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Internal Assessment Test II

Sub	ENGINEERING MATHEMATICS IV (REG)					Code	15MAT41
Date	08 / 05 / 2017	Duration	90 mins	Max Marks	50	Sem IV	Branch	TCE A,B

	Question 1 is compulsory. Answer any SIX questions from the rest.	**Marks**	**OBE**	
	REGULAR		**CO**	**RBT**
1.	In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation of the distribution, given F(0.5)=0.19, F(1.4)=0.42	**08**	**401.4**	**L3**
2.	The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1. If 10 lines are chosen at random, what is the probability that a) no line is busy b) all lines are busy c) at least one line is busy d) at most 2 lines are busy.	**07**	**401.4**	**L3**
3.	The probability that a newsreader commits no mistake in reading the news is $1/e^3$. Find the probability that, on a particular news broadcast, he commits a) only 2 mistakes b) more than 3 mistakes c) at most 3 mistakes.	**07**	**401.4**	**L3**
4.	Derive the mean and variance of exponential distribution.	**07**	**401.4**	**L2**
5.	The pdf of a continuous random variate **x** is given by $f(x) = kx^2$, $0 < x < 3$ and f(x) = 0 elsewhere. Find k. P(x > 1), P(x < 1), P(1 < x < 2).Also find the mean, variance and standard deviation of the distribution.	**07**	**401.4**	**L3**

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7.	Derive Rodrigue's formula	07	401.2	L2
8.	Prove that (i) $J_n(-x) = (-1)^n J_n(x) = J_{-n}(x)$ and (ii) $\frac{d}{dx}\left[x^n J_n(x)\right]=x^n J_{n-1}(x)$	07	401.2	L3
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Scheme & Solutions of II - IAT. Engineering Mathematics - IV (1)

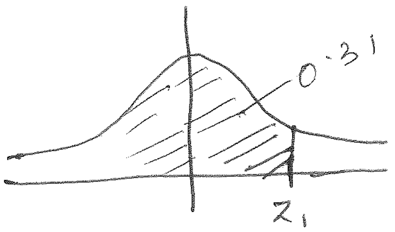
TCE A&B - 08-05-2017. (15MAT41). (D. PRATHAP)

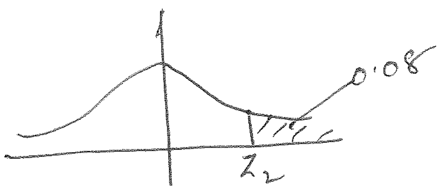
Solns: 1. Let μ & σ be the Mean & SD. of Normal Distribution
x be the r.v. of Normal distribution

Given $P(x < 45) = 31\% = 0.31$ & $P(x > 64) = 8\% = 0.08$ → (1M)

We know $Z = \frac{x-\mu}{\sigma}$; So let $\frac{45-\mu}{\sigma} = z_1$ & $\frac{64-\mu}{\sigma} = z_2$.

$\therefore P(z < z_1) = 0.31$ & $P(z > z_2) = 0.08$





→ (1M)

$0.5 + P(0 < z < z_1) = 0.31$

$\Rightarrow P(0 < z < z_1) = -0.19$

From Std Normal Tables

$\phi(z_1) = P(0 < z < z_1) = -\phi(0.5)$

$\because \phi(0.5) = 0.1915 \approx 0.19$

$\Rightarrow z_1 = -0.5$

$\Rightarrow \frac{45-\mu}{\sigma} = -0.5$

$\Rightarrow \mu - 0.5\sigma = 45$ → (1) (2M)

$0.5 - P(0 < z < z_2) = 0.08$

$P(0 < z < z_2) = 0.5 - 0.08 = 0.42$ → (2M)

$\phi(z_2) = P(0 < z < z_2) = 0.42$
$= \phi(1.4)$

$\because \phi(1.4) = .4192 \approx 0.42$

$\therefore z_2 = 1.4$

$\Rightarrow \frac{64-\mu}{\sigma} = 1.4$

$\Rightarrow \mu + 1.4\sigma = 64$ → (2) (1M)

