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Internal Assesment Test – I Scheme and Solutions

Sub:	Fluid Mechanics							Code:	15ME44
Date:	28/03/2017	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	Mechanical

	Marks	Marks Split up
PART – A (Answer any one full question)		
1 a) Distinguish between	[06]	
i) Mass density and specific weight		2 Marks
ii) Newtonian Fluid and Non-Newtonian Fluid		2 Marks
iii) Absolute Viscosity and Kinematic Viscosity		2 Marks

1 a) Distinguish between

i) Mass density and specific weight

Mass density	Specific weight
Mass density is defined as the ratio of mass of the fluid to the Volume of fluid. $\boxed{\rho = \frac{m}{V}\ Kg/m^3}$	Specific weight is defined as ratio of weight of the liquid to the Volume of the liquid. $w = \frac{W}{V} = \frac{(m \times g)}{V} = \rho g$. $\boxed{w = \rho g}$

ii) Newtonian Fluid and Non-Newtonian Fluid

Newtonian fluid	Non-Newtonian fluid.
• The fluids which obey's ~~law~~ Newton law of Viscosity is called Newtonian fluid. • Shear stress will be directly proportional to the rate of shear strain	• The fluids which doesn't obey law of Viscosity is called Non-newtonian fluid. • Shear stress (τ) will not be directly proportional to the rate of shear strain

iii) Absolute Viscosity and Kinematic Viscosity

Absolute Viscosity	Kinematic Viscosity
• It is defined as the resistance offered the fluid from one layer to other layer. - Unit = Ns/m	• It is defined as the ratio of Absolute Viscosity to the density of fluid. $K = \frac{V}{\rho} = m^2/s$

b) Explain the effect of variation of temperature on viscosity of liquid and gases [04]

Variation of Viscosity in Liquid →

when the temperature of the liquid increases viscosity of the liquid decreases

$$\text{temperature} \propto \frac{1}{\text{viscosity}}$$

• This is mainly due to the cohensive forces in the liquid particles. As the temperature increases cohesive force decrease hence viscosity also decreases

2 Marks

Variation of Viscosity in Gas →

when the temperature of liquid increases the viscosity of gases also increases.

• This is mainly due to the molecular momentum of the gases.
when the temperature increases, the Kinetic Energy of gas molecules also increases hence molecular momentum also increases
∴ ~~Kinemat~~ Viscosity also increases

2 Marks

2 a) Differentiate between [06]

i) Steady and Unsteady flow

Steady flow is one in which the properties of the fluid do not vary with time. i.e, $\frac{dv}{dt} = 0$.
While an unsteady flow is one in which the properties of the fluid vary with time. i.e, $\frac{dv}{dt} \neq 0$

2 Marks

ii) Uniform and non-uniform flow

Uniform flow is one in which the properties of the fluid do not vary with the surface. i.e, $\frac{dv}{ds} = 0$.
While a non-uniform flow is one in which properties of the fluid varies with respect to the surface.
i.e, $\frac{dv}{ds} \neq 0$.

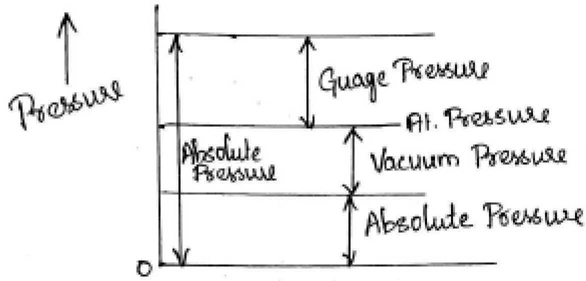
2 Marks

iii) Laminar and Turbulent flow

Laminar flow is one in which the fluid flows in a specified path. i.e, the streamlines flow parallel to each other & in a specific direction.
While Turbulent flow is one where the fluid flows in zig-zag direction ~~since~~ since the particles flow in an zig-zag direction.

2 Marks

b) Establish a relationship among absolute, gauge and atmospheric pressure with sketch [04]



2 Marks

Guage Pressure is measured above the atmospheric pressure.

Vaccum Pressure is the pressure measured below the atmospheric pressure.

