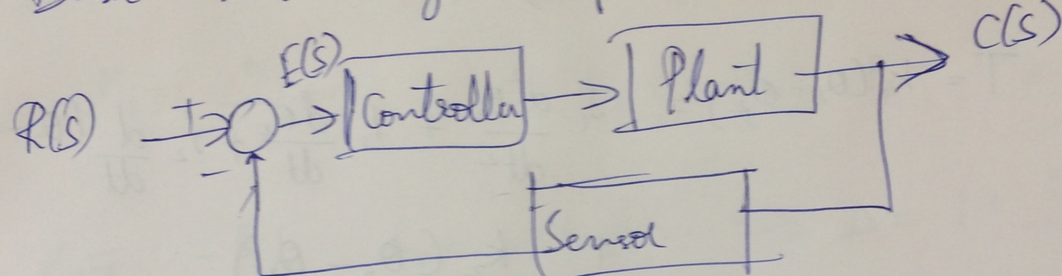
Solutions for 15EC43-(ECE-A and TCE-A)

April 10, 2017

1a.) A control system is an electro-mechanical system with the objective of regulating/controlling a physical quantity.

Basic block diagram of a control system



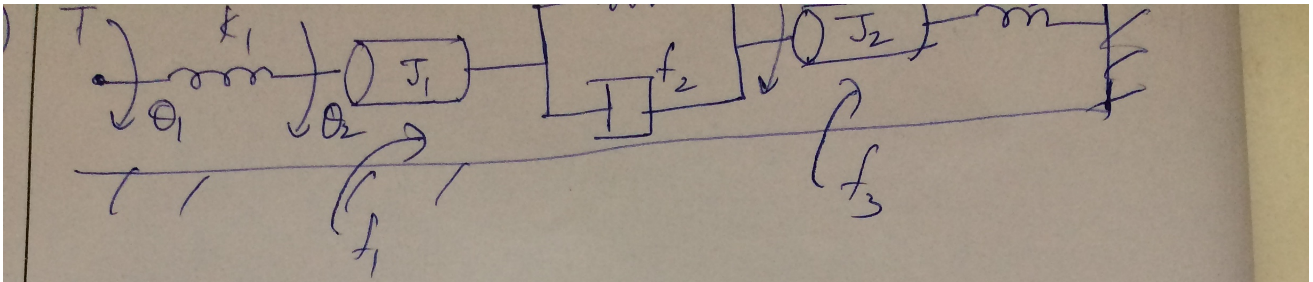
Reference [R(s)] :- The desired ~~state~~ state of the physical quantity.

Controlled quantity C(s) :- The actual state of the physical quantity.

Plant :- The device that affects the physical quantity being controlled.

Sensor :- Senses the physical quantity so that can be compared with the reference.

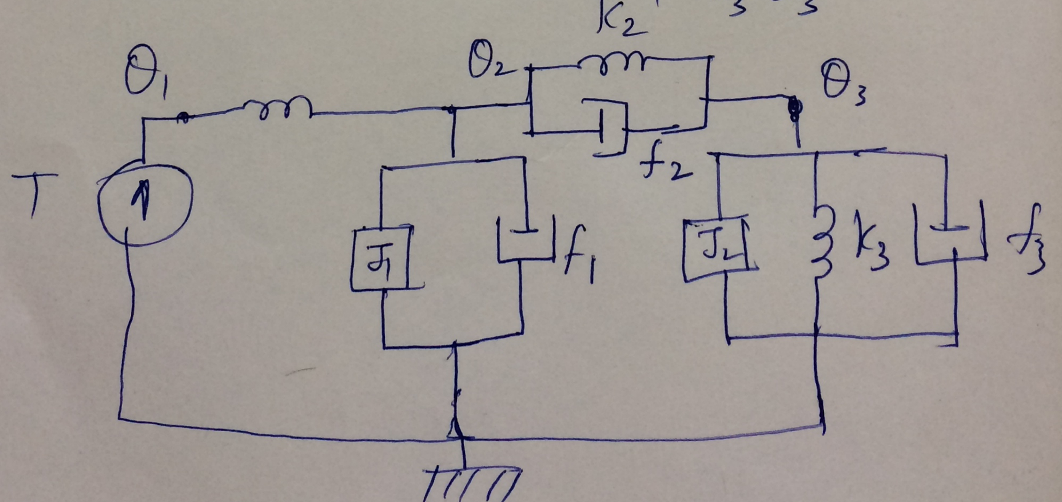
Controller :- Electrical/Electronic system that the plant ~~~~ to the desired state.

