CMR Institute of Technology

IAT - 1

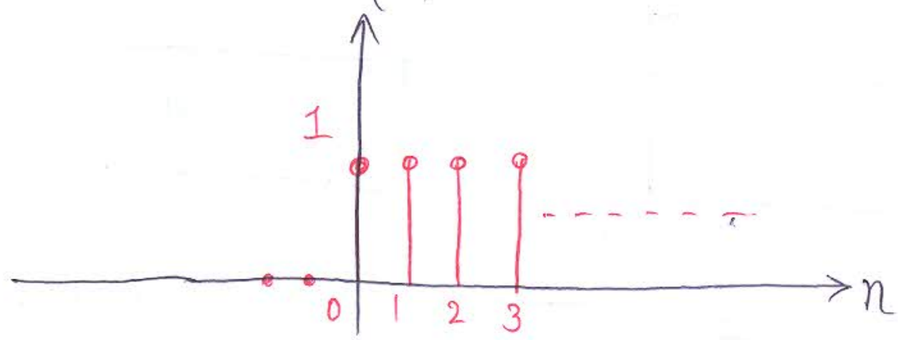
Signals & Systems (15EC44)

1. a) Check whether the following signals are periodic or not. If they are periodic, find their fundamental period.

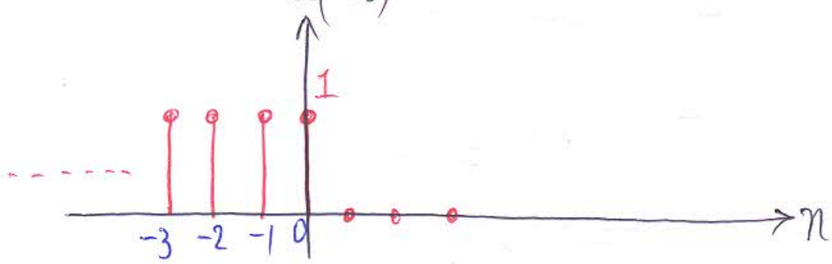
i) $x(n) = u(n) + u(-n)$

$$u(n) = \begin{cases} 1 & , \quad n \geq 0 \\ 0 & , \quad n < 0 \end{cases}$$

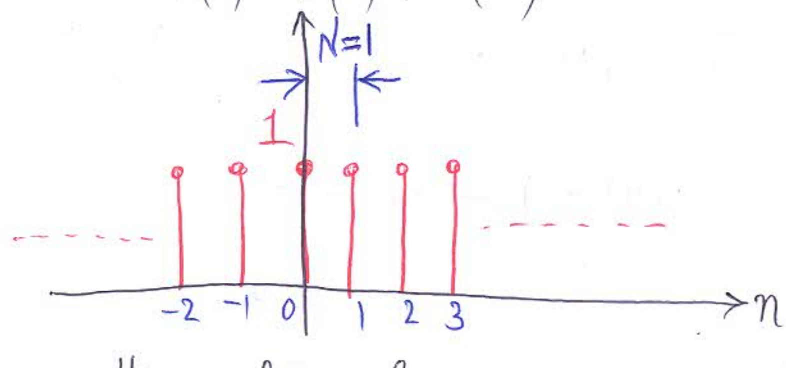
$u(n)$



$u(-n)$



$x(n) = u(n) + u(-n)$



$\boxed{N = 1}$

From the above figure, we can observe that sequence $x(n)$ repeats for every 1 sample.

ii) $x(t) = \left[2\cos^2\left(\frac{\pi t}{2}\right) - 1\right] \sin\pi t \cos\pi t$

Sol

$$x(t) = \left[\cos 2\left(\frac{\pi t}{2}\right)\right] \frac{1}{2} \sin(2\pi t)$$

$$x(t) = \frac{1}{2}(\cos\pi t) \sin(2\pi t)$$

$$x(t) = \frac{1}{2}\left\{\frac{1}{2}\left[\sin(3\pi t) - \sin(-\pi t)\right]\right\}$$

$$x(t) = \frac{1}{4}\sin(3\pi t) + \frac{1}{4}\sin(\pi t)$$

$\omega_1 = 3\pi$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{3\pi}$

$\boxed{T_1 = \frac{2}{3} \text{ sec}}$

$\omega_2 = \pi$

$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\pi}$

$\boxed{T_2 = 2 \text{ sec}}$

$$\frac{T_2}{T_1} = \frac{2}{2/3} = \frac{6}{2} = \frac{3}{1} \text{ (rational no)}$$

$$T = 3T_1 = T_2$$

$$T = 3 \times \frac{2}{3} = 2$$

$$\boxed{T = 2 \text{ sec}}$$

$\therefore$ The given signal $x(t)$ is periodic.

iii) $x(t) = 2\cos(10t+1) - \sin(4t-1)$

$\omega_1 = 10$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{10}$

$\omega_2 = 4$

$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{4}$

$\boxed{T_1 = \frac{\pi}{5} \text{ sec}}$ $\boxed{T_2 = \frac{\pi}{2} \text{ sec}}$

$$\frac{T_2}{T_1} = \frac{\pi/2}{\pi/5} = \frac{5}{2} \left(\text{rational no}\right)$$

$$T = 2T_2 = 5T_1$$

$$T = 2 \times \frac{\pi}{2} = 5 \times \frac{\pi}{5}$$

$$\boxed{T = \pi \text{ sec}}$$

Hence $x(t)$ is periodic signal

iv) $x(t) = 2e^{j16\pi t} + 3e^{-j7\pi t}$

Sol

$\omega_1 = 16\pi$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{16\pi}$

$T_1 = \frac{1}{8}$ sec

$\omega_2 = 7\pi$

$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{7\pi}$

$T_2 = \frac{2}{7}$ sec

$$\frac{T_2}{T_1} = \frac{2/7}{1/8} = \frac{2}{7} \times 8$$

$$\frac{T_2}{T_1} = \frac{16}{7} \left(\text{rational no}\right)$$

$$T = 7T_2 = 16T_1$$

$$T = 7 \times \frac{2}{7} = 16 \times \frac{1}{8}$$

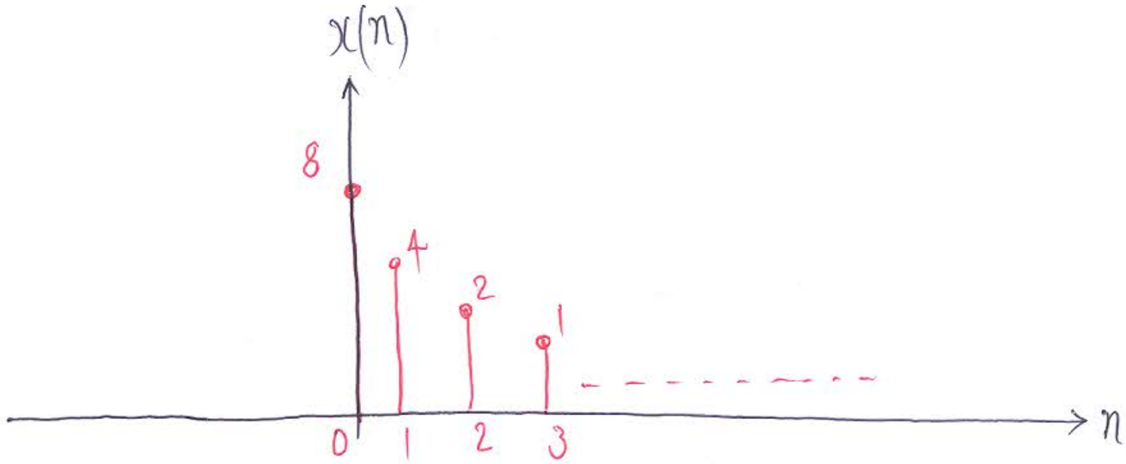
$$\boxed{T = 2 \text{ sec}}$$

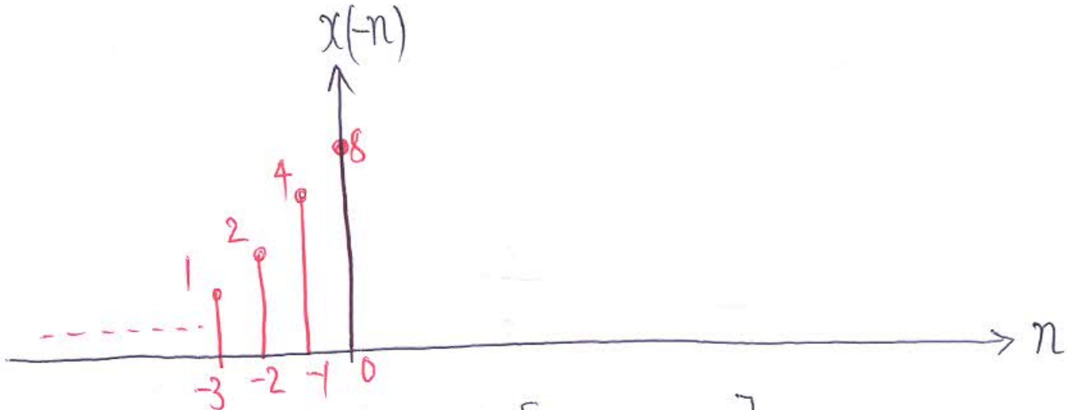
Hence the signal $x(t)$ is periodic signal

2) a) Sketch the even & odd components of the following signals.