Absolute Pressure : It is the pressure measured from ~~0 pressure~~. zero pressure

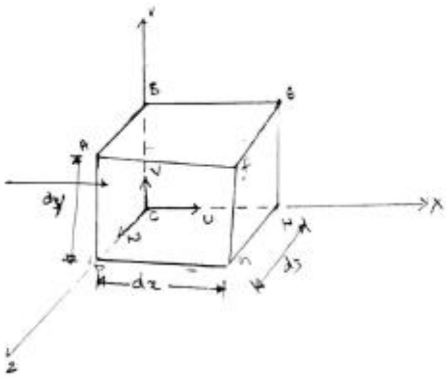
$\therefore$ $P_{abs} = P_{atm} + P_{guage}$

$P_{abs} = P_{atm} - P_{vacuum}$

2 Marks

PART – B (Answer any two questions)

3 Derive with usual notations, the continuity equation for 3D flow and modify the equation for steady and incompressible flow. [10]



3 Marks

Consider a fluid Element whose dimensions are dx, dy and dz along $x, y, \& z$ axis respective.

let $u, v, \& w$ be the velocities in x, y and z direction.

The mass/time of fluid Entering the surface ABCD along x-direction is given by

$$\dot{m}_{ABCD} = \rho \times dy\,dz \times u \quad \text{—(1)}$$

It's observed that, the density and velocity changes when it travels through the surface

Density changes by $\rho + \frac{\partial \rho}{\partial x} . dx$

Velocity changes

$$u + \frac{\partial u}{\partial x} dx$$

$$\therefore \dot{m}_{EFGH} = \rho + \frac{\partial \rho}{\partial x} dx + dy dz + u + \frac{\partial u}{\partial x} dx.$$

$$= dx\left[\rho + \frac{\partial \rho}{\partial x} + u + \frac{\partial u}{dx}\right] + dy dz.$$

$$= dx\left[\rho + u + \frac{\partial (\rho u)}{\partial x}\right] + dy\, dz.$$

$$\dot{m}_{ABCD} - \dot{m}_{EFGH}$$

$$\therefore \left[\rho * dy dz \times u\right] - \left[\rho \times dy dz - u\right] - dx\left[+\frac{\partial}{\partial x}(\rho u) dy dz\right]$$

$$= -dx\left[\frac{\partial}{\partial x}(\rho u)\ dy dz\right] - ②$$

III^{ly} along y- direction.

$$= -dx\left[\frac{\partial}{\partial y}(\rho v)\ dx dz\right] - ③$$

$$= -dz\left[\frac{\partial}{\partial y}(\rho w)\ dx dz\right] - ③.$$

The mass per time of fluid is = Total mass of the fluid Element

$$\therefore m = [\rho dx.dy.dz]$$

$$\dot{m} = \frac{\partial}{\partial t}[dx.dy\, dz].$$

$$\therefore \frac{\partial}{\partial t}\rho[dx dy dz] = -\left[dx dy dz\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial}{\partial z}(\rho w)\right]\right.$$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

5 Marks

The above Equation, is Called Continuity Equation in 3-D.

~~For compressible li.~~

For Non, Compressible liquid.

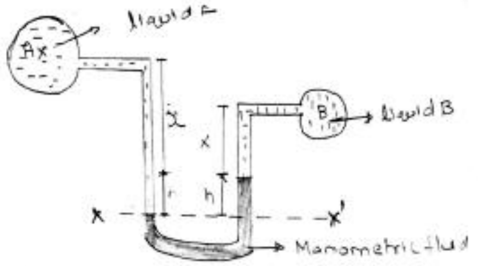
density remain Constant

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

2 Marks

4 With a neat diagram, explain U-tube differential manometers and inverted U-tube differential manometers [10]

2 Marks



i) U-tube diffrential Manometer.

diffrential Manamometer is Used to find the Pressure at diffrent points

Let, x-x' be the refrence line

x → be the distance of liquid A from refrence line

y → be the distance of liquid B from refrence line.

P_A → Pressure of liquid A ~~be~~ with density ρ_A

P_B → Pressure of liquid B with density ρ_B

h → height of Manometric fluid from refrence line

ρ → density of manometric fluid

The manometric fluid which is used should have density higher than the other two liquids.

Applying Pascals law to Right limb

$P_B + \rho_2 g y + \rho g h = 0$ — (1)

Applying Pascal's law to left limb.

$P_A + \rho_1 g x = 0$ — (2)

(eq) (2) - eq (1)

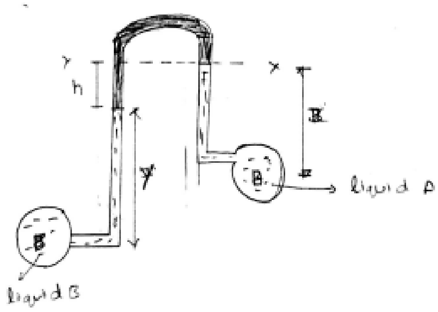
$P_A - P_B = -\rho_2 g y - \rho g h + \rho_1 g x$

$$\boxed{P_A - P_B = \rho_1 g x - \rho_2 g y - \rho g h}$$

3 Marks

b) Inverted U-tube diffrential Manometer

2 Marks



Let,

x - be the distance of liquid A from xx'

