}\,\Delta x\,\Delta y + \left(\cancel{\tau_{zx}} + \frac{\partial \tau_{zx}}{\partial z}\right)\Delta y\,\Delta x\,+$$

$$X\,\Delta x\,\Delta y\,\Delta z = 0$$

Dividing throughout by $\Delta x \cdot \Delta y \; \Delta z$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + X = 0 \quad \text{——} (1)$$

Similarly,

~~Now~~ $\quad \Sigma f(y) = 0.$

~~Now~~
$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z} + Y = 0 \quad \text{——} (2)$$

& $\quad \Sigma f(z) = 0$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + Z = 0 \quad \text{——} (3)$$

These are the equations of equilibrium in cartesian co-ordinate.

when body forces are absent, the equation is reduces to,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \quad \text{—— (4)}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z} = 0 \quad \text{—— (5)}$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0 \quad \text{—— (6)}$$

In 2-Dimensional, all z compound are 'zero'

$\therefore$ equation of equilibrium in 2-Dimensional in presence of body forces are :-

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + X = 0 \quad \text{—— (i)}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + Y = 0 \quad \text{—— (ii)}$$

In ~~presence~~

When body forces are absent the equation reduces to,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0$$

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Q3. Given data, $u = (4x^2 + 3y^2 + 4z^3) \times 10^{-3}$

$v = (3x^2 + 6y^3 + 4z^2) \times 10^{-3}$

$w = (4x^3 + 8y^2 + 4z^2) \times 10^{-3}$.

$E = 200\ GPa$.

$= 2 \times 10^5\ N/mm^2$.

$\mu = 0.3$

To find strain component at point $(2, 3, -4)$

Solve:-

$$\varepsilon_x = \frac{\partial u}{\partial x} = 8x \times 10^{-3}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = 18y^2 \times 10^{-3}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = 8z \times 10^{-3}$$

&

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = (6y + 6x) \times 10^{-3}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = (8z + 16y) \times 10^{-3}.$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = (12x^2 + 12z^2) \times 10^{-3}$$

P.T.O

$\therefore$ Substituting (x, y, z) as $(2, 3, -4)$ in above equations.

we get,

$$\varepsilon_x = 8x \times 10^{-3} = 0.016$$

$$\varepsilon_y = 18y^2 \times 10^{-3} = 0.162$$

$$\varepsilon_z = 8z \times 10^{-3} = -0.032$$

$$\gamma_{xy} = (6y + 6x) \times 10^{-3} = 0.03$$

$$\gamma_{yz} = (8z + 16y) \times 10^{-3} = 0.016$$

$$\gamma_{zx} = (12x^2 + 12z^2) \times 10^{-3} = -0.144$$

Now, stress strain relation,

$$\left.\begin{aligned} \varepsilon_x &= \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} \\ \varepsilon_y &= \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E} \\ \varepsilon_z &= \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} \end{aligned}\right] \text{—— (1)}$$

Substituting values in equation (1) to obtain σ_x, σ_y & σ_z.

$\therefore$

$$\cancel{0.16} \quad 0.016 = \frac{\sigma_x}{200} - \frac{0.3\sigma_y}{200} - \frac{0.3\sigma_z}{200}$$

$$0.162 = \frac{\sigma_y}{200} - \frac{0.3\sigma_x}{200} - \frac{0.3\sigma_z}{200}$$

$$-0.032 = \frac{\sigma_z}{200} - \frac{0.3\sigma_x}{200} - \frac{0.3\sigma_y}{200}$$

Using Calculator,

$$\sigma_x = 19307.69$$

$$\sigma_y = 41769.23$$

$$\sigma_z = \cancel{11.9}\ 11923.07$$

Now,

$$\cancel{E = 2G(1+\mu)}$$

$$\cancel{G = \frac{E}{2(1+\mu)}}$$

$$\tau = \frac{E \times \gamma}{2(1+\mu)} \qquad \left[\because G = \frac{\tau}{\gamma}\right]$$

$$\therefore \quad \tau_{xy} = \frac{E}{2(1+\mu)} \times \gamma_{xy} = \frac{2\times10^5}{2(1+0.3)} \times 0.03 = \underline{2307.69\ N/mm^2}$$

$$\tau_{yz} = \frac{E}{2(1+\mu)} \times \gamma_{yz} = \frac{2\times10^5}{2(1+0.3)} \times 0.016 = \underline{1230.769\ N/mm^2}$$

$$\tau_{zx} = \frac{E}{2(1+\mu)} \times \gamma_{zx} = \frac{2\times10^5}{2(1+0.3)} \times (-0.144) = \underline{-11076.92}\ N/mm^2$$