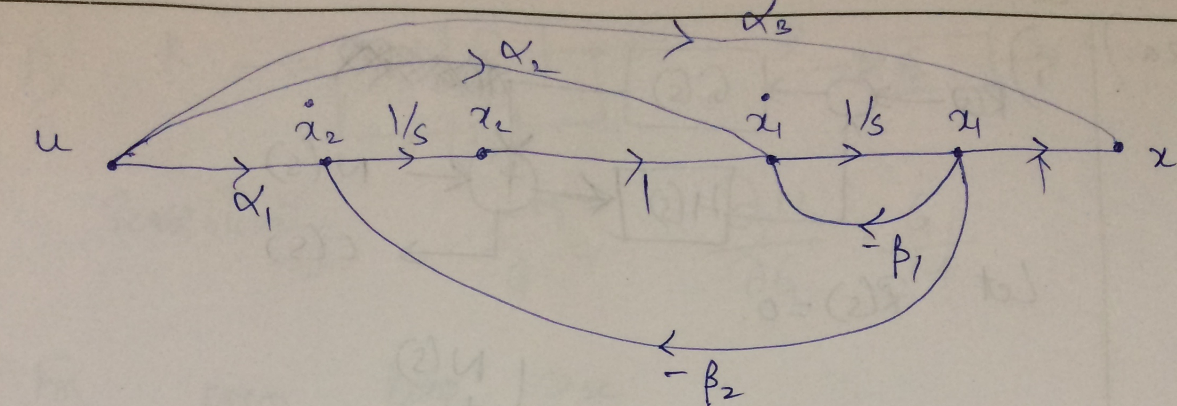


$$T = k_1(\theta_1 - \theta_2)$$

$$T = k_1(\theta_1 - \theta_2) = J_1 \frac{d^2\theta_2}{dt^2} + f_1 \frac{d\theta_2}{dt} + f_2 \frac{d}{dt}(\theta_2 - \theta_3) + k_2(\theta_2 - \theta_3).$$

$$f_2 \frac{d}{dt}(\theta_2 - \theta_3) + k_2(\theta_2 - \theta_3) = J_2 \frac{d^2\theta_3}{dt^2} + f_3 \frac{d\theta_3}{dt} + k_3\theta_3$$





$$P_1 = \frac{\alpha_1}{s^2}, \quad P_2 = \frac{\alpha_2}{s}, \quad P_3 = \alpha_3.$$

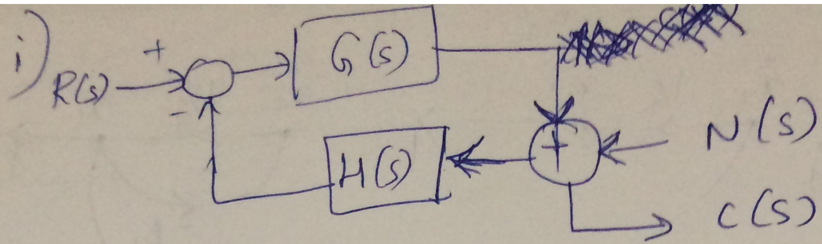
$$L_1 = -\beta_1/s, \quad L_2 = -\beta_2/s^2 \implies \Delta = 1 + \frac{\beta_1}{s} + \frac{\beta_2}{s^2}$$