Y - be the distance of liquid B from 'xx''

$h \rightarrow$ be the distance of Monometric from for 'xx''

P_A & $P_B \rightarrow$ Pressure of liquid A and B with density ρ_A and ρ_B

Applying Pascal's law to left limb

$$P_B - \rho_2 g y \mp \rho g h = 0 \quad -(1)$$

Applying Pascal's law to Right limb

$$P_A - \rho_1 g x = 0 \quad -(2)$$

Eq (2) - (1)

$$\boxed{P_A - B_B = \rho_2 g y + \rho g h - \rho_1 g x}$$

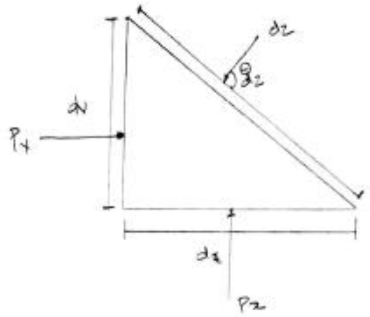
3 Marks

5 State and prove Pascal law and also show that the Pressure at any point in fluid depends on height of the fluid [10]

Pascal's law

$\rightarrow$ The pressure of the liquid at any point will be same from all direction

2 Marks



Consider a Widge liquid Element of dimension dx, dy and z let, P_x P_y and P_z be the Pressure applied in $x \rightarrow y$ and in Inclined z direction.

Resolving along x - axis

$$P_x dx(1) - P_z dz \sin(90-\theta) = 0$$

$$P_x dx - P_z dx = 0$$

$$dx(P_x - P_z) = 0$$

$$P_x - P_z = 0 \quad -(1)$$

Resolving in y-direction

$$P_y\,dy(1) - P_z\,ds\cos(90-\theta) = 0$$

$$P_y\,dy - P_z\,dy = 0$$

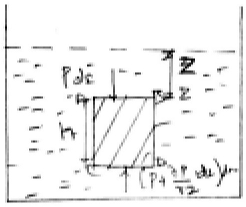
$$dy\left[P_y - P_z\right] = 0$$

$$P_y - P_z = 0 \quad -(2)$$

3 Marks

from equation (1) and (2)

$$P_y = P_z = P_x.$$



2 Marks

Consider a liquid Element placed in a beaker at a distance Z from the top Surface.

The pressure Exerted on Surface AB = PdA

The pressure Exerted on Surface CD =

$$\left(P + \frac{\partial P}{\partial z}dz\right).dA$$

$$\therefore\ PdA - \left(P + \frac{\partial P}{\partial z}dz\right) + \rho g A.dz$$

$$-\frac{\partial P}{\partial z}dz = -\rho g\,dz.$$

$$\therefore\ -\partial p = -\rho g\,\partial z$$

$$= \partial p = \rho g\,\partial z$$

$$\therefore\ \boxed{P = \rho g Z}$$

The above Equation tells ρg is Constant value and pressure is depend on 'Z' (i.e height of fluid).

3 Marks

PART – C (Answer all questions)

6 A U-tube differential manometer containing mercury is connected on one side to pipe A containing carbon tetrachloride (Sp.Gr 1.6) under a pressure of 120kPa, and on the other side to pipe B containing oil (Sp.Gr 0.8) under a pressure of 200kPa. The pipe A lies 2.5m above pipe B and the mercury level in the limb communicating with pipe A lies 4m below the pipe A. Determine the difference in the levels of mercury in the two limbs of the manometer. [10]

$P_A = 120\,kPa = 120\times10^3\,Pa$

$P_B = 200\,kPa = 200\times10^3\,Pa$

$\rho_A = Sp.Gr \times \rho_{std}$

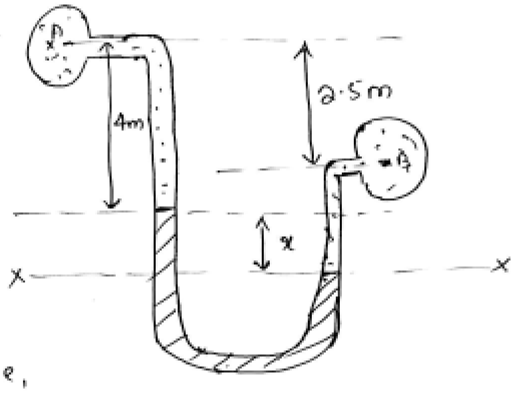
$\rho_A = 1.6\times1000$

$= 1600\,kg/m^3$

$\rho_B = 0.8\times\rho_{std}$

$= 800\,kg/m^3.$

$\rho = 13600\,kg/m^3.$



From figure we have,

$$P_A + (\rho_A\times g\times 4) + \rho g x = P_B + \rho_B g\,((4-2.5)+x)$$

$$120\times10^3 + (1600\times9.8\times4) + (13600\times9.8\times x)$$
$$= 200\times10^3 + [800\times9.8\,(1.5+x)]$$