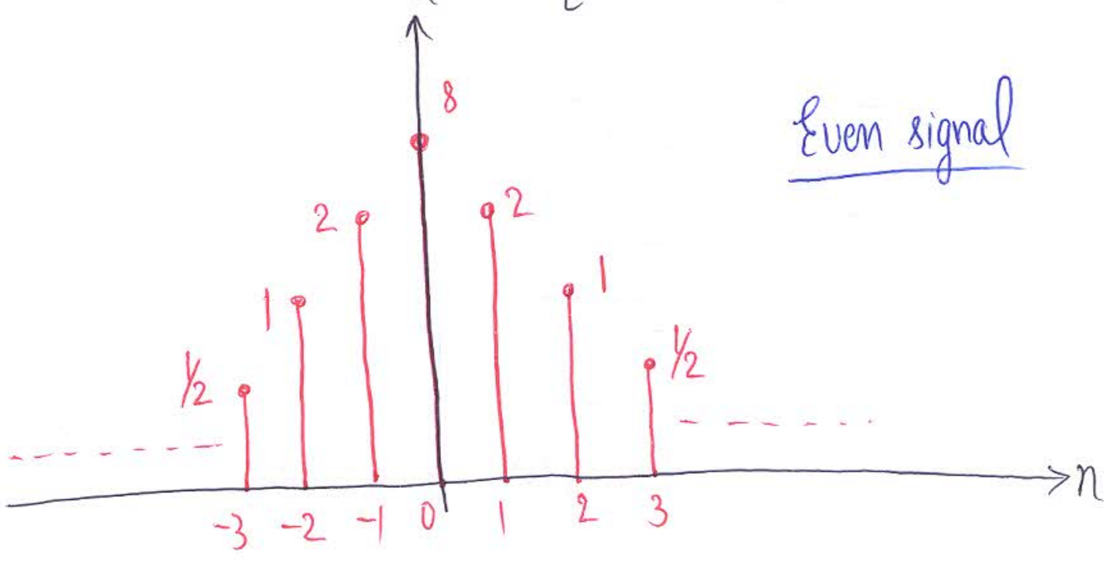
(i) $x(n) = 8\,(0.5)^n\,u(n)$

$$x(n) = \begin{cases} 8(0.5)^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