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

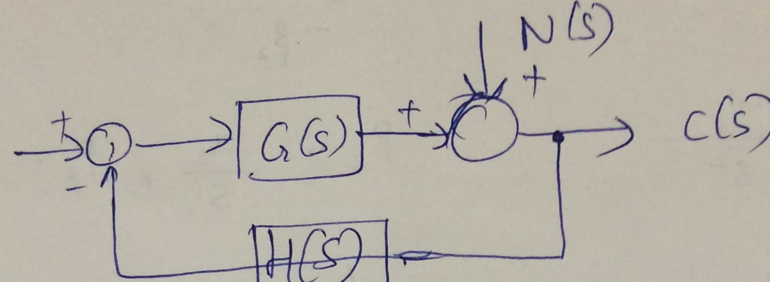
$$T = \frac{\alpha_1/s^2 + \alpha_2/s + \alpha_3\left\{1 + \frac{\beta_1}{s} + \frac{\beta_2}{s^2}\right\}}{1 + \frac{\beta_1}{s} + \frac{\beta_2}{s^2}}$$

$$T = \frac{\alpha_1 + \alpha_2 s + \alpha_3\left(s^2 + \beta_1 s + \beta_2\right)}{s^2 + \beta_1 s + \beta_2}$$

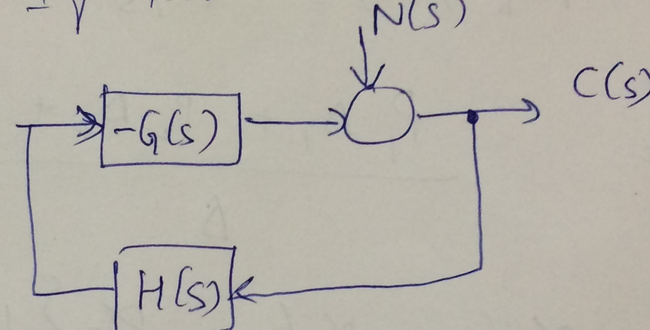
2a.) i)



Let $R(s) = 0$.



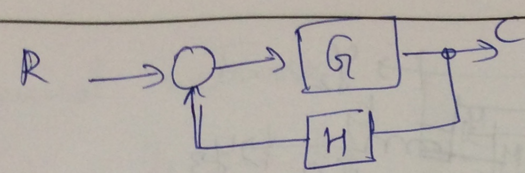
If $R(s) = 0$

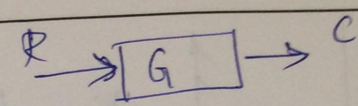


$$\frac{C(s)}{N(s)} = \frac{1}{1 + GH}$$

If there was no feed back then $\frac{C(s)}{N(s)} = 1$. Thus noise can be supressed if GH is chose wisely.

ii)





Sensitivity $S_G^M = \frac{\partial M}{\partial G} \times \frac{G}{M}$

For open loop case

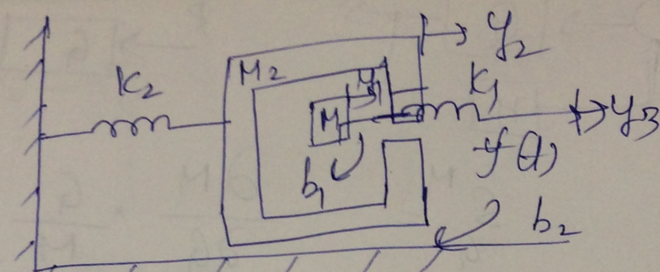
$$S_G^M = 1$$

For closed loop case

$$S_G^M = \frac{1}{1+GH}.$$

Again if GH is chosen wisely the overall system can be made insensitive to the changes in 'G'.