$$\cancel{1200000}\ \cancel{130}\ 133280x - 7840x - 11760$$
$$= 200000 - 120000 - 62720$$

$$\Rightarrow \quad 125440x = 29040$$

$$\Rightarrow \quad x = 0.2315\,m.$$

Data- 2 Marks

Diagram- 2 Marks

Solution- 6 Marks

7 Is the motion $u=x^2y$; $v= 2yz-xy^2$; $w= x^2-z^2$ kinematically possible for steady flow of an incompressible fluid. [10]

$u = x^2 y$

$v = 2yz - xy^2$

$w = x^2 - z^2$

for Compressible fluid we need to check

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad -(1)$$

$\therefore \; u = x^2 y$

Partially diff w.r.t x.

$$\frac{\partial u}{\partial x} = 2xy \quad -(2)$$

$v = 2yz - xy^2$

Partially diff w.r.t y

$$\frac{\partial v}{\partial y} = 2z - 2xy \quad -(3)$$

~~Partiall~~ $w = x^2 - z^2$

Partially diff w.r.t z

$$\frac{\partial w}{\partial z} = -2z \quad -(4)$$

Substituting eq (2), (3), (4) in Eq (1)

$$\therefore \; 2xy + [2z - 2xy] + [-2z]$$

$$\cancel{2xy} + \cancel{2z} - \cancel{2xy} - \cancel{2z}$$

$$= 0$$

$\therefore$ Hence the Continuity Equation is Satisfied. ~~for~~ ~~Incompr~~

Hence the given motion, is possible for Steady flow for in Compressible fluid

OR

Identifying the continuity equation- 2 Marks

Differentiation – 5 Marks

Substitution and Proof- 3 Marks

Given $u = \frac{kx}{(x^2+y^2+z^2)^{3/2}}$ and $= \frac{ky}{(x^2+y^2+z^2)^{3/2}}$, find an expression for velocity component w in a three-dimensional incompressible fluid flow.

[10]

Differentiation – 4 Marks

Substitution and identification – 3 Marks

Integration – 3 Marks

$$u = \frac{kx}{(x^2+y^2+z^2)^{3/2}} \qquad V = \frac{ky}{(x^2+y^2+z^2)^{3/2}} \qquad w = ?$$

$$\frac{\partial u}{\partial x} = k\left[\frac{(x^2+y^2+z^2)^{3/2}(1) - x\left(\frac{3}{2}\right)(x^2+y^2+z^2)^{1/2} \times 2x}{\left((x^2+y^2+z^2)^{3/2}\right)^2}\right]$$

$$= (x^2+y^2+z^2)^{1/2} k\left[\frac{x^2+y^2+z^2 - \frac{3x}{2}(2x)}{(x^2+y^2+z^2)^3}\right]$$

$$= \frac{k\left[(x^2+y^2+z^2) - 3x^2\right]}{(x^2+y^2+z^2)^{5/2}} \quad \text{— (i)}$$

$$\frac{\partial v}{\partial y} = k\left[\frac{(x^2+y^2+z^2)^{3/2}(1) - \frac{3y}{2}(x^2+y^2+z^2)^{1/2}\, 2y}{\left((x^2+y^2+z^2)^{3/2}\right)^2}\right]$$

$$= k\left[\frac{x^2+y^2+z^2 - 3y^2}{(x^2+y^2+z^2)^{5/2}}\right]$$

We have $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ for an incompressible fluid

$$\therefore \frac{\partial w}{\partial z} = -k\left(\frac{x^2+y^2+z^2-3x^2}{(x^2+y^2+z^2)^{5/2}} + \frac{x^2+y^2+z^2-3y^2}{(x^2+y^2+z^2)^{5/2}}\right)$$

$$= -k\left[\frac{-x^2-y^2+2z^2}{(x^2+y^2+z^2)^{5/2}}\right]$$

$$= \frac{k(x^2+y^2-2z^2)}{(x^2+y^2+z^2)^{5/2}}$$

$$\frac{\partial w}{\partial z} = \frac{k(x^2+y^2+z^2-3z^2)}{(x^2+y^2+z^2)^{5/2}} \quad \text{— (1)}$$

Integrating the eqn (1) we get

$$\boxed{w = \frac{kz}{(x^2+y^2+z^2)^{3/2}}}$$