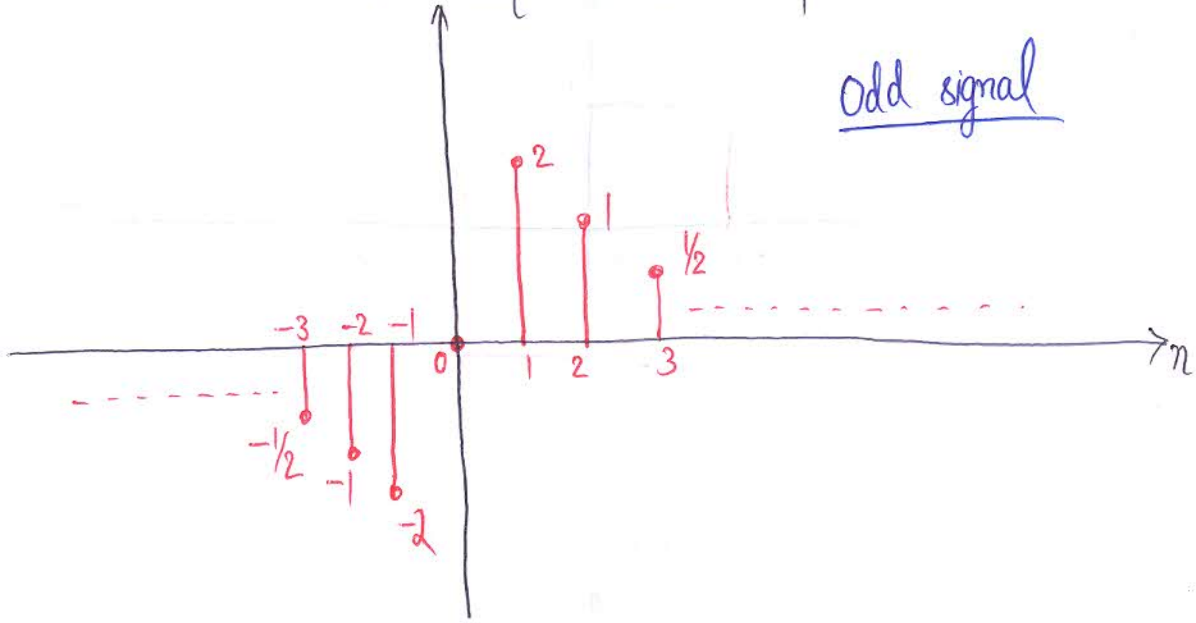


$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$

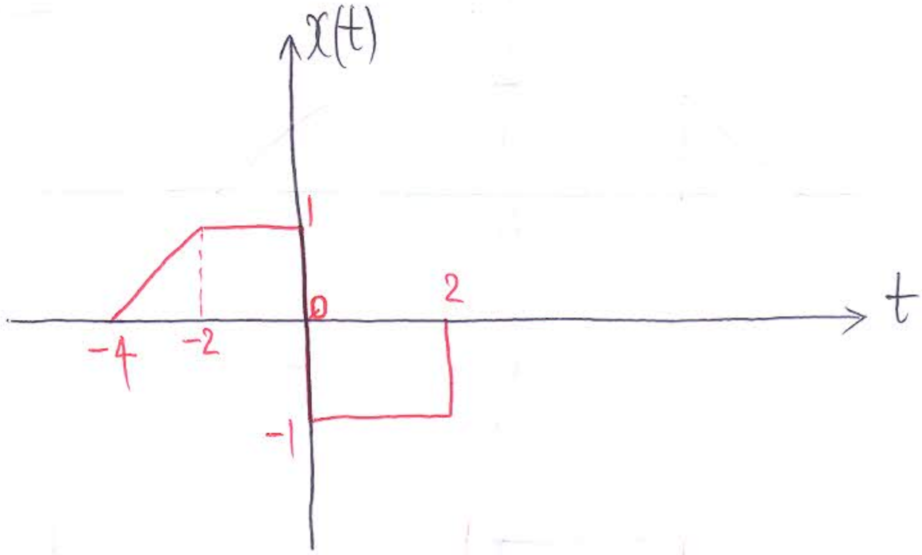


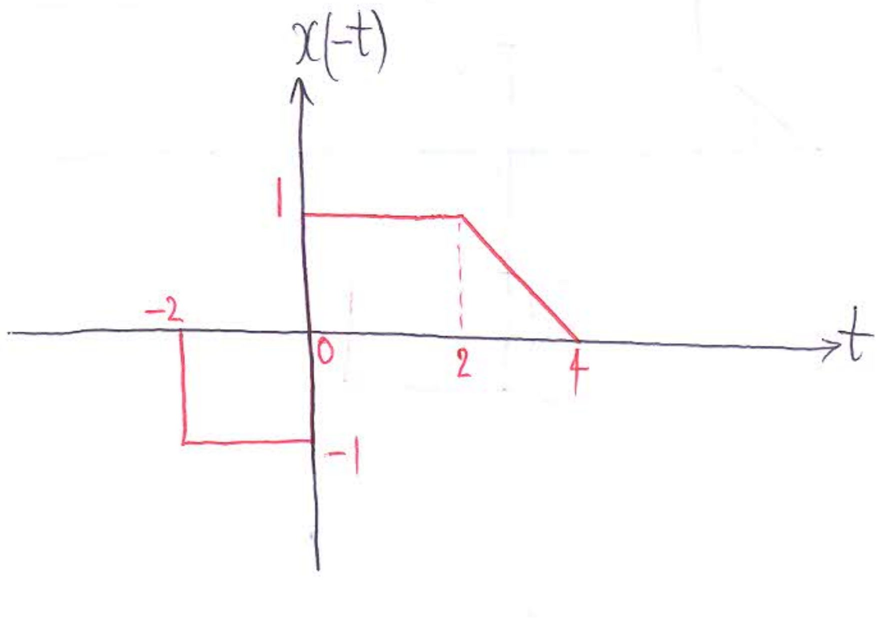
Even signal

$$x_o(n) = \frac{1}{2}\left\{x(n) + \left[-x(-n)\right]\right\}$$

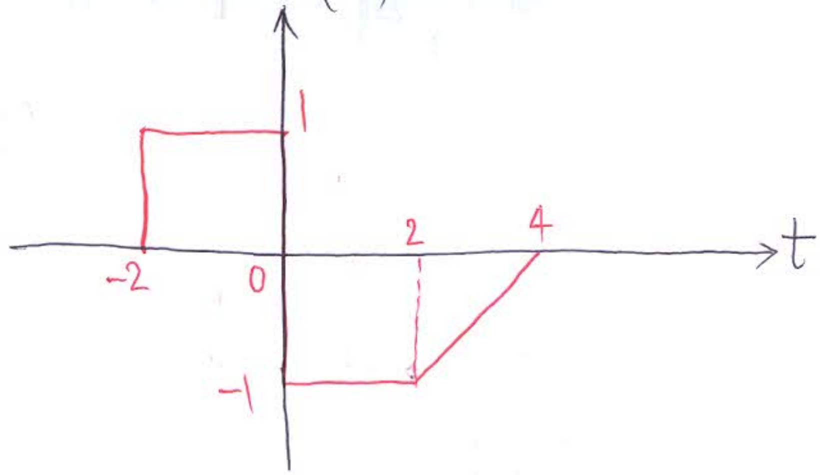


(ii)

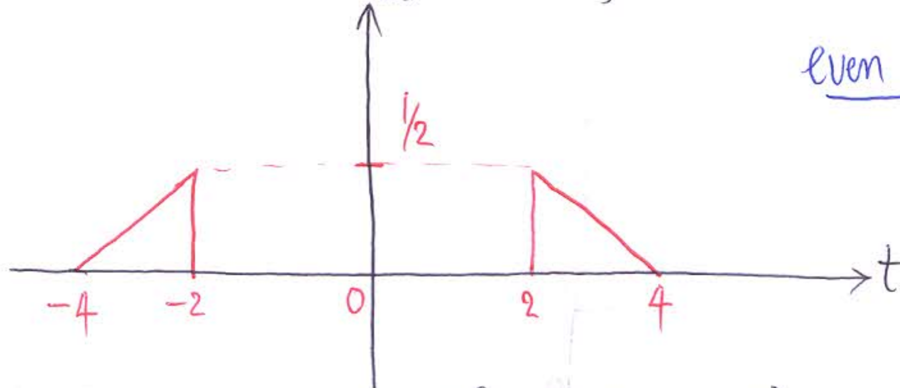




$-x(-t)$

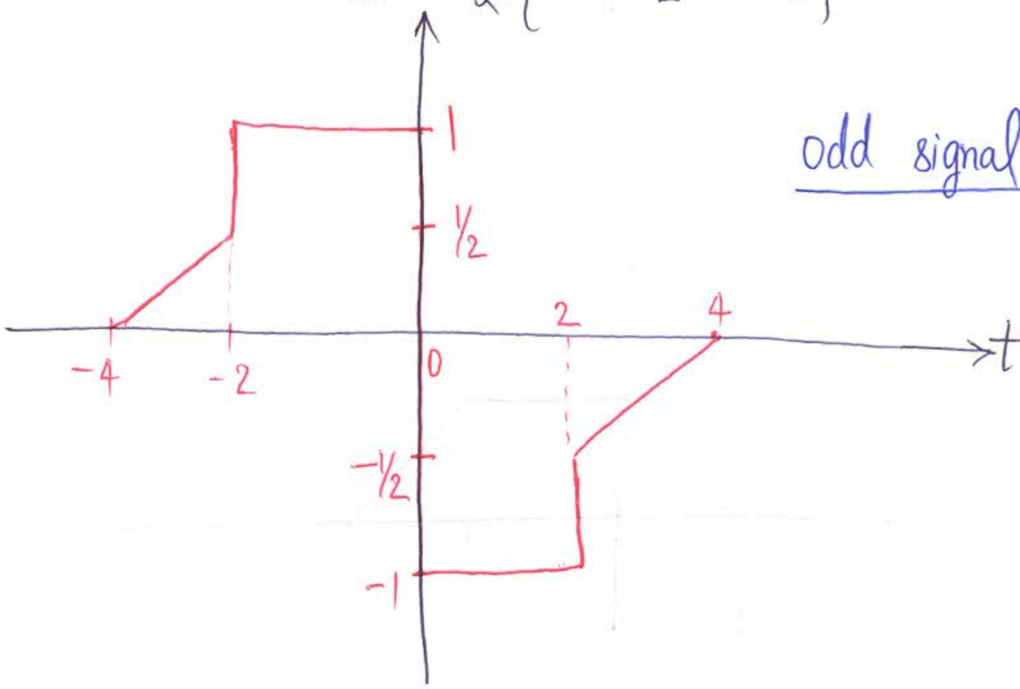


$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$



even signal

$$x_o(t) = \frac{1}{2}\left\{x(t) + \left[-x(-t)\right]\right\}$$



odd signal

6) Prove that

i) If two signals are even, then their product is even

ii) If two signals are odd, then their product is even

iii) If one signal is even signal & the other is odd then their product is odd

iv) If one signal is odd & the other is even, then their product is odd.

Sol:- let $x_1(t)$ & $x_2(t)$ be two CT signals then their product is given by

$$y(t) = x_1(t) \cdot x_2(t) \longrightarrow (1)$$

i) If $x_1(t) \to$ even & $x_2(t) \to$ even

$$x_1(-t) = x_1(t)$$
$$x_2(-t) = x_2(t)$$
$$y(-t) = x_1(-t) \cdot x_2(-t)$$
$$y(-t) = x_1(t) \cdot x_2(t)$$
$$y(-t) = y(t)$$

Hence the product is also even signal

ii) If $x_1(t) \to$ odd & $x_2(t) \to$ odd

$$x_1(-t) = -x_1(t)$$
$$x_2(-t) = -x_2(t)$$
$$y(-t) = x_1(-t) \cdot x_2(-t)$$
$$y(-t) = \{-x_1(t)\}\{-x_2(t)\}$$
$$y(-t) = x_1(t) \cdot x_2(t)$$
$$y(-t) = y(t)$$

Hence the product is also even signal

iii) If $x_1(t) \rightarrow$ even & $x_2(t) \rightarrow$ odd

$$x_1(-t) = x_1(t) \quad \&$$
$$x_2(-t) = -x_2(t)$$
$$y(-t) = x_1(-t)\, x_2(-t)$$
$$y(-t) = x_1(t) \{-x_2(t)\}$$
$$y(-t) = -x_1(t) \cdot x_2(t)$$
$$y(-t) = -y(t)$$