2b.)



ii)

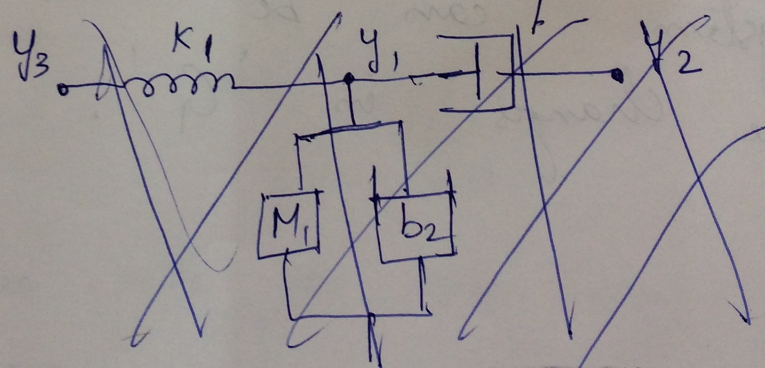
$$f(t) = k_1 (y_3 - y_1).$$

$$f(t) = k_1(y_3 - y_1) = M_1 \frac{d^2 y_1}{dt^2} + b_1 \frac{d}{dt}(y_1 - y_2)$$

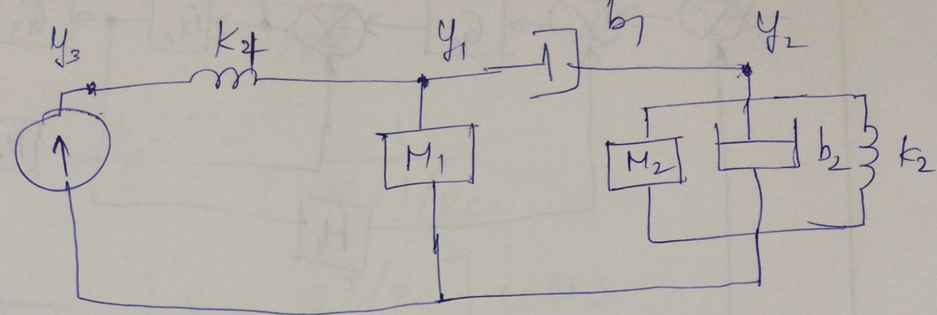
$$\cancel{b_2 \frac{dy}{dt}}$$

$$b_1 \frac{d}{dt}(y_1 - y_2) = M_2 \frac{d^2 y_2}{dt^2} + b_2 \frac{dy_2}{dt} + k_2 y_2 .$$

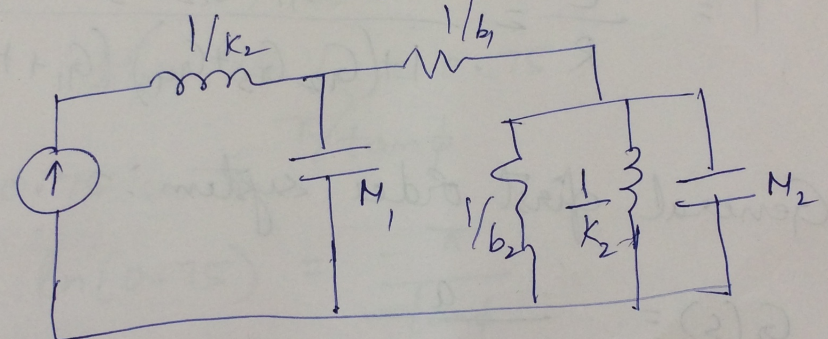
i)