Solving (1) & (2) we get $\mu = 50$ & $\sigma = SD = 10$ → (1M)

2. Let x be the r.v that denote the no of telephone lines busy. Given $p = 0.1$ $q = 1-p = 0.9$; $n = 10$. → (1M)

We know $p(x) = nC_x \, p^x q^{n-x} = 10C_x \, p^x q^{10-x}$ ($\because n = 10$).

a) Prob. that no line is busy $= P(x=0) = P(0) = 10C_0 p^0 q^{10} = (0.9)^{10}$
$= 0.3487$ → (2M)

b) Prob that all lines are busy $= P(10) = P(x=10) = 10C_{10} p^{10} q^0$ → (1M)
$= (0.1)^{10} =$

c) Prob that atleast one line is busy $= 1 - P(x=0)$

$= 1 -$ Prob. that no line is busy

$= 1 - p(x=0) = 1 - 0.3487 = 0.6513 \longrightarrow$ (1M)

(d) Prob. that atmost 2 lines busy $= P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= 10C_0 \, p^0 q^{10} + 10C_1 \, p^1 q^9 + 10C_2 \, p^2 q^8$

$= (0.9)^{10} + 10(0.1)(0.9)^9 + 45(0.1)^2(0.9)^8 = 0.9298$

$\longrightarrow$ (2M).

3) Let x be the r.v that denotes the no of mistakes of a newsreader during a broadcast.

We know $P(x) = \dfrac{m^x e^{-m}}{x!}$, Given $P(x=0) = e^{-3}$

$\Rightarrow \dfrac{m^0 e^{-m}}{0!} = e^{-3}$

$\Rightarrow e^{-m} = e^{-3} \Rightarrow \boxed{m=3}$

$\longrightarrow$ (2M)

i) Prob. of comitting only 2 mistakes

$= P(x=2) = P(2) = \dfrac{3^2 e^{-3}}{2!} = 0.2240 \longrightarrow$ (1M)

ii) P (commiting more than 3 mistakes) $= P(x > 3)$

$= 1 - P(x \leq 3) = 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3)]$

$= 1 - \left[\dfrac{m^0 e^{-m}}{0!} + \dfrac{m^1 e^{-m}}{1!} + \dfrac{m^2 e^{-m}}{2!} + \dfrac{m^3 e^{-m}}{3!}\right] = 1 - \left[e^{-3} + 3e^{-3} + \dfrac{9e^{-3}}{2} + \dfrac{27e^{-3}}{6}\right]$

$= 1 - e^{-3}\left(1 + 3 + \dfrac{9}{2} + \dfrac{27}{6}\right) = 0.3528 \longrightarrow$ (2M)

iii) Prob of Committing at most 3 mistakes $= P(x \leq 3)$

$= 1 - P(x > 3) = 1 - 0.3528 = 0.6472 \longrightarrow$ (2M)

3

4) We know Exponential distribution has $f(x) = \begin{cases} \alpha e^{-\alpha x}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$

where $\alpha > 0$

is the prob. density fn $\longrightarrow$ (1M)

Its Mean $\mu = \int_{-\infty}^{\infty} x f(x) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{\infty} x f(x)\, dx$

$$= 0 + \int_0^{\infty} x \cdot \alpha e^{-\alpha x}\, dx = \alpha \cdot \left[x \cdot \left(\frac{e^{-\alpha x}}{-\alpha} \right)_0^{\infty} - \int_0^{\infty} \frac{e^{-\alpha x}}{-\alpha} \cdot dx \right]$$

$$= \alpha \left[\frac{-1}{\alpha} \left[x e^{-\alpha x} \right]_0^{\infty} + \frac{1}{\alpha} \left(\frac{e^{-\alpha x}}{-\alpha} \right)_0^{\infty} \right] = \alpha \left[0 - \frac{1}{\alpha^2} (0 - 1) \right] = \frac{1}{\alpha}$$

$\therefore \boxed{\mu = \frac{1}{\alpha}}$ $\because$ $x / e^{\alpha x} \to 0$ as $x \to \infty$ by L'Hospital's rule