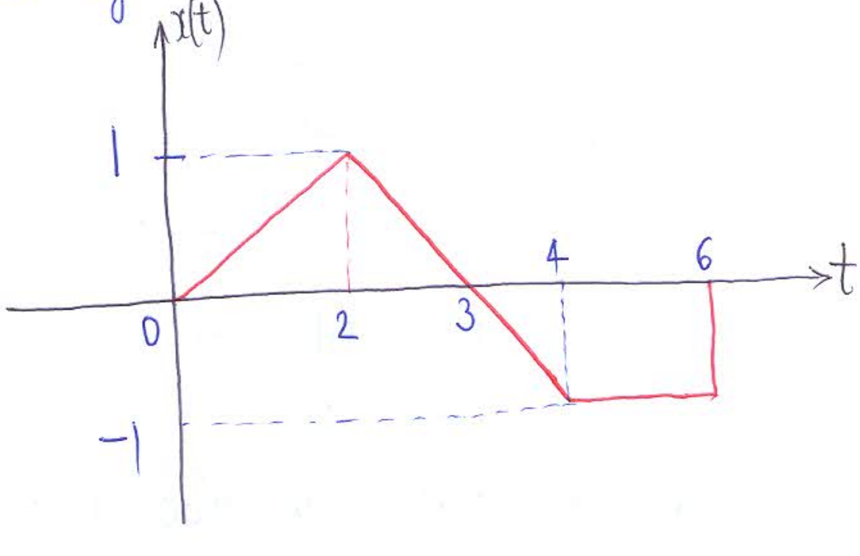
Hence the product is odd signal

iv) If $x_1(t) \rightarrow$ odd & $x_2(t) \rightarrow$ even

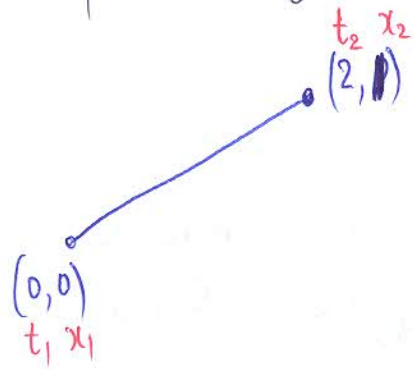
$$x_1(-t) = -x_1(t)$$
$$x_2(-t) = x_2(t)$$
$$y(-t) = x_1(-t) \cdot x_2(-t)$$
$$y(-t) = \{-x_1(t)\} \cdot x_2(t)$$
$$y(-t) = -x_1(t) \cdot x_2(t)$$
$$y(-t) = -y(t)$$

Hence the product is odd signal.

3. a) Determine whether the signal $x(t)$ shown below is an energy / power signal.

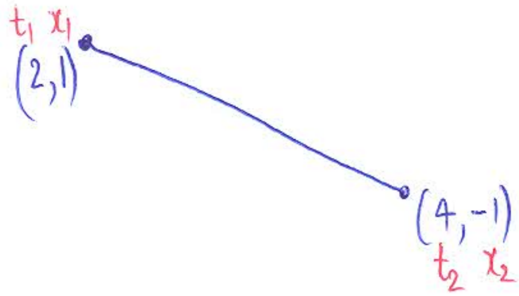


Sol $\therefore$ The given signal $x(t)$ is a finite duration aperiodic signal. Hence it is a "ENERGY SIGNAL"



$$x - x_1 = \frac{x_2 - x_1}{t_2 - t_1}(t - t_1)$$

$$\boxed{x = \frac{1}{2}t} \quad 0 \le t \le 2$$



$$x - 1 = \frac{-1-1}{4-2}(t-2)$$

$$x - 1 = \frac{-2}{2}(t-2)$$

$$x - 1 = 2 - t$$

$$\boxed{x = 3 - t} \quad 2 \le t \le 4$$

$$\therefore \quad x(t) = \begin{cases} t/2, & 0 \le t \le 2 \\ 3-t, & 2 \le t \le 4 \\ -1, & 4 \le t \le 6 \end{cases}$$

Energy,
$$E = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = \int_0^2 \frac{t^2}{4} dt + \int_2^4 (3-t)^2 dt + \int_4^6 (-1)^2 dt$$

$$E = \frac{1}{4}\left[\frac{t^3}{3}\right]_0^2 + \int_2^4 (9 + t^2 - 6t)\, dt + \int_4^6 dt$$

$$E = \frac{1}{12}[8] + \left[9t + \frac{t^3}{3} - 3t^2\right]_2^4 + 2$$

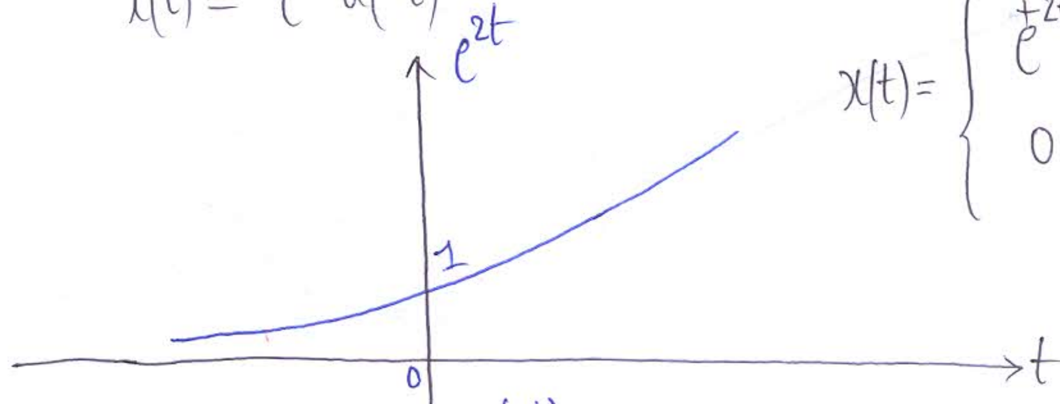
$$E = \frac{2}{3} + \left\{ 9(2) + \frac{1}{3}(56) - 36 \right\} + 2$$

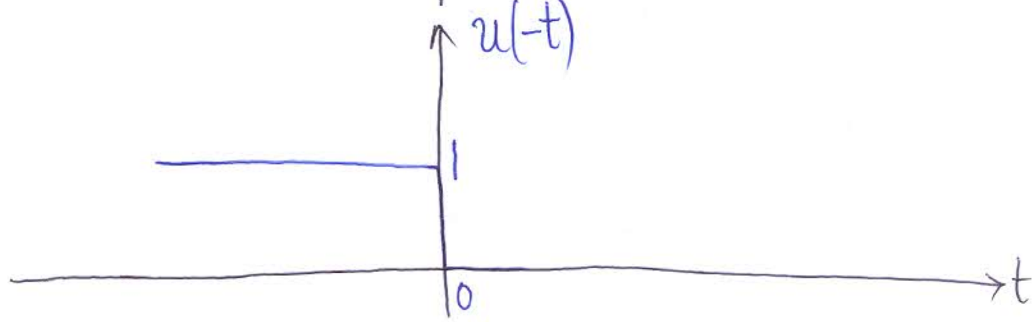
$$\boxed{E = \cancel{9}\,\frac{10}{3} \text{ Joules}}$$

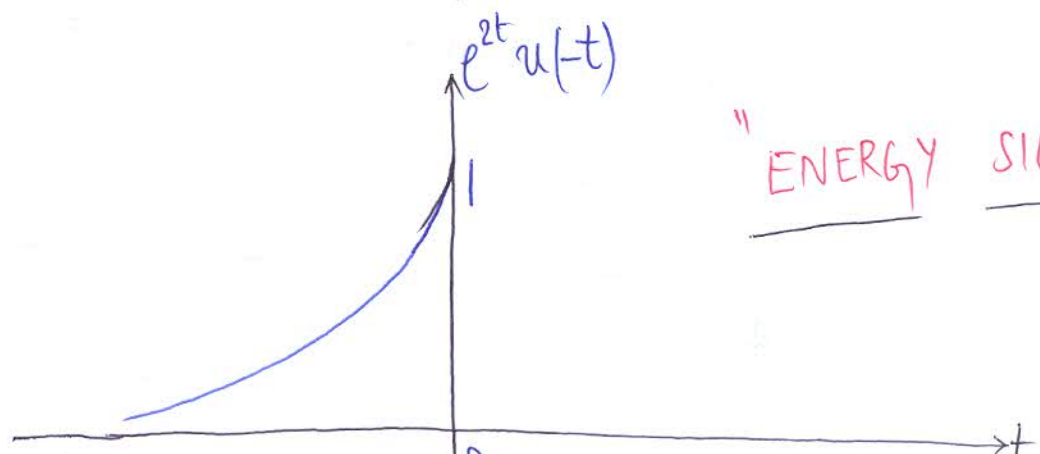
b) Categorize each of the following signals as an energy/power signals. Find the energy/power of signals.

i) $x(t) = e^{2t}u(-t)$

$$x(t) = \begin{cases} e^{+2t}, & t \le 0 \\ 0, & t > 0 \end{cases}$$







"ENERGY SIGNAL"