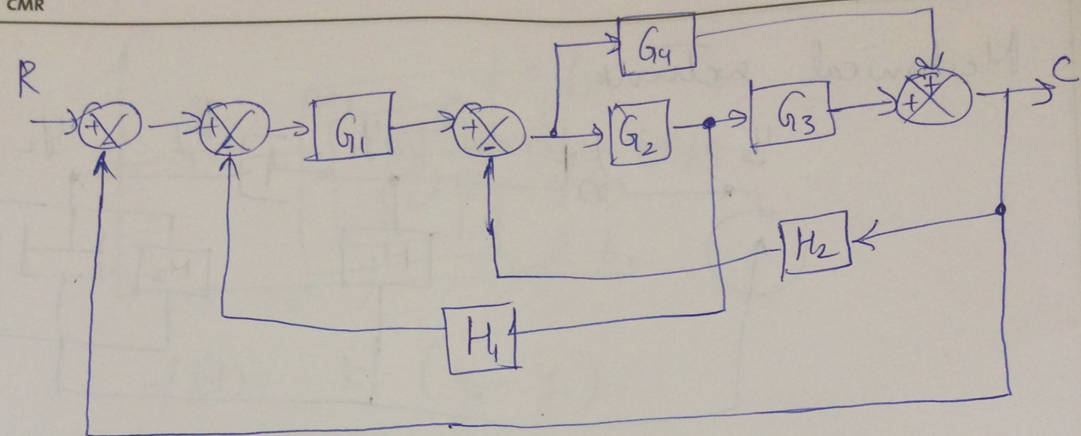
i) Mechanical network



ii) Force - Current analogy.



2c.)

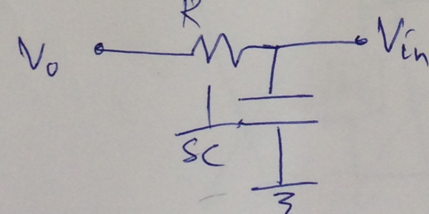


$$T = \frac{C}{R} = \frac{G_1(G_2 G_3 + G_4)}{1+(G_2 G_3 + G_4)(G_1 + H_2) + G_1 H_1 G_2}$$

3 a.) General first order system :-

$$G(s) = \frac{a}{s+a}$$

RC network



$$\frac{V_o(s)}{V_{in}(s)} = \frac{1/RC}{s + 1/RC}$$

$$V_0(s) = \frac{1}{s} \times \frac{1/RC}{s+1/RC}$$

$$V_0(s) = \frac{1}{s} - \frac{1}{s+1/RC}$$

$$v_0(t) = \left[1 - e^{-t/RC}\right] u(t).$$

b) Given $G(s) = \dfrac{K}{s(s+\alpha)}$.

i) Let $M_p = 75\% = 0.75$

$$0.75 = e^{-\pi/\tan\phi}$$

$$\ln(0.75) = \frac{-\pi}{\tan\phi}$$

$$0.2877 = \frac{\pi}{\tan\phi}$$

$$\zeta = \cos(\phi) = 0.0912$$

$$\zeta = 0.0912.$$

$$T(s) = \frac{k}{s^2+\alpha s+k}, \quad \omega_n = \sqrt{k}, \quad \zeta = \frac{\alpha}{2\sqrt{k}}.$$

$$\frac{\alpha}{2\sqrt{k}} = 0.0912.$$

CMR

Let $M_r = 0.25$ for modified k.

$$\ln(0.25) = \frac{-\pi}{\tan\phi_{new}}$$

$$\tan\phi_{new} = \frac{\pi}{1.3863}$$

$$\zeta_{new} = 0.4037$$

$$\zeta_{new} = \frac{\alpha}{2\sqrt{k_{new}}} = 0.4037.$$

$$\frac{\alpha/2\sqrt{k}}{\alpha/2\sqrt{k_{new}}} = \frac{0.0912}{0.4037}$$

$$\sqrt{\frac{k_{new}}{k}} = 0.2259.$$

$$k_{new} = 0.051\,k$$

$\zeta_{required} = 0.6$, $\omega_d = 8$ rad/sec.