$\longrightarrow$ (3M)

$$\text{Variance} = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx = \alpha \int_0^{\infty} (x-\mu)^2 e^{-\alpha x}\, dx$$

$$= \alpha \left[(x-\mu)^2 \left(\frac{e^{-\alpha x}}{-\alpha} \right) - 2(x-\mu) \left(\frac{e^{-\alpha x}}{(-\alpha)^2} \right) + 2 \left(\frac{e^{-\alpha x}}{(-\alpha)^3} \right) \right]_0^{\infty}$$

$$= \alpha \left[\frac{-1}{\alpha} (0 - \mu^2) - \frac{2}{\alpha^2} (0 - (-\mu)) - \frac{2}{\alpha^3} (0 - 1) \right].$$

$$= \alpha \left[\frac{\mu^2}{\alpha} - \frac{2\mu}{\alpha^2} + \frac{2}{\alpha^3} \right] = \frac{1}{\alpha^2} \quad \boxed{\because \mu = \frac{1}{\alpha}}.$$

$\therefore \sigma^2 = \frac{1}{\alpha^2} \Rightarrow \boxed{\sigma = \frac{1}{\alpha}}.$ $\longrightarrow$ (3M)

5) $\because f(x)$ is a prob. density fn we've $\int_{-\infty}^{\infty} f(x)\,dx = 1$ $\longrightarrow$ (1M)

$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = 1 \quad \Rightarrow 0 + \int_{0}^{3} kx^2\,dx + 0 = 1$

$\Rightarrow k\cdot\left(\frac{x^3}{3}\right)_0^3 = 1 \quad \Rightarrow \frac{k}{3}(27-0) = 1 \quad \Rightarrow \boxed{k = \frac{1}{9}}$ $\longrightarrow$ (1M)

i) $P(x>1) = \int_{1}^{\infty} f(x)\,dx = \int_{1}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = \int_{1}^{3} \frac{1}{9}x^2\,dx =$

$\frac{1}{9}\left(\frac{x^3}{3}\right)_1^3 = \frac{1}{27}(27-1) = \frac{26}{27}$ $\longrightarrow$ (1M)

ii) $P(x<1) = \int_{-\infty}^{1} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx = 0 + \int_{0}^{1} \frac{1}{9}x^2\,dx$

$= \frac{1}{9}\left(\frac{x^3}{3}\right)_0^1 = \frac{1}{9}\left(\frac{1}{3} - 0\right) = \frac{1}{27}$ $\longrightarrow$ (1M)

iii) $P(1<x<2) = \int_{1}^{2} f(x)\,dx = \int_{1}^{2} \frac{1}{9}x^2\,dx = \frac{1}{9}\left(\frac{x^3}{3}\right)_1^2 = \frac{1}{27}(8-1) = \frac{7}{27}$ $\longrightarrow$ (1M)

iv) Mean $= \mu = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{3} x f(x)\,dx + \int_{3}^{\infty} x f(x)\,dx$

$= 0 + \int_{0}^{3} x\cdot\frac{1}{9}x^2\,dx + 0 = \frac{1}{9}\left(\frac{x^4}{4}\right)_0^3 = \frac{1}{36}(81-0) = \frac{81}{36}$ $\longrightarrow$ (1M)

v) Variance $= \sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2 = \int_{-\infty}^{0} x^2 f(x)\,dx + \int_{0}^{3} x^2 f(x)\,dx + \int_{3}^{\infty} x^2 f(x)\,dx - \left(\frac{81}{36}\right)^2$

$= 0 + \int_{0}^{3} \frac{1}{9}x^2\cdot x^2\,dx - \left(\frac{9}{4}\right)^2 = \frac{1}{9}\left(\frac{x^5}{5}\right)_0^3 - \left(\frac{9}{4}\right)^2 = \frac{1}{45}(3^5-0) - \left(\frac{9}{4}\right)^2$

$= \frac{243}{45} - \frac{81}{16} = \frac{81}{15} - \frac{81}{16} = 81\left(\frac{1}{15} - \frac{1}{16}\right) = 81\cdot\frac{1}{240} = \frac{27}{80}$