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \Rightarrow E = \int_{-\infty}^{0} |x(t)|^2 dt$$

$$E = \int_{-\infty}^{0} e^{4t}\,dt = \left.\frac{e^{4t}}{4}\right|_{-\infty}^{0} = \frac{1}{4}\left[e^{0} - e^{-\infty}\right]$$

$$\boxed{E = \frac{1}{4}\,J}$$

but Avg Power $\Rightarrow P = \lim_{T\to\infty} \frac{E}{T} = 0$

Hence the given signal is energy signal

ii) $x(t) = 5\cos\pi t + \sin 5\pi t$

Sol

Energy :- $E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\,dt$

$$E = \lim_{T\to\infty} \left\{ 25\int_{-T}^{T} \cos^2\pi t\,dt + \int_{-T}^{T} \sin^2 5\pi t\,dt + 10\int_{-T}^{T} \cancel{\cos\pi t\,\sin 5\pi t}^{\,0}\,dt \right\}$$

$$E = \lim_{T\to\infty} \left\{ \frac{25}{2}\left[\int_{-T}^{T} dt + \cancel{\int_{-T}^{T} \cos 2\pi t\,dt}^{\,0}\right] + \frac{1}{2}\left[\int_{-T}^{T} dt - \cancel{\int_{-T}^{T} \cos 10\pi t\,dt}^{\,0}\right] \right\}$$

$$\boxed{E = \infty}$$

Power :- $P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt$

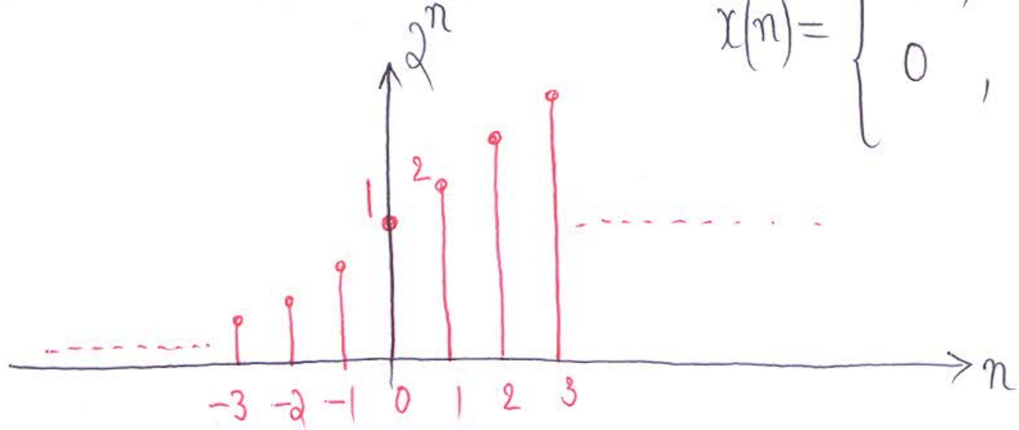
$$P = \frac{25}{2} + \frac{1}{2}$$

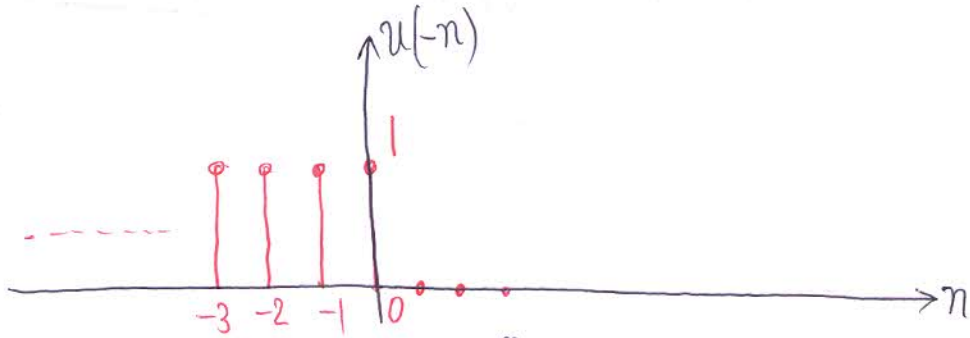
$$\boxed{P = \frac{26}{2}\ \text{Watts}}$$

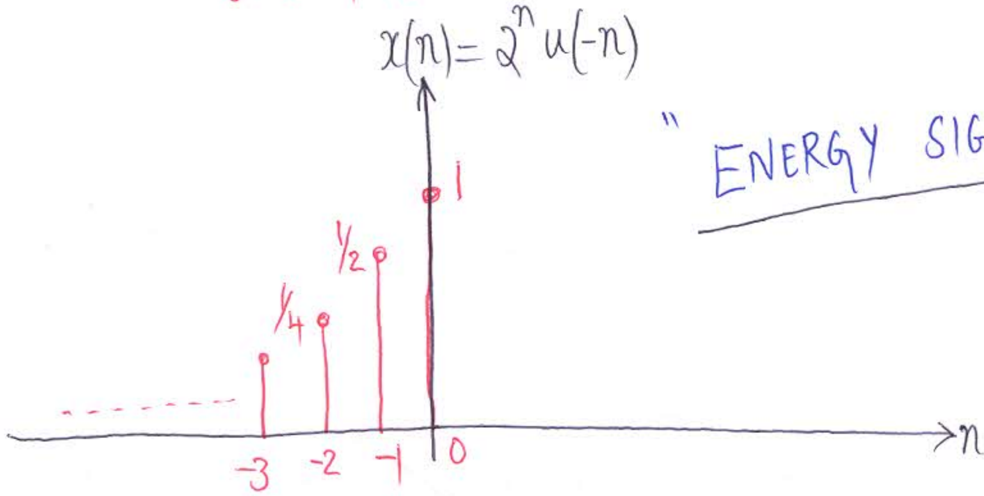
Hence the given signal is Power signal.

iii) $x(n) = 2^n u(-n)$

$$x(n) = \begin{cases} 2^n, & n \leq 0 \\ 0, & n > 0 \end{cases}$$







"ENERGY SIGNAL"

Energy:

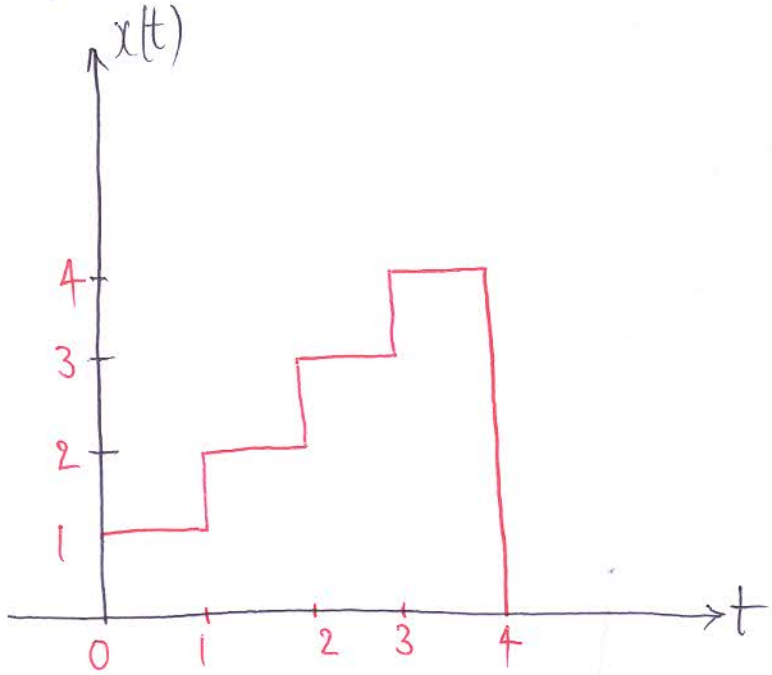
$$E = \lim_{N \to \infty} \sum_{n=-N}^{N} |x(n)|^2 = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

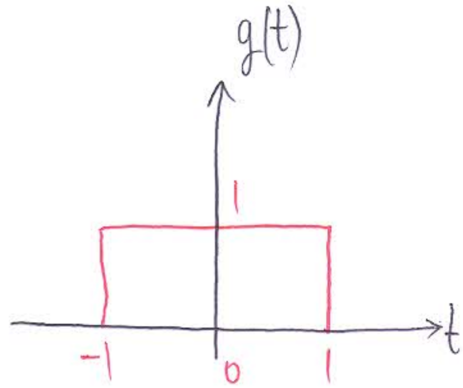
$$E = \sum_{n=-\infty}^{0} [2^n]^2 = \sum_{n=-\infty}^{0} 4^n$$

$$E = 4^0 + 4^{-1} + 4^{-2} + \dots\dots$$

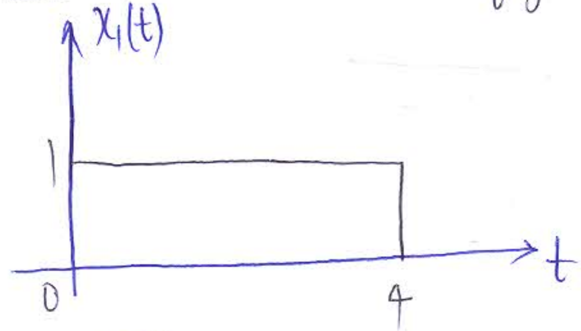
$$E = 1 + \frac{1}{4} + \frac{1}{4^2} + \dots\dots$$

