| CMR INSTITUTE OF TECHNOLOGY | USN | | | | | | | | | | |
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Internal Assesment Test - I

| Sub: | ANTENNAS AND PROPAGATION | | | | | | Code: | 10EC64 |
|---|---|---|---|---|---|---|---|---|
| Date: | 28 / 03 / 2017 | Duration: | 90 mins | Max Marks: | 50 | Sem: VI | Branch: | ECE(C,D) |

Answer Any FIVE FULL Questions

| | | Marks | OBE CO | OBE RBT |
|---|---|---|---|---|
| 1. | Explain the following terms as related to antenna system: (i) Directivity, (ii) Effective Height, (iii) Beam Area,(iv) Effective Aperture | [10] | CO1 | L1 |

(i)

The directivity of an antenna is equal to the ratio of the maximum power density $P(\theta, \phi)_{max}$ (watts/m$^2$) to its average value over a sphere as observed in the far field of an antenna

$$D = \frac{P(\theta, \phi)_{max}}{P(\theta, \phi)_{av}} \rightarrow \text{Directivity from pattern.}$$

The directivity is dimensionless ratio $\geq 1$.

The average power density over a sphere,

$$P(\theta, \phi)_{av} = \frac{1}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P(\theta, \phi) \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{4\pi} \iint_{4\pi} P(\theta, \phi) \, d\Omega \quad (W \, sr^{-1})$$

$$\therefore D = \frac{P(\theta, \phi)_{max}}{\left(\frac{1}{4\pi}\right) \iint_{4\pi} P(\theta, \phi) \, d\Omega} = \frac{1}{\frac{1}{4\pi} \iint_{4\pi} \left[\frac{P(\theta, \phi)}{P(\theta, \phi)_{max}}\right] d\Omega}$$

$$= \frac{4\pi}{\iint_{4\pi} P_n(\theta, \phi) \, d\Omega} = \frac{4\pi}{\Omega_A} \quad \text{Directivity from beam area } \Omega_A$$

where, $P_n(\theta, \phi) \, d\Omega = \frac{P(\theta, \phi)}{P(\theta, \phi)_{max}}$ = normalized power pattern

(ii)

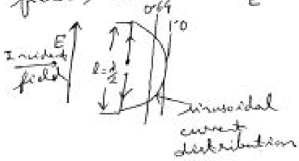
$\frac{2m}{\pi}$ ...

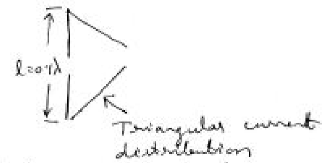
Effective Height

multiplying the effective height by the incident field E of the same polarization gives the voltage V induced.

$\therefore$ $V = hE$

Accordingly, the effective height may be defined as the ratio of the induced voltage to the incident field, or $h = \frac{V}{E}$.




consider, a vertical dipole of length $l = \lambda/2$ immersed in an incident field E.

→ If the current distribution was uniform, effective height would be $l$.

→ But actual current distribution is nearly sinusoidal with an average value $\frac{2}{\pi} = 0.64$ (of the maximum)

$\therefore$ The effective height ~~=~~ $h = 0.64l$.

Assumed antenna is oriented for maximum response.

Now say $l = 0.1\lambda$

The current tapers almost linearly from the central fed point to ~~the~~ zero at the ends in a triangular distribution.

The average current is $\frac{1}{2}$ of the maximum.

$\therefore$ effective height $= 0.5l$.

∴ Effective height can also be defined, by considering the transmitting case of the antenna and equating the effective height to the physical height (or length $l$) and multiplying by the normalized average current.

$$h_e = \frac{1}{I_0}\int_0^{h_p} I(z)\,dz = \frac{I_{av}}{I_0}\,h_p \ (m)$$

where, $h_e$ = effective height (m)

$h_p$ = physical " (m)

$I_{av}$ = Average current (A)

Effective height can also be expressed as a vector quantity, For linear polarization,

$$V = \vec{h_e}\cdot\vec{E} = h_e E \cos\theta$$

$h_e \rightarrow$ effective height and ~~polarization angle of anten~~

$\vec{E} \rightarrow$ Field Intensity " " " of incident wave

$\theta \rightarrow$ angle b/w polarization angles of antenna and and wave in degree.