$\Rightarrow \sigma^2 = \frac{27}{80}$ & $\sigma = \sqrt{\frac{27}{80}}$ $\longrightarrow$ (1M)

(6) Series soln for Bessel's Differential Equation

Soln:- Bessel's DE of order n is $x^2y'' + xy' + (x^2-n^2)y = 0 \longrightarrow (1)$

Using Frobenius method, we assume $y = \sum_{r=0}^{\infty} a_r x^{k+r}$ as a soln, $a_0 \neq 0$ $\longrightarrow (2)$

$\Rightarrow y' = \sum a_r (k+r) x^{k+r-1} \longrightarrow (3)$; $y'' = \sum a_r (k+r)(k+r-1) x^{k+r-2} \longrightarrow (4)$

Using (2), (3), (4) in (1) we get $\longrightarrow$ (1M)

$$\sum a_r (k+r)(k+r-1) x^{k+r} + \sum a_r (k+r) x^{k+r} + \sum a_r x^{k+r+2} - n^2 \sum a_r x^{k+r} = 0$$

$$\Rightarrow \sum a_r x^{k+r} \left\{ (k+r)(k+r-1) + (k+r) - n^2 \right\} + \sum a_r x^{k+r+2} = 0$$

$$\Rightarrow \sum a_r x^{k+r} \left((k+r)\{(k+r-1+1\} - n^2 \right) + \sum a_r x^{k+r+2} = 0$$

$$\Rightarrow \sum a_r x^{k+r} \left((k+r)^2 - n^2 \right) + \sum a_r x^{k+r+2} = 0 \longrightarrow \text{(1M)}$$

Equating coefft of lowest degree term in x to zero, we get. (ie x^k)

$$a_0 (k^2 - n^2) = 0 \quad (\because a_0 \neq 0), \quad k = \pm n . \longrightarrow \text{(1M)}$$

Equating Coefft of x^{k+1} to zero

$$a_1 \left((k+1)^2 - n^2 \right) = 0 \Rightarrow a_1 = 0 \because k+1 \neq \pm n \longrightarrow \text{(1M)}$$

Illly equating coefft of x^{k+r} to zero we get

$$a_r \left((k+r)^2 - n^2 \right) + a_{r-2} = 0 \Rightarrow \boxed{a_r = \frac{-a_{r-2}}{(k+r)^2 - n^2}}, \; r \geq 2 \longrightarrow (5)$$

When $k = n$ (5) $\Rightarrow a_r = \dfrac{-a_{r-2}}{(n+r)^2 - n^2} \Rightarrow \boxed{a_r = \dfrac{-a_{r-2}}{2nr + r^2}}$

$\Rightarrow a_2 = \dfrac{-a_0}{4(n+1)}$; $a_3 = \dfrac{-a_1}{6n+9} = 0 \; (\because a_1 = 0)$ $a_4 = \dfrac{-a_2}{8(n+2)}$; $a_5 = 0 \; \ldots$ $\longrightarrow$ (1M)

So if the soln when $k=n$ is denoted by y_1 then

$$y_1 = x^k\left(a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots\right), \quad k=n$$

$$= x^k\left[a_0 - \frac{a_0}{4(n+1)}x^2 + \frac{a_0}{32(n+1)(n+2)}x^4 - \cdots\right], \quad k=n$$

$$= a_0 x^n\left[1 - \frac{x^2}{2^2(n+1)} + \frac{x^4}{2^5(n+1)(n+2)} - \cdots\right].$$

|||ly if y_2 denotes the soln when $k=-n$ then

$$y_2 = a_0 x^{-n}\left[1 - \frac{x^2}{2^2(-n+1)} + \frac{x^4}{2^5(-n+1)(-n+2)} - \cdots\right].$$

$\therefore$ The general soln of ① is given by $\boxed{y = Ay_1 + By_2}$ $\longrightarrow$ 2M

(OR)