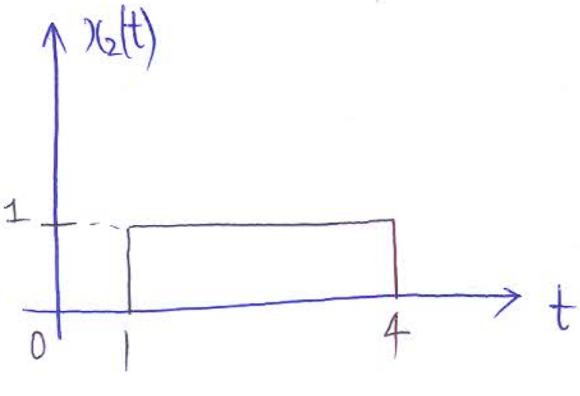
4) The staircase signal $x(t)$ is shown in fig (4a) may be viewed as superposition of 4 rectangular pulses. Starting with rectangular pulse $g(t)$, construct waveform $x(t)$ & express $x(t)$ in terms of $g(t)$.





Sol: $x(t)$ is viewed as superposition of 4 rectangular pulses as shown in figure below.





ie $x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t) \longrightarrow (1)$

$$E = \frac{1}{1-\frac{1}{4}}$$

$$\boxed{E = \frac{4}{3}\ J}$$

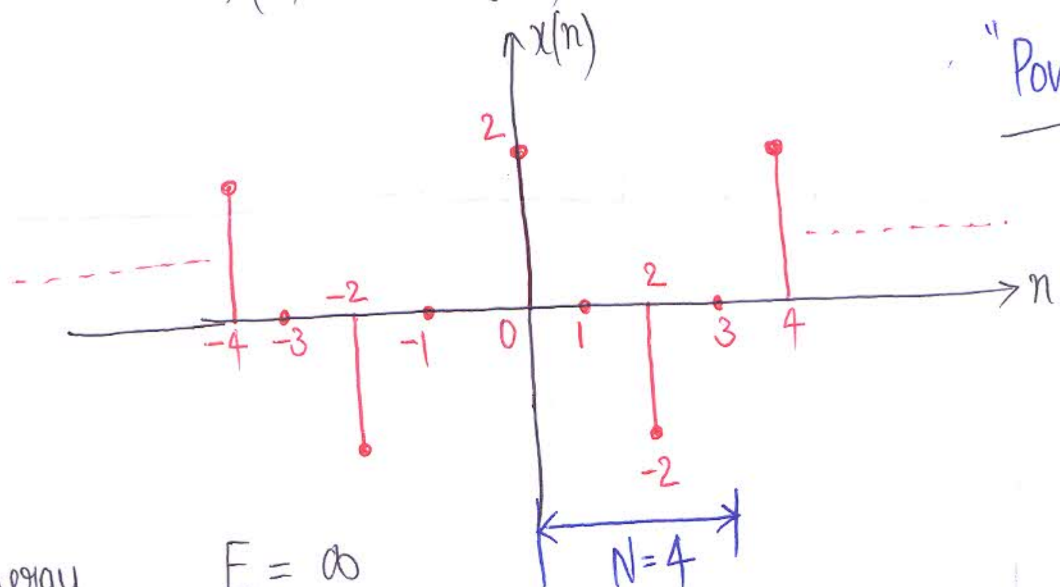
Hence given signal is energy signal.

Avg Power $P = 0$

iv) $x(n) = j^{n} + j^{-n}$ W.K.T $j = e^{j\frac{\pi}{2}}$

$$x(n) = e^{j\frac{\pi n}{2}} + e^{-j\frac{\pi n}{2}}$$

$$x(n) = 2\cos\left(\frac{\pi n}{2}\right)$$



"POWER SIGNAL"

Energy, $E = \infty$

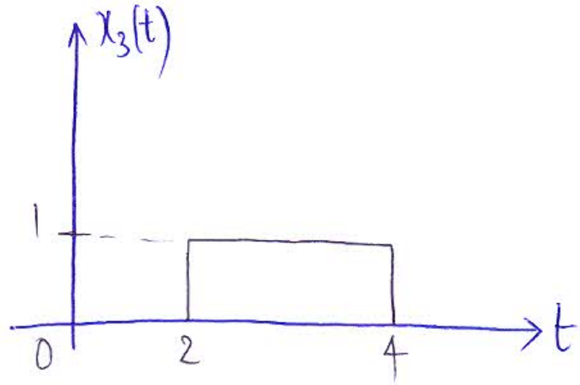
Avg Power in one Period, is given by

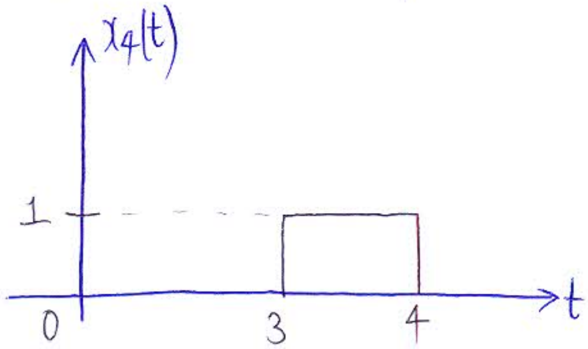
$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

$$P = \frac{1}{4} \sum_{n=0}^{3} |x(n)|^2 = \frac{1}{4}\left[4+0+0+4\right]$$

$$\boxed{P = 2\ \text{Watts}}$$

Hence the given signal is Power signal.





$$x(t) = g\left(\frac{t}{2} - 1\right) + g\left(\frac{2}{3}t - \frac{5}{3}\right) + g(t-3) + g(2t-7)$$