$$\zeta = 0.6 = \frac{\alpha}{2\sqrt{K}}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 8$$

$$\omega_n = \frac{8}{\sqrt{1-\zeta^2}} = 10 \text{ rad/sec}$$

$$\boxed{K = \omega_n^2 = 100.}$$

$$\therefore \quad \alpha = 0.6 \times 2\sqrt{K}$$

$$\boxed{\alpha = 12}$$

The peak value of the response when the system is excited by 2 V is

$$C(t_p) = 2\left[1 - \frac{e^{-\zeta\omega_n t_p}}{\sqrt{1-\zeta^2}}\sin(\omega_d t_p + \phi)\right]$$

$$= 2\left[1 + \frac{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}{\cancel{\sqrt{1-\zeta^2}}}\cancel{\sin\phi}\right]$$

$$C(t_p) = 2\left[1 + e^{-\pi/\tan\phi}\right] = 2\left[1 + e^{-\pi/1.333}\right]$$

$$C(t_p) = 2.1896 \text{ V.}$$

3c.) $GH(s) = \frac{100}{s^3(s^2+8s+100)}$

i) Type of the system - Type 3

ii) Error constants

$$K_p = \lim_{s\to 0} G(s) = \infty$$

$$K_v = \lim_{s\to 0} sG(s) = \infty$$

$$K_a = \lim_{s\to 0} s^2 G(s) = \infty$$

iii) Steady state error for an input

$r(t) = 2 + t + 2t^2$.

$$e_{ss} = \lim_{s\to 0} \frac{s \times R(s)}{1+G(s)}$$

$$= \lim_{s\to 0} \frac{\cancel{s} \times \left\{ \frac{2}{\cancel{s}} + \frac{1}{s^{\cancel{2}}} + \frac{4}{s^{2}} \right\}}{1 + \frac{100}{s^3(s^2+8s+100)}}$$

$$= \lim_{s\to 0} \frac{(2s^2 + s + 4)}{s^2 + \frac{100}{s(s^2+8s+100)}}$$

$$= \frac{4}{\lim_{s \to 0} \frac{100}{s(s^2+8s+100)}} = 0.$$

$$\boxed{e_{ss} = 0}.$$

4a) Impulse signal :-

$$\delta(t) = \begin{cases} \neq 0 & t = 0 \\ = 0 & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

Step signal :-

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0. \end{cases}$$

4b) $G(s) = \dfrac{50}{s(s+5)}$, $\omega_n^2 = 50$, $\zeta = \dfrac{5}{2\sqrt{50}}$

$\omega_n = 7.071$ rad/s, $\zeta = 0.3536$

t_p (peak Time) $= \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

$= \dfrac{\pi}{6.6143} = 0.475$ sec

$c(t_p) = \left[1 + e^{-\pi/\tan\phi}\right]$

$= \left[1 + e^{-\pi/2.6454}\right]$

$\boxed{c(t_p) = 1.3050}$

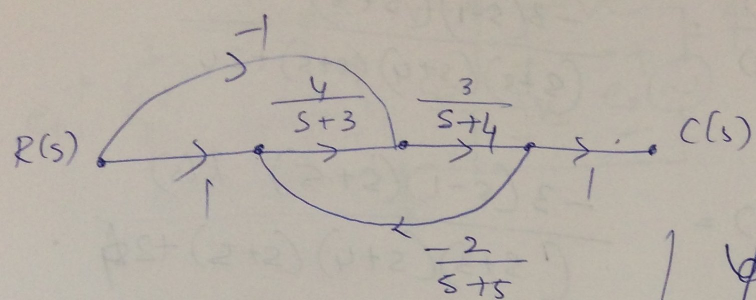
Percentage overshoot $= 30.5\%$

Settling Time $t_s = \dfrac{4}{\zeta\omega_n}$ (for 2% tolerance)

$= \dfrac{4}{0.3536 \times 7.071}$

$\boxed{t_s = 1.5998 \text{ sec}}$

4c.)