By defn

$$J_n(x) = \sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{\Gamma(n+r+1)\, r!}$$

taking $n = 1/2$ we get $J_{1/2}(x) = \dfrac{\sum (-1)^r \left(\frac{x}{2}\right)^{\frac{1}{2}+2r}}{\Gamma\left(\frac{1}{2}+r+1\right)\cdot r!}$

$$\Rightarrow J_{1/2}(x) = \sum (-1)^r \sqrt{\frac{x}{2}} \cdot \left(\frac{x}{2}\right)^{2r} \cdot \frac{1}{\Gamma\left(r+\frac{3}{2}\right) r!}$$

$$\Rightarrow J_{1/2}(x) = \sqrt{\frac{x}{2}}\left[\frac{1}{\Gamma\left(\frac{3}{2}\right)} - \left(\frac{x}{2}\right)^2 \frac{1}{\Gamma\left(\frac{5}{2}\right) 1!} + \left(\frac{x}{2}\right)^4 \cdot \frac{1}{\Gamma\left(\frac{7}{2}\right)\cdot 2!} - \cdots\right]$$

we know $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$; $\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2}$; $\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{3}{4}\sqrt{\pi}$

So $J_{1/2}(x) = \sqrt{\frac{x}{2}}\left[\frac{2}{\sqrt{\pi}} - \frac{x^2}{4}\cdot\frac{4}{3\sqrt{\pi}} + \frac{x^4}{16}\cdot\frac{8}{15\sqrt{\pi}\cdot 2} - \cdots\right]$ $\longrightarrow$ 4M

So $J_{1/2}(x) = \sqrt{\frac{x}{2}}\frac{1}{\sqrt{\pi}}\left[2 - \frac{x^2}{3\sqrt{\pi}} + \frac{x^4}{4}\cdot\frac{1}{15\sqrt{\pi}} - \cdots\right]$ (7)

$$= \frac{\sqrt{x}}{\sqrt{2\pi}}\cdot\frac{2}{x}\left[x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots\right]$$

$$= \sqrt{\frac{2}{\pi x}}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right) = \sqrt{\frac{2}{\pi x}}\sin x.$$

$$J_{-1/2}(x) = \sum (-1)^r \left(\frac{x}{2}\right)^{-\frac{1}{2}+2r}\frac{1}{\Gamma(r+\frac{1}{2})\,r!} = \sqrt{\frac{2}{x}}\sum (-1)^r\left(\frac{x}{2}\right)^{2r}\cdot\frac{1}{\Gamma(r+\frac{1}{2})\,r!}$$

$$= \sqrt{\frac{2}{x}}\left[\frac{1}{\Gamma(\frac{1}{2})} - \left(\frac{x}{2}\right)^2\cdot\frac{1}{\Gamma(\frac{3}{2})\cdot 1!} + \left(\frac{x}{2}\right)^4\cdot\frac{1}{\Gamma(\frac{5}{2})\cdot 2!} - \cdots\right]$$

$$= \sqrt{\frac{2}{x}}\left[\frac{1}{\sqrt{\pi}}\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right)\right]$$

$$= \sqrt{\frac{2}{\pi x}}\cos x. \longrightarrow \text{(3M)}$$

(7) Rodrigues formula

Let $u = (x^2-1)^n$ $\longrightarrow$ (1) & $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ be the Legendre DE $\longrightarrow$ (2)

$\longrightarrow$ (1M)

$\Rightarrow \frac{du_1}{dx} = n(x^2-1)^{n-1}2x \Rightarrow (x^2-1)\frac{du}{dx} = n(x^2-1)^n 2x$

$\Rightarrow (x^2-1)u_1 = 2nxu.$

$\Rightarrow (x^2-1)u_2 + 2xu_1 = 2n(xu_1 + u)$

D'ting n times using Leibnitz for the n^{th} derivative we get

$$(x^2-1)u_{n+2} + 2nxu_{n+1} + (n^2-n)u_n + 2xu_{n+1} + 2nu_n = 2nxu_{n+1} + 2n^2u_n + 2nu_n.$$

$\Rightarrow (x^2-1)u_{n+2} + 2xu_{n+1} - n(n+1)u_n = 0$

$\Rightarrow (1-x^2)u_{n+2} - 2xu_{n+1} + n(n+1)u_n = 0 \longrightarrow$ (3)