5) Sketch the following functions for the signal given

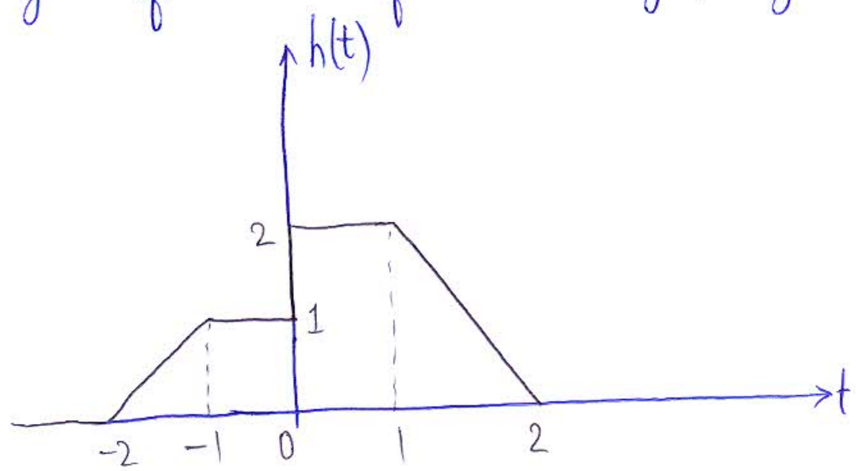
i) $h(t)\,u(t)$

ii) $h(t)\,u(-t)$

iii) $h(t)\,u(t-2)$

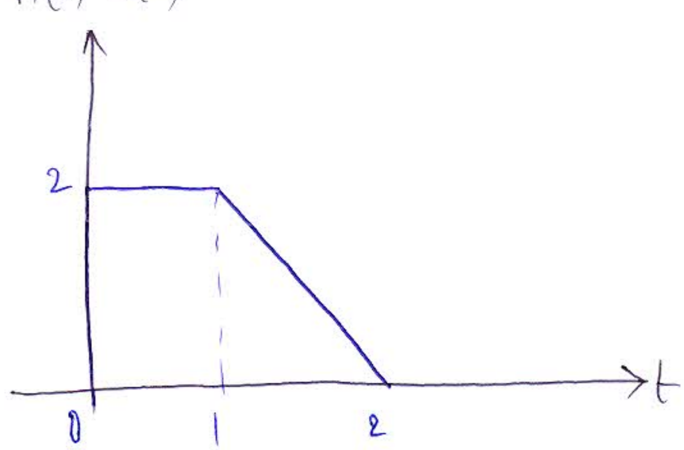
iv) $h(t)\,u(t+2)$

v) $h(t-2)\,u(t)$

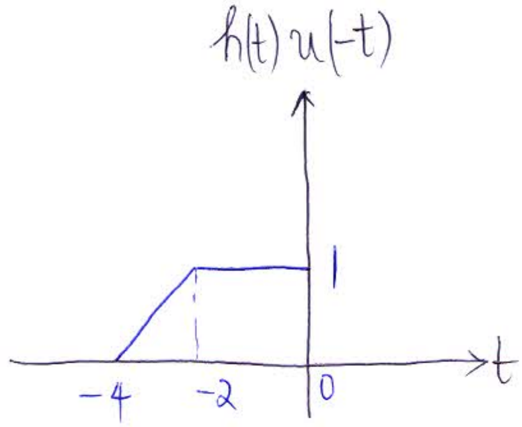


Sol

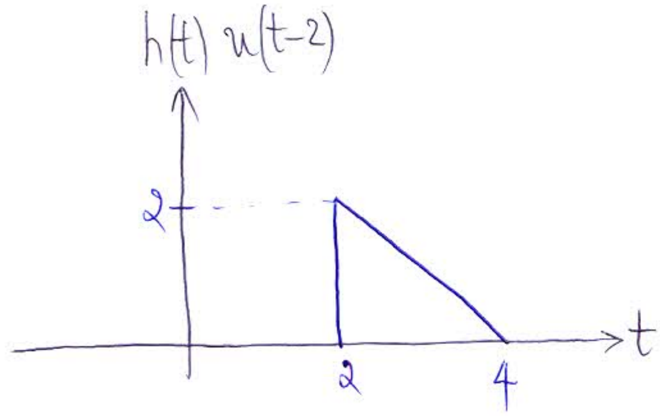
i) $h(t)\,u(t)$



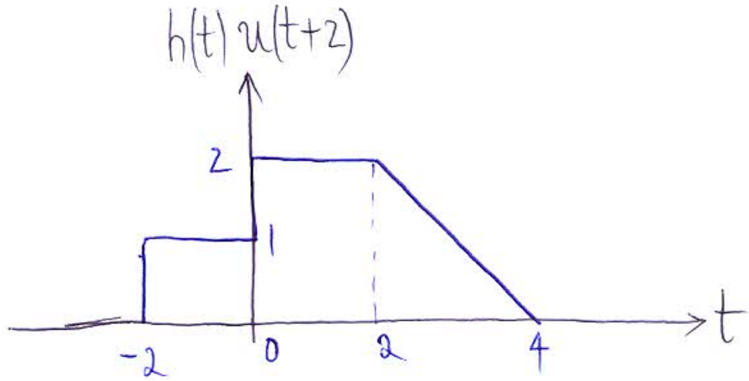
ii)



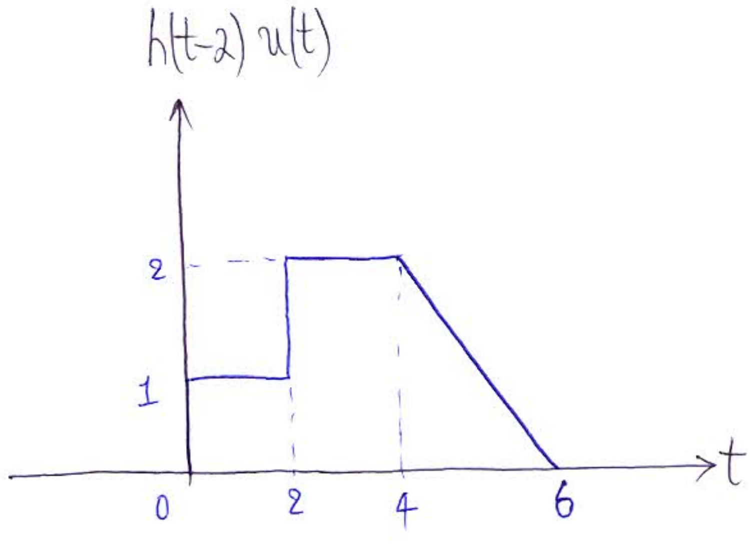
iii)



iv)

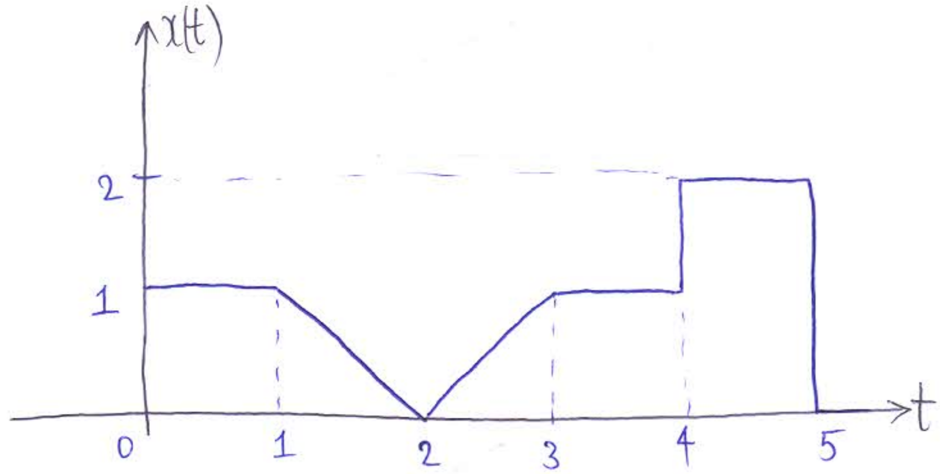


v)



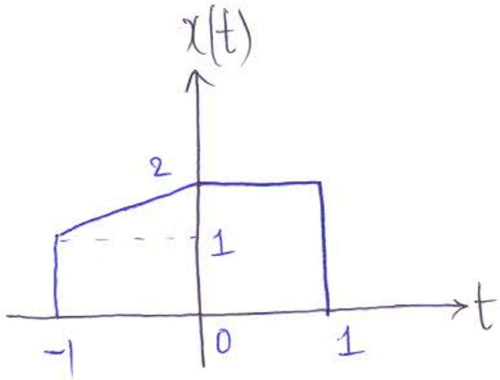
6) a) Sketch the waveform of following signal.

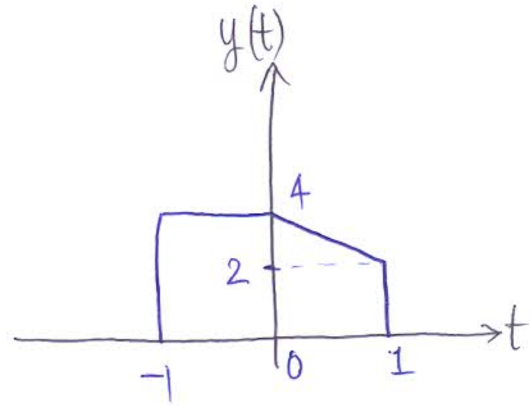
$$x(t) = u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$$



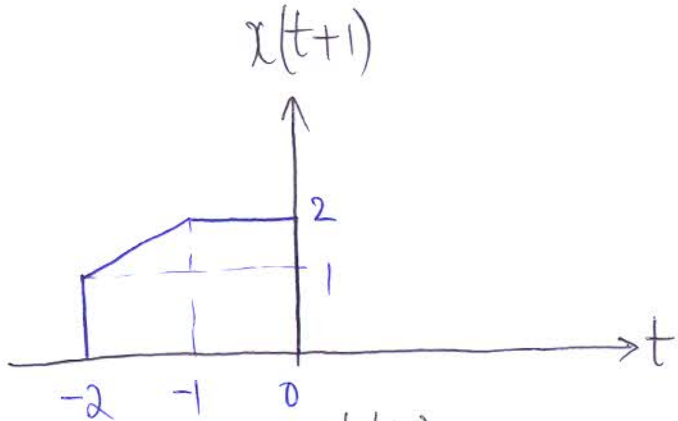
b) If $x(t)$ & $y(t)$ are shown in fig below, sketch