(iii)

Beam area or beam solid angle $\Omega_A$ of an antenna is given by the integral of the normalized power pattern over a sphere ($4\pi$ sr)

$$\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

and $$\Omega_A = \iint_{4\pi} P_n(\theta,\phi)\, d\Omega \ (sr)$$

where, $d\Omega$ = $\sin\theta\, d\theta\, d\phi$, sr.

The beam area $\Omega_A$ is the solid angle through which all of the power radiated by the antenna would stream if $P(\theta,\phi)$ maintained its maximum value over $\Omega_A$ and was zero elsewhere.

$\boxed{\therefore \text{ The power radiated } = P(\theta,\phi)\,\Omega_A \text{ watts.}}$

The beam area ~~can~~ is given approximately in terms of the angles subtended by the half-power points of the main lobe in the two principal planes.

∴ Beam are $\cong \Omega_A \cong \theta_{HP}\,\phi_{HP}$.

where $\theta_{HP}$ and $\phi_{HP}$ are the HPBW in the two principal planes, minor lobes being neglected.

(iv)

Antenna ...

consider a receiving antena which is a rectangular horn immersed in the field of a uniform plane wave.


plane wave incident on electromagnetic horn of physical aperture $A_p$.

Let the Poynting vector, or power density, of the plane wave be S watts/$m^2$

Let the area, or physical aperture of the horn be $A_p$ $m^2$.

If the horn extracts all the power from the wave over its entire physical aperture. Then the total power P absorbed from the wave is,

$$P = \frac{E^2}{Z} \cdot A_p = S A_p \ W \quad (1)$$

Thus, the em horn considered to have an aperture, (the ~~the~~ total power it extracts from passing wave being proportional to the aperture or area of its mouth.

— But the field response of the horn is not uniform across the aperture A.

→ As $\vec{E}$ at the sidewalls must be zero.

∴ The effective aperture $A_e$ of the horn is less than the physical aperture $A_p$.

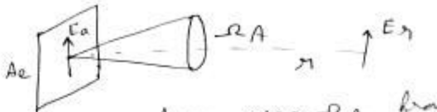
$\varepsilon_{AP} = \frac{A_e}{A_p}$ (dimensionless) Aperture efficiency

For horn and parabolic reflector antenna,

$\varepsilon_{AP}$ in the range of 50 to 80%

$(0.5 \le \varepsilon_{ap} \le 0.8)$

– Large dipole or patch arrays with uniform field to the edge of physical aperture may attain higher aperture efficiencies approaching 100%.




Radiation over beam area $\Omega_A$ from aperture $A_e$

Consider an antenna with effective aperture $A_e$.

It radiates all its power in a conical pattern of beam area $\Omega_A$.

Assuming uniform field $E_a$ over the aperture, the power radiated is,

$$P = \frac{E_a^2}{Z_0} A_e \text{ (W)} \quad \text{...(1)}$$

where $Z_0$ = Intrinsic impedance of medium (377 Ω for free-space)

Assuming a uniform field $E_r$ in the far field at a distance r, the power radiated is,

$r^2\Omega_A$ = area.

$$P = \frac{E_r^2}{Z_0} r^2 \Omega_A \text{ (W)} \quad \text{...(2)}$$

$$\therefore \frac{E_a^2}{Z_0} A_e = \frac{E_r^2}{Z_0} r^2 \Omega_A$$

Now $E_r = \frac{E_a A_e}{r\lambda}$

$$\Rightarrow \frac{E_r^2}{E_a^2} = \frac{A_e^2}{r^2\lambda^2}$$

$$\Rightarrow \cancel{A_e = \frac{A_e^2}{r^2\lambda^2} r^2 \Omega_A}$$

$\Rightarrow$

$$A_e = \frac{E_r^2}{E_a^2} r^2 \Omega_A$$

$$= \frac{A_e^2 \cdot E_a^2}{E_a^2 r^2 \lambda^2} r^2 \Omega_A$$

$$= \frac{A_e \Omega_A}{\lambda^2}$$

$$\Rightarrow A_e = \frac{\lambda^2}{\Omega_A}$$

$$\Rightarrow \boxed{\lambda^2 = A_e \Omega_A}$$

Aperture beam area relation.