Given $e(t) = r(t) - y(t)$

$Y(s) = T(s)\,R(s)$

$E(s) = R(s) - Y(s)$

$P_1 = \frac{12}{(s+3)(s+4)}$, $\Delta_1 = 1$

$E(s) = R(s) - T(s)R(s)$

$P_2 = \frac{-3}{(s+4)}$, $\Delta_2 = 1$

$E(s) = R(s)[1 - T(s)]$

$e_{ss} = \lim_{t \to \infty} e(t)$

$\Delta = 1 + \frac{24}{(s+3)(s+4)(s+5)}$

$= \lim_{s \to 0} sE(s)$

$= \lim_{s \to 0} sR(s)[1 - T(s)]$

$$T = \frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{\{12 - 3(s+3)\} / (s+3)(s+4)}{\{(s+3)(s+4)(s+5) + 24\} / \cancel{(s+3)}\,\cancel{(s+4)}\,(s+5)}$$

$$T = \frac{(-3s+3)(s+5)}{(s+3)(s+4)(s+5) + 24} = \frac{-3(s-1)(s+5)}{(s+3)(s+4)(s+5) + 24}$$

CMR

$$\frac{Y(s)}{R(s)} = \frac{-3(s-1)(s+5)}{(s+3)(s+4)(s+5)+24}$$

$$Y(s) = \frac{-3(s-1)(s+5)\ R(s)}{(s+3)(s+4)(s+5)+24}.$$

$$E(s) = R(s) - Y(s)$$

$$= \frac{1}{s}\left[1 + \frac{3(s-1)(s+5)}{(s+3)(s+4)(s+5)+24}\right]$$

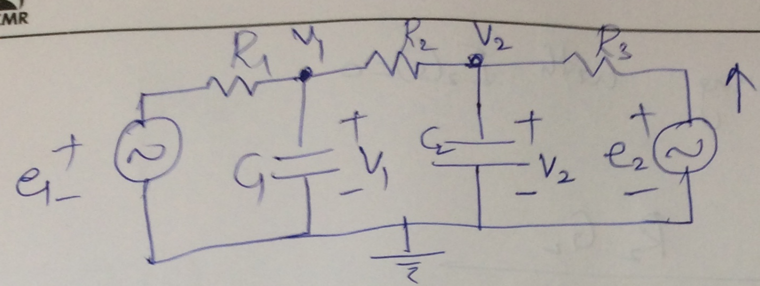
Assuming boundedness.

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = e_{ss}$$

$$e_{ss} = \lim_{s\to 0}\left[1 + \frac{3(s-1)(s+5)}{(s+3)(s+4)(s+5)+24}\right]$$

$$= \left[1 - \frac{15}{84}\right] = 0.824$$

It is a type-0, 3rd order system.



$$\frac{V_1 - e_1}{R_1} + \frac{V_1 - V_2}{R_2} + C_1 \frac{dv_1}{dt} = 0 \longrightarrow (1)$$

$$\frac{V_2 - V_1}{R_2} + C_2 \frac{dv_2}{dt} + \frac{V_2 - e_2}{R_3} = 0 \longrightarrow (2)$$

$$V_1 \left\{ \frac{1}{R_1} + \frac{1}{R_2} \right\} + C_1 \frac{dv_1}{dt} - \frac{e_1}{R_1} = \frac{V_2}{R_2}$$

$$V_1(s) \left[sC_1 + \frac{1}{R_1} + \frac{1}{R_2} \right] - \frac{E_1(s)}{R_1} = \frac{V_2(s)}{R_2}.$$

$$V_2(s) \left[sC_2 + \frac{1}{R_2} + \frac{1}{R_3} \right] - \frac{E_2(s)}{R_3} = \frac{V_1(s)}{R_2}$$

Let $G_2 = \left[sC_2 + \frac{1}{R_2} + \frac{1}{R_3} \right]$, $G_1 = \left[sC_1 + \frac{1}{R_1} + \frac{1}{R_3} \right]$