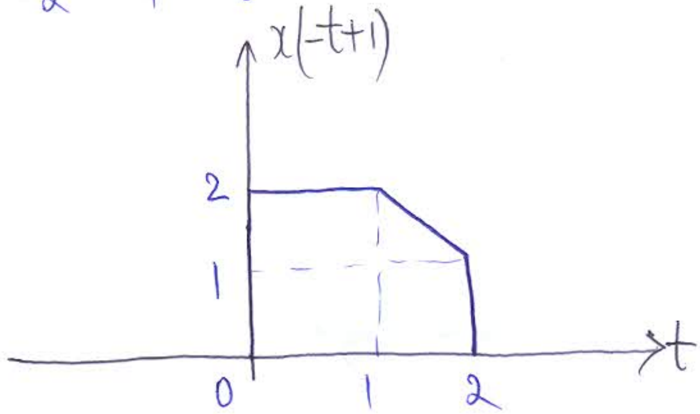
$x(1-t)\, y(t/2)$

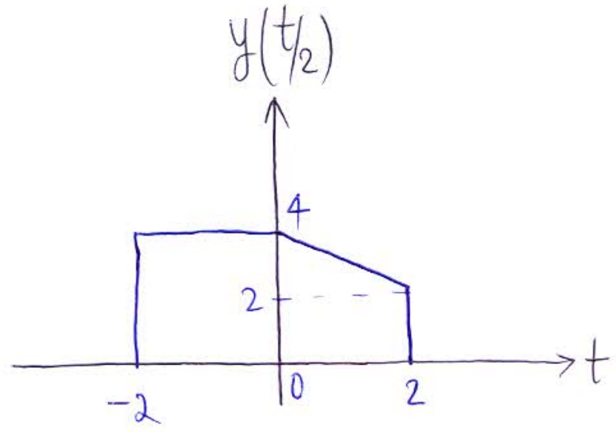


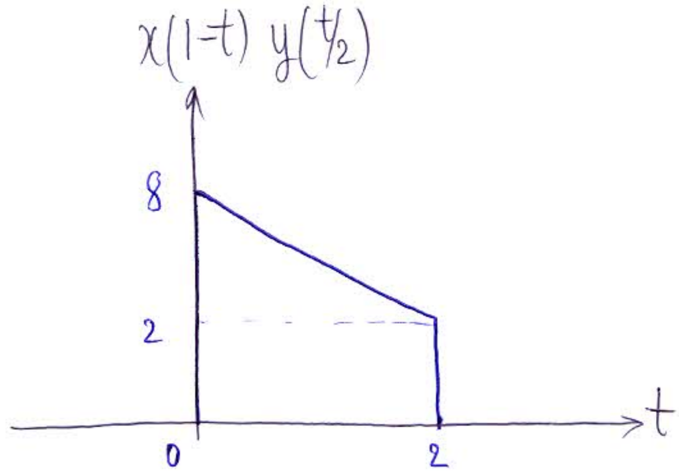


Sol









7) Determine whether following systems are (i) Memoryless (ii) Causal (iii) Stable (iv) Linear & (v) Time invariant

a) $y(t) = (\cos 3t)\, x(t)$

<u>Linear</u> :- If $x_1(t) \longrightarrow y_1(t) = (\cos 3t)\, x_1(t)$

$x_2(t) \longrightarrow y_2(t) = (\cos 3t)\, x_2(t)$

let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$x_3(t) \longrightarrow y_3(t) = (\cos 3t)\, x_3(t)$

$$y_3(t) = (\cos 3t)\left[a_1 x_1(t) + a_2 x_2(t)\right] \longrightarrow (1)$$

$$y_3(t) = a_1 y_1(t) + a_2 y_2(t)$$

Hence system is linear

(ii) Time invariance

If $x_1(t) \longrightarrow y_1(t) = (\cos 3t)\, x_1(t)$

let $x_2(t) = x_1(t-t_0)$

$x_2(t) \longrightarrow y_2(t) = (\cos 3t)\, x_2(t)$

$= (\cos 3t)\, x_1(t-t_0)$

but $y_2(t) \neq y_1(t-t_0)$

Hence System is not time invariant

(iii) Stability

If i/p is bounded ie

$M_x \leq |x(t)| < \infty$

then $|y(t)| = |\cos 3t|\, |x(t)|$ [cos max & min value is 1 & −1]

hence o/p is also bounded $M_y \leq |y(t)| < \infty$

∴ System is stable

(iv) Causal

$y(t) = (\cos 3t)\, x(t)$

$t = 0 \Rightarrow y(0) = (\cos 0)\, x(0)$

Output at present time depends on input at present time. Hence system is Causal

v) Memoryless

System is Memoryless

6) $y(n) = n + x(n)$

i) Linearity :- If $x_1(n) \longrightarrow y_1(n) = n + x_1(n)$ &

$x_2(n) \longrightarrow y_2(n) = n + x_2(n)$

let $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

$x_3(n) \longrightarrow y_3(n) = n + x_3(n)$

$y_3(n) = n + [a_1 x_1(n) + a_2 x_2(n)]$

$a_1 y_1(n) + a_2 y_2(n) = a_1[n + x_1(n)] + a_2[n + x_2(n)]$

$\therefore y_3(n) \neq a_1 y_1(n) + a_2 y_2(n)$

Hence system is non linear

ii) Time invariance

If $x_1(n) \longrightarrow y_1(n) = n + x_1(n)$

let $x_2(n) = x_1(n-n_0)$

$x_2(n) \longrightarrow y_2(n) = n + x_2(n)$

$y_2(n) = n + x_1(n-n_0)$

$y_1(n-n_0) = (n-n_0) + x_1(n-n_0)$

$\therefore y_2(n) \neq y_1(n-n_0)$

Hence System is not time- invariant

iii) Causal $y(n) = n + x(n)$

Present o/p depends on present input

Hence system is Causal

iv) Memoryless

System is Memoryless since current o/p doesnot depend on either past input or future i/p.

v) Stability

If i/p is bounded ie

$M_x \leq |x(n)| < \infty$ then

$|y(n)| = n + |x(n)|$

As $n \longrightarrow \infty$, $|y(n)| \longrightarrow \infty$ (output becomes unbounded)

Hence system is unstable.

8. a) Show that an arbitrary signal $x(n)$ can be expressed as sum of weighted and shifted discrete time-impulses,

$$x(n) = \sum_{K=-\infty}^{\infty} x(K)\, \delta(n-K)$$

and hence derive expression for convolution sum for a discrete time LTI system.

Sol

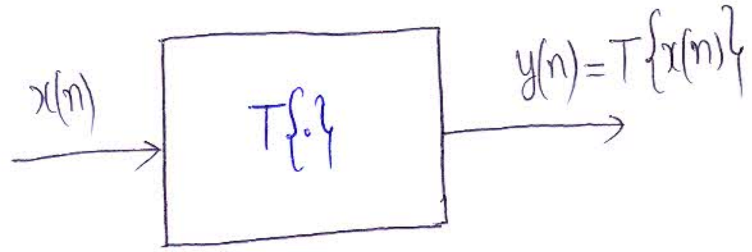


fig 1

Consider a Linear-Time invariant system as shown in fig 1. Let the system be excited with an arbitrary i/p as shown in fig 2.

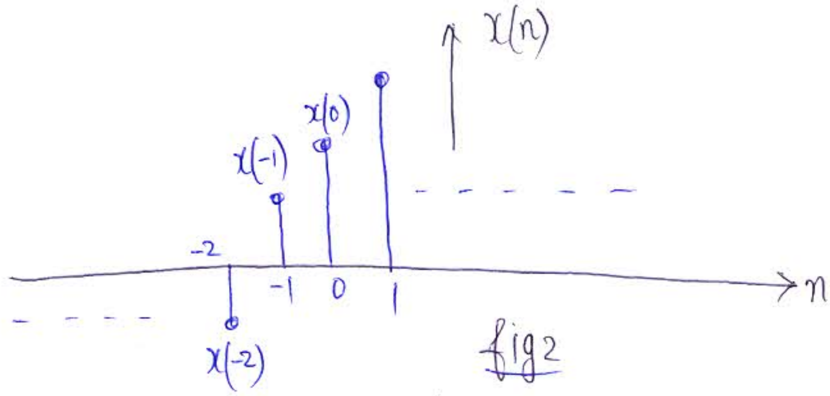


fig 2

From fig ②, we can write

$$x(n) = \cdots\cdots + x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + \cdots\cdots$$

$$\boxed{x(n) = \sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)} \longrightarrow ①$$

The response of discrete time LTI system is given by

$$y(n) = T\{x(n)\}$$

$$y(n) = T\left\{\sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)\right\}$$

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\; T\{\delta(n-K)\}$$

$$\boxed{y(n) = \sum_{K=-\infty}^{\infty} x(K)\,h(n-K)} \longrightarrow ②$$

Equation ② is known as "CONVOLUTION SUM"

Symbolically we can write

$$y(n) = x(n) * h(n)$$