Now, $D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\frac{\lambda^2}{A_e}} = 4\pi \frac{A_e}{\lambda^2}$

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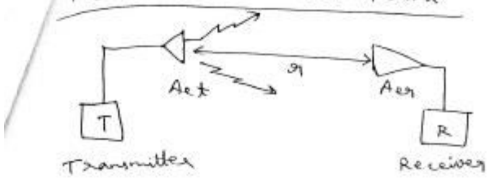
| 2.(a) | State and prove Friis transmission formula. | [04] | CO1 | L1 |

The radio Communication link




Friis Transmission formula

Assume lossless, matched antennas.

let, Tx feed a power $P_t$ to a transmitting antenna of effective aperture $A_{et}$.

At a dist. $r$ a receiving antenna is placed.

$A_{er} \rightarrow$ effective aperture of receiving antenna.

$\therefore$ $A_{er}$ intercepts some power radiated by the transmitting antenna of effective aperture $A_{et}$ and delivers to the receiver R.

Assume transmitting antenna is isotropic.

$\therefore$ Power / area available at the receiving antenna is,

$$S_r = \frac{P_t}{4\pi r^2} \; W/m^2 \quad - (1)$$

Now say antenna has gain $G_t$

The power available / unit area,

$$S_r = \frac{P_t G_t}{4\pi r^2} \; W/m^2 \quad (2)$$

Now the power collected by the lossless matched receiving antenna of effective aperture $A_{er}$

$$P_r = S_r A_{er} = \frac{P_t G_t}{4\pi r^2} A_{er} \quad - (3)$$

Now, gain of the transmitting antenna can be expressed as,

$$G_t = \frac{4\pi A_{et}}{\lambda^2} \quad \left[ \text{if antenna is lossless, } e_{cd}=1 \Rightarrow G = e_{cd} D = D \right]$$

$$\therefore P_r = \frac{P_t A_{er}}{4\pi r^2} \cdot \frac{4\pi \cdot A_{et}}{\lambda^2}$$

$$\Rightarrow \boxed{\frac{P_r}{P_t} = \frac{A_{er} A_{et}}{r^2 \lambda^2} \; (\text{dimensionless})}$$

Friis Transmission formula.

where, $P_r \rightarrow$ received power W

$P_t \rightarrow$ transmitted " W

$A_{et} \rightarrow$ effective aperture of receiving antenna $m^2$

$r$ = distance b/w antennas

$\lambda$ = wavelength.

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| 2.(b) | Calculate the directivity for the following sources with power patterns:<br>(i) $U = U_m \sin^2\theta \sin^2\varphi$; for $0 \le \theta \le \pi$ and $0 \le \varphi \le \pi$<br>$= 0$ ; elsewhere | [06] | CO1 | L3 |

| | | | | |
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| | (ii) $U = U_m \sin^2\theta \sin^3\varphi$; for $0 \le \theta \le \pi$ and $0 \le \varphi \le \pi$<br>$= 0$ ; elsewhere | | | |

2.(b) (i) $U = U_m \sin^2\theta \sin^2\phi$

$\therefore \Omega_A = \iint P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi$

$\therefore D = \frac{4\pi U_m}{P_{rad}}$

$$P_{rad} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{\pi} U_m \sin^2\theta \sin^2\phi \cdot \sin\theta \, d\theta \, d\phi$$

$$= U_m \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta \int_{\phi=0}^{\pi} \sin^2\phi \, d\phi$$

$$= U_m \cdot \frac{4}{3} \cdot \frac{\pi}{2} = \frac{4\pi}{6} U_m$$

$$\therefore \boxed{D = \frac{4\pi U_m}{\frac{4\pi}{6} U_m} = 6}$$

Now, $\int_{\theta=0}^{\pi} \sin^3\theta \, d\theta$

$[\sin 3\theta = 3\sin\theta - 4\sin^3\theta$
$\Rightarrow 4\sin^3\theta = 3\sin\theta - \sin 3\theta$
$\Rightarrow \sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$

$$= \int_{\theta=0}^{\pi} \frac{3}{4} \sin\theta \, d\theta - \frac{1}{4} \int_{\theta=0}^{\pi} \sin 3\theta \, d\theta$$