$\Rightarrow$ u_n is a soln of Legendre's Diff Eqn $\longrightarrow$ (2M)

$\because$ u is a polynomial of degree $2n$, u_n is a polynomial of degree n. $\longrightarrow$ 1M

Also $P_n(x)$ which satisfies Legendre's DE is a polynomial of degree n. $\therefore$ $P_n(x) = k u_n$ for some const. k

$$\Rightarrow P_n(x) = k\left[(x^2-1)^n\right]_n \Rightarrow P_n(x) = k\left[(x-1)^n(x+1)^n\right]_n \longrightarrow \text{1M}$$

Applying again Leibnitz thm on the RHS we've

$$P_n(x) = k\left[(x-1)^n\{x+1\}_n + n(n)(n-1)^{n-1}\{(1+1)^n\}_{n-1} + \frac{n(n+1)}{2}n(n-1)x^{n-2}\{(x+1)^n\}_{n-2} + \cdots + \{(x-1)^n\}_n(x+1)^n\right]$$

Let $z = (x-1)^n$ then $z_1 = n(n-1)x^{n-1}$ $z_2 = n(n-1)x^{n-2}$

$\cdots z_n = n(n-1)(n-2)\cdots 2.1\,(x-1)^{n-n} \Rightarrow z_n = n!\,(x-1)^0 = n!$.

$$\therefore \{(x-1)^n\}_n = n!$$

$\therefore$ $P_n(1) = k \cdot n!\, 2^n$ & $P_n(1) = 1 \Rightarrow k = \dfrac{1}{n!\,2^n}$

$$\therefore \boxed{P_n(x) = \frac{1}{n!\,2^n}\frac{d^n}{dx^n}(x^2-1)^n} \longrightarrow \text{2M}$$

(a) We've $J_n(x) = \displaystyle\sum_{r=0}^{\infty}(-1)^r\left(\frac{x}{2}\right)^{n+2r}\frac{1}{\Gamma(n+r+1)\,r!}$ $\longrightarrow$ 1M

$$\Rightarrow J_n(-x) = \sum_{r=0}^{\infty}(-1)^r\left(-\frac{x}{2}\right)^{n+2r}\cdot\frac{1}{\Gamma(n+r+1)\cdot r!} = \sum_{r=0}^{\infty}(-1)^r(-1)^{n+2r}\left(\frac{x}{2}\right)^{n+2r}\cdot\frac{1}{\Gamma(n+r+1)\,r!}$$

$$= (-1)^n\sum\left((-1)^3\right)^r\left(\frac{x}{2}\right)^{n+2r}\frac{1}{\Gamma(n+r+1)\,r!} = (-1)^n J_n(x) \longrightarrow \text{4M}$$

also $J_n(x) = (-1)^n J_{-n}(x)$ so $J_n(-x) = J_{-n}(x)$

$$\therefore J_n(-x) = (-1)^n J_n(x) = J_{-n}(x) \longrightarrow \text{2M}$$

9 Let $f(x) = x^4 - 3x^2 + x$

We know $x^4 = \frac{8}{35} P_4(x) + \frac{4}{7} P_2(x) + \frac{1}{5} P_0(x)$

$x^2 = \frac{1}{3} P_0(x) + \frac{2}{3} P_2(x)$ → 3M

$x = P_1(x)$

Substituting these in $f(x)$ we get

$$f(x) = x^4 - 3x^2 + x = \frac{8}{35} P_4(x) + \frac{4}{7} P_2(x) + \frac{1}{5} P_0(x) - 3\left[\frac{1}{3} P_0(x) + \frac{2}{3} P_2(x)\right] + P_1(x)$$

$$= \frac{8}{35} P_4(x) + \left(\frac{4}{7} - 2\right) P_2(x) + P_1(x) + \left(\frac{1}{5} - 1\right) P_0(x)$$

$$= \frac{8}{35} P_4(x) - \frac{10}{7} P_2(x) + P_1(x) - \frac{4}{5} P_0(x).$$

→ 4M