Upon Solving with $E_2(s)=0$

$$\frac{V_1(s)}{E_1(s)} = \frac{R_2 G_2}{R_1 R_2 G_1 G_2 - 1}$$

$$= \frac{R_2 \left\{ sC_2 + \frac{1}{R_2} + \frac{1}{R_3} \right\}}{R_1 R_2 G_1 G_2 - 1}$$

$$\left.\frac{V_1(s)}{E_1(s)}\right|_{E_2=0} = \frac{sC_2 R_2 R_3 + R_2 + R_3}{(sC_1 R_1 \cancel{R_2} + R_1 + R_2)(sC_2 R_1 R_2 + R_2 + R_3) - R_1 R_3}$$

$$\left.\frac{V_2(s)}{E_2(s)}\right|_{E_1=0} = \frac{R_1 R_2^2 (sC_1 R_1 R_2 + R_1 + R_2)}{(sC_1 R_1 R_2 + R_1 + R_2)(sC_2 R_2 R_3 + R_2 + R_3) - R_1 R_2^2 R_3}$$

6)
$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

$$T(s) = \frac{G}{1+G} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$T(s) = \frac{C(s)}{R(s)}$$

$$C(s) = T(s)R(s)$$

$$= \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$$

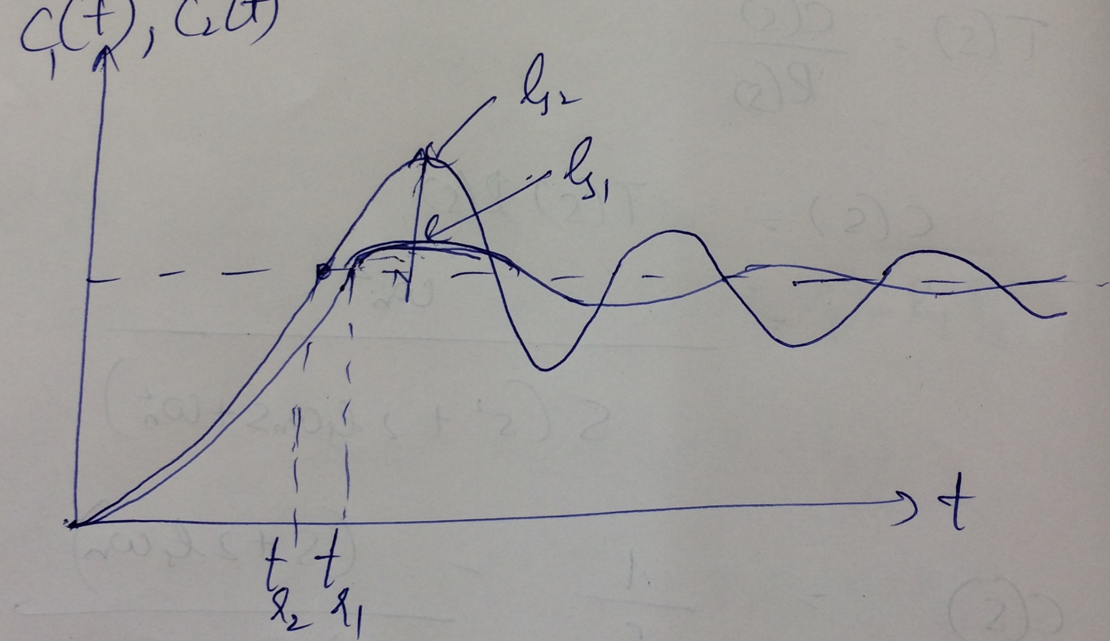
$$C(s) = \frac{1}{s} - \frac{(s+2\zeta\omega_n)}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$$c(t) = \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi) \right]$$

where $\omega_d = \omega_n \sqrt{1-\xi^2}$

$\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$.



$t_{r_1} > t_{r_2}$, $M_{r_1} < M_{r_2}$ for $\xi_1 > \xi_2$.