$$= \frac{3}{4}\left[-\cos\theta\right]_0^{\pi} + \frac{1}{4}\left[\frac{\cos 3\theta}{3}\right]_0^{\pi}$$

$$= \frac{3}{4}\left[\cos\theta\right]_{\pi}^{0} + \frac{1}{12}\left[\cos 3\pi - \cos 0\right]$$

$$= \frac{3}{4}\left[1+1\right] + \frac{1}{12}\left[-1-1\right]$$

$$= \frac{6}{4} - \frac{2}{12} = \frac{18-2}{12} = \frac{16}{12} = \frac{4}{3}$$

Also, $\int_{\phi=0}^{\pi} \sin^2\phi \, d\phi = \frac{1}{2}\int_{\phi=0}^{\pi} (1 - \cos 2\phi) \, d\phi$

$[\cos 2\theta = 1 - 2\sin^2\theta$
$\Rightarrow 2\sin^2\theta = 1 - \ldots$
$\Rightarrow \sin^2\theta = \frac{1}{2}(1 - \ldots$

$$= \frac{1}{2}[\pi] - \frac{1}{2}\left[\frac{\sin 2\phi}{2}\right]_0^{\pi} = \frac{\pi}{2}.$$

2 (b)(ii) $U = U_m \sin^2\theta \sin^3\phi$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{\pi} P_n(\theta,\phi) \sin\theta \, d\theta \, d\phi}$$

$$= \frac{4\pi}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{\pi} \sin^2\theta \sin^3\phi \sin\theta \, d\theta \, d\phi}$$

$$= \frac{4\pi}{\int_{\theta=0}^{\pi} \sin^3\theta \, d\theta \int_{\phi=0}^{\pi} \sin^3\phi \, d\phi} = \frac{4\pi}{\frac{4}{3} \times \frac{4}{3}} = \frac{9\pi}{4}$$

$$= 7.06$$

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| 3.(a) | Show that maximum effective aperture of short dipole is $0.119\lambda^2$. | [06] | CO3 | L3 |

Soln: The maximum effective aperture of an antenna is,

$$A_{em} = \frac{V^2}{4SR_r}$$

where, $V = El$

where, $E \rightarrow$ effective electric field intensity

$l \rightarrow$ length of the dipole.

The radiation resistance $R_r$ of a short dipole of length $l$ with uniform current is,

$$R_r = \frac{80\pi^2 l^2}{\lambda^2}\left(\frac{I_{av}}{I_0}\right)^2$$

$$= 790\left(\frac{I_{av}}{I_0}\right)^2\left(\frac{l}{\lambda}\right)^2$$

where $\lambda \rightarrow$ wavelength

$I_{av} \rightarrow$ average current

$I_0 \rightarrow$ terminal current

The power density, or Poynting vector, of the incident wave at the dipole is related to the field intensity by,

$$S = \frac{E^2}{Z} = \frac{E^2}{120\pi}$$

where, $Z \rightarrow$ intrinsic impedance of the medium.

Here medium is free space.

$\therefore Z = 120\pi\,\Omega$

$$\therefore A_{em} = \frac{V^2}{4SR_r} = \frac{V^2}{4}\cdot\frac{120\pi}{E^2}\cdot\frac{\lambda^2}{80\pi^2 l^2}\cdot\left(\frac{I_0}{I_{av}}\right)^2$$

The maximum effective aperture of a short dipole (for $I_{av} = I_0$),

$$A_{em} = \frac{\cancel{E^2}\cancel{l^2}\cdot 120\pi}{4\,\cancel{E^2}}\cdot\frac{\lambda^2}{80\pi^2\cancel{l^2}}$$

$$= \frac{120\pi\lambda^2}{320\pi^2} = \frac{\cancel{12}3}{8\,\cancel{32}\pi}\lambda^2 = 0.119\lambda^2$$

(b) $D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi\times 0.119\cancel{\lambda^2}}{\cancel{\lambda^2}} = 1.5$

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| 3.(b) | Show that the directivity for unidirectional operation is 2(n+1) for an intensity variation of $U = U_m cos^n\theta$. | [04] | CO1 | L3 |

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| | Ans. $U = U_m \cos^n\theta, \quad D = \dfrac{4\pi}{\iint P_n(\theta,\phi)\,d\Omega}$ <br> $= \dfrac{4\pi}{\int_{\theta=0}^{\pi/2}\int_{\phi=0}^{2\pi} \cos^n\theta \sin\theta \, d\theta \, d\phi}$ <br> $= \dfrac{\cancel{4\pi}\ 2}{\cancel{2\pi}\int_0^{\pi/2} \cos^n\theta \sin\theta \, d\theta}$ <br> $= \dfrac{2}{\cancel{\frac{\cos(n+1)\theta}{(n+1)}\Big\vert_0^{\pi/2}}\ \frac{1}{(n+1)}} = 2(n+1)$ <br> $\int_0^{\pi/2} \cos^n\theta \sin\theta \, d\theta$. <br> Let, $\cos\theta = z$ <br> $\Rightarrow -\sin\theta \, d\theta = dz$ <br> $\theta = 0 \quad z = 1$ <br> $\theta = \pi/2, \ z = 0$ <br> $\therefore \int_0^{\pi/2} \cos^n\theta \sin\theta \, d\theta = -\int_1^0 z^n dz$ <br> $= \left[\dfrac{z^{n+1}}{n+1}\right]_0^1$ <br> $= \dfrac{1}{(n+1)}$ | | | |
| | | | | |
| 4. | Derive the expression for total field for two isotropic point sources with same amplitude and opposite phase. Plot the field pattern when the two isotropic sources are spaced $\lambda/2$ apart. <br> $E = E_0 e^{+j\psi/2} - E_0 e^{-j\psi/2}$ <br> $= 2E_0 \cdot \frac{1}{2}\left(\cos\psi/2 + j\sin\frac{\psi}{2} - \cos\psi/2 - j\sin\psi/2\right)$ <br> $= 2E_0 \cdot \frac{1}{\cancel{2}} 2j \sin\psi/2 \quad = 2E_0 j \sin\psi/2$ <br> $= 2j E_0 \sin\left(\frac{d_r}{2}\cos\phi\right)$ --- (1) <br> The presence of $j$ in (1) indicates the phase reversal of ~~the~~ one of the sources results in a $90°$ phase shift of the total field as compared with the total field for case 1 <br> Taking $2jE_0 = 1$ and considering the ~~the~~ special case of $d = \lambda/2$, <br> $E = \sin\left(\pi/2 \cos\phi\right)$ --- (2) | [10] | CO2 | L3 |

The directions $\phi_m$ of maximum field are obtained when,

$$\frac{\pi}{2} \cos\phi_m = \pm(2k+1)\,\pi/2$$

where, $k = 0,$ ~~...~~ $1, 2, 3 \ldots$

when, ~~...~~

$$\frac{\pi}{2} \cos\phi_m = \pm \pi/2$$

$$\Rightarrow \cos\phi_m = \pm 1$$

$$\therefore \phi_m = 0° \text{ and } 180° \quad \ldots 3(a)$$

The null directions $\phi_0$ are given by,

$$\frac{\pi}{2}\cos\phi_0 = \pm k\pi$$

for $k = 0$, ~~...~~ $\frac{\pi}{2}\cos\phi_0 = 0$

$$\Rightarrow \phi_0 = \pm 90° \quad \ldots 3(b)$$

The half power directions are given by, $\sin\left(\frac{\pi}{2}\cos\phi\right) = \frac{1}{\sqrt{2}}$ considering $2E_0 = 1$

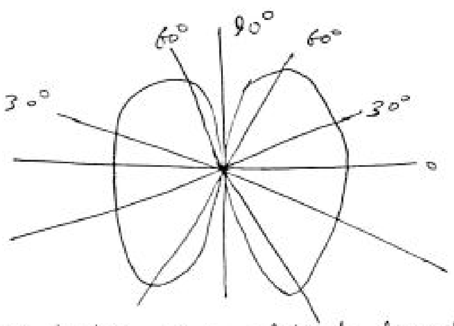
$$\frac{\pi}{2}\cos\phi = \pm(2k+1)\frac{\pi}{4}$$

for $k=0$, $\frac{\pi}{2}\cos\phi = \pm\frac{\pi}{4}$

$$\Rightarrow \cos\phi = \pm\frac{1}{2}$$

$$\Rightarrow \phi = \pm 60°, \pm 120° \quad - (3c)$$

The field pattern as in (3) are shown in fig. 1. The pattern is a relatively broad figure-of-eight with the maximum field in the same direction as the line joining the sources (x axis).




The pattern is a relatively broad figure-of-eight ~~revolution of~~ with the maximum field in the same direction as the line joining the two sources (x axis)

– The space pattern is a fig. of revolution around the x-axis.

– The two sources can be described as ~~the~~ a simple-type of "end-fire" array.

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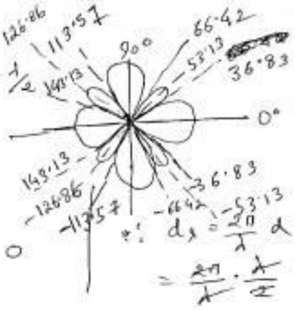
| 5. | Draw the field pattern of a broadside array of 5 elements. Each element is at a distance of $\lambda/_2$ from its neighbour. Assume $\delta = -\pi$. | [10] | CO2 | L3 |

1) Broadside array. $n = 5$, $d = \lambda/2$

$\delta = 0$.




$\because d_r = \frac{2\pi}{\lambda} d = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = \pi$

$\therefore$ Condition for maxima

$\psi = 0$

$\Rightarrow \pi\cos\phi = 0$

$\Rightarrow \phi = 0$

$\Rightarrow \boxed{\phi = \pm 90°}$

Condition for null :

$\frac{n\psi}{2} = \pm k\pi$

$\frac{5}{2}(\pi\cos\phi) = \pm k\pi$

$\Rightarrow \cos\phi = \pm\frac{2k}{5}$

$\Rightarrow \phi = \pm\cos^{-1}\left[\pm\frac{2k}{5}\right]$

for $k = 1$, $\phi = \pm\cos^{-1}\left[\pm\frac{2}{5}\right]$

$\Rightarrow \boxed{\phi = \pm 66.42}$

and $\boxed{\phi = \pm 113.57}$

for $k = 2$, $\phi = \pm\cos^{-1}\left[\pm\frac{4}{5}\right]$

$\Rightarrow \boxed{\phi = \pm 36.869}$

and $\boxed{\phi = \pm \pm 143.13}$

for $k = 3$, $\rightarrow$ discarded.

Direction of side lobes

$\frac{n\psi}{2} = \pm(2k+1)\frac{\pi}{2}$

$\Rightarrow \frac{5}{2}\pi\cos\phi = \pm(2k+1)\frac{\pi}{2}$

$\Rightarrow 5\cos\phi = \pm(2k+1)$

$\Rightarrow \cos\phi = \pm\frac{1}{5}(2k+1)$

$\Rightarrow \phi = \pm\cos^{-1}\left[\pm\frac{1}{5}(2k+1)\right]$

for $k=1$, $\phi = \pm\cos^{-1}\left[\pm\frac{1}{5}(3)\right]$

$\Rightarrow \boxed{\phi = \pm\cos^{-1}\left(\frac{3}{5}\right) = \pm 53.130}$

and $\boxed{\phi = \pm\cos^{-1}\left(-\frac{3}{5}\right) = \pm 126.86}$

for $k = 2$, $\phi = \pm\cos^{-1}[\pm 1]$

$\Rightarrow \boxed{\phi = \pm\cos^{-1}[1] = 0}$

and $\boxed{\phi = = \pm\cos^{-1}[-1] = 180°}$

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| 6. | Find peaks, nulls and beam-width of a given array of two point sources with equal amplitude and equal phase. Given d = λ/2 and n=4. Draw the field pattern. | [10] | CO2 | L3 |

For major lobe, $\psi = 0$

Here, $d = \frac{\lambda}{2}$, $n = 4$, $\delta = 0$ $\therefore \psi = \frac{2\pi}{\lambda} d\cos\phi + \delta$

$= \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\phi + 0$

$= \pi\cos\phi$

For ~~any~~ major lobe, $\psi = 0$

$\Rightarrow \pi\cos\phi = 0$

$\Rightarrow \phi = \pm 90°$

For minima, $\frac{n\psi}{2} = \pm k\pi$

$\Rightarrow \frac{4}{2}\pi\cos\phi = \pm k\pi$

$\Rightarrow \pi\cos\phi = \pm \frac{k\pi}{2}$

for $k=1$, $\phi = \pm 60°, \pm 120°$

for $k=2$, $\phi = \pm 0°, 180°$

For side-lobes,

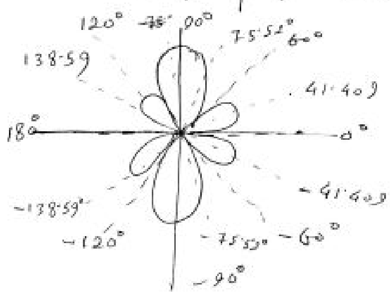
$\frac{n\psi}{2} = \pm(2k+1)\frac{\pi}{2}$

$\Rightarrow \cos\phi = \pm(2k+1)\frac{1}{4}$

for $k=0$, $\phi = \pm 75.52°$

for $k=1$, $\phi = \pm 41.409°, \pm 138.59°$

for $k=2$, discarded




BWFN = 60°

HPBW = 30°

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| 7. | Derive an expression for the radiation resistance of a short dipole. | [10] | CO3 | L1 |

The average Poynting vector is given by,

$\vec{S} = \frac{1}{2} \mathrm{Re}(\vec{E} \times \vec{H}^*)$ --(1)

The far-field components are $E_\theta$, $H_\phi$.

$\therefore$ The radial component of the Poynting vector

$S_r = \frac{1}{2} \mathrm{Re}\, E_\theta H_\phi^*$ --(2)

Where $E_\theta$ and $H_\phi^*$ are complex.

Now, $\frac{E_\theta}{H_\phi} = Z$

$\Rightarrow E_\theta = H_\phi Z = H_\phi \sqrt{\frac{\mu}{\epsilon}}$.

$\therefore$ Equation (2) becomes,

$S_r = \frac{1}{2} \mathrm{Re}\, Z H_\phi H_\phi^*$

$= \frac{1}{2} |H_\phi|^2 \mathrm{Re}\, Z$

$= \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu}{\epsilon}}$ --(3)

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| | The total power P radiated is,<br>$P = \iint S_r\, ds = \frac{1}{2}\sqrt{\frac{\mu}{\epsilon}} \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \lvert H_\phi \rvert^2 r^2 \sin\theta\, d\theta\, d\phi$ . (4)<br>Now, $\lvert H_\phi \rvert = \frac{\omega I_o L \sin\theta}{4\pi c r}$ . (5)<br>~~Sub~~ Substituting this into (4),<br>$P = \frac{1}{2}\sqrt{\frac{\mu}{\epsilon}} \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\omega^2 I_o^2 L^2}{16\pi^2 c^2 r^2} \sin^2\theta \cdot r^2 \sin\theta\, d\theta\, d\phi$<br>$\cancel{= \frac{1}{2} 2\pi \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{\omega^2 I_o^2 L^2}{16\pi^2 c^2} \int_{\theta=0}^{\pi} \sin^3\theta\, d\theta}$<br>$= \frac{1}{2}\sqrt{\frac{\mu}{\epsilon}} \cdot \frac{\beta^2 I_o^2 L^2}{16\pi^2} \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin^3\theta\, d\theta\, d\phi$<br>$= \frac{1}{\cancel{32}\,4}\sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_o^2 L^2}{\cancel{\pi^2}\,\pi} \cdot 2\pi \cdot \frac{4}{3}$ .<br>$= \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_o^2 L^2}{12\pi}$ . (6)<br>This is the average power or rate at which energy is streaming out of a sphere surrounding the dipole.<br>∴ This is equal to the power radiated. Assuming no losses, it is also equal to the power delivered to the dipole.<br>$I_o \rightarrow$ maximum current,<br>∴ The corresponding r.m.s current $= \frac{I_o}{\sqrt{2}}$<br>$\therefore \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_o^2 L^2}{12\pi} = \left(\frac{I_o}{\sqrt{2}}\right)^2 R_r$<br>$\Rightarrow \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 L^2}{6\pi} = R_r$<br>$\Rightarrow \boxed{R_r = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 L^2}{6\pi}}$<br>For air or vacuum, $\sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_o}{\epsilon_o}} = 377 = 120\pi\ \Omega$<br>∴ Radiation resistance of a dipole with uniform current is,<br>$R_r = \cancel{120\pi}\,20 \cdot \frac{\beta^2 L^2}{\cancel{6\pi}}$<br>$= 20 \cdot \frac{(2\pi)^2}{\lambda^2} \cdot L^2 = 80\pi^2 \left(\frac{L}{\lambda}\right)^2$<br>$= 80\pi^2 L_\lambda^2 = 790 L_\lambda^2\ (\Omega)$ | | | |

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| | - It is assumed that with end-loading the dipole current is uniform.<br>- If no end loading dipole current is zero at the ends.<br>- If dipole is short, the current tapers almost linearly from a max. at the center to zero at the ends.<br>Modifying eqn. (6) for this general case,<br>$P = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_{av}^2 L^2}{12\pi}$ (W) (7).<br>The power delivered to the dipole is as before,<br>$P = \frac{1}{2} I_0^2 R_r$.<br>where, $I_0 \rightarrow$ amp. of terminal current of center-fed dipole (peak value in time)<br>$\therefore \frac{1}{2} I_0^2 R_r = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 I_{av}^2 L^2}{12\pi}$<br>$\therefore R_r = \sqrt{\frac{\mu}{\epsilon}} \frac{\beta^2 L^2}{6\pi} \left(\frac{I_{av}}{I_0}\right)^2$<br>$= 790 \left(\frac{I_{av}}{I_0}\right)^2 L_\lambda^2$ $(\Omega)$<br>For a short dipole without end-loading,<br>$I_{av} = \frac{I_0}{2}$.<br>$\therefore R_r = 197 (L_\lambda)^2$ | | | |

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| | Scheme of solution<br>2017 IAT-1 AWP<br><br>1. (i) Directivity — 3m<br>(ii) Effective Height — 2m<br>(iii) Beam area — 2m<br>(iv) Effective Aperture — 3m<br><br>2. (a) Diagram - 1m<br>Derivation & final formula - 3m<br><br>2. (b) (i) Approach - 2m<br>Final answer - 1m<br>(ii) Approach - 2m<br>Final answer - 1m<br><br>3. (a) Diagram - 1m<br>Derivation - 5m<br><br>3. (b) Approach - 2m<br>Proof - 3m<br><br>4. Diagram - 1m<br>Expression for total field - 3m<br>Field pattern - 1m<br>Maximum, null, side-lobe directions - 5m<br><br>5. Approach - 2m<br>Maximum-direction - 2m<br>Null - direction - 2m<br>Field pattern - 2m<br>Side-lobe direction - 2m<br><br>6. Approach - 2m<br>Maximum (major-lobe direction) - 2m<br>Null - direction - 2m<br>Side - lobe direction - 2m<br>- Field pattern - 2m<br><br>7. Expression for $H_\phi$ - 2m<br>Derivation - 6m<br>Final result - 2m | | | |

| Course Outcomes | | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 | PO9 | PO10 | PO11 | PO12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CO1: | Explain the radiation mechanism of various types antennas. | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CO2: | Construct array of two point sources and array of n-isopropic point sources and interpret the radiation pattern for various arrays | 3 | 2 | 2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CO3: | Analyze dipole antennas and thin linear | 3 | 2 | 2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| | antennas and derive field components in far field antenna zone. | | | | | | | | | | | | |
| CO4: | Derive the field components for loop, slot and horn antennas, assess the antennas and hence interpret their operation in broadcasting applications. | 3 | 2 | 2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CO5: | Analyze the performances of high frequency antennas and hence rank the antennas according to the bandwidth requirement, resolution, use in remote sensing applications and broadcasting. | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CO6: | Analyze ground wave, space wave, surface wave and ionospheric propagation, derive the field equations and summarize the performance in various regions of atmosphere. | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| Cognitive level | KEYWORDS |
|---|---|
| L1 | List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc. |
| L2 | summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend |
| L3 | Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover. |
| L4 | Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer. |
| L5 | Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize. |

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*