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Internal Assessment Test II

Sub	ENGINEERING MATHEMATICS IV							Code:	15MAT41
Date:	8 / 5/ 2017	Duration	90 mins	Max Marks	50	Sem	IV	Branch	CSE-A,B, ISE-A

First question is compulsary. Answer ANY SIX questions from Q2 toQ8

Question	Marks	OBE CO	OBE RBT
1. Discuss the transformation $w = z^2$.	[08]	CO3	L3
2. Use Cauchy's residue theorem to evaluate the integral $\oint_C \frac{z^2}{(z-1)^2(z+2)}dz, where\ C:\|Z\|=3.$	[07]	CO3	L3
3. Find the mean and standard deviation of Binomial distribution.	[07]	CO4	L3
4. In a certain factory turning out razor blades, there is a small chance of 0.002 for a blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing i) no defective ii) one defective iii) two defective blades, in a consignment of 10,000 packets.	[07]	CO4	L3

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5. The marks of 1000 students in an examination follow a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) between 65 and 70 (iii) more than 75, given $\Phi(1)=.3413$. [07] CO4 L3

6. The sales per day in a shop are exponentially distributed with the average sale amounting to Rs. 100 and net profit 8%. Find the probability that the net profit exceeds Rs. 30 on two consecutive days. [07] CO4 L3

7. A fair coin is tossed thrice. The random variables X and Y are defined as follows: X = 0 or 1 according as head or tail occurs on the first; Y = Number of heads. Determine (i) the distribution of X and Y (ii) joint distribution of X and Y. [07] CO4 L3

8. The joint probability distribution for two random variables X and Y is given below: [07] CO4 L3

X \ Y	-2	-1	4	5
1	0.1	0.2	0.0	0.3
2	0.2	0.1	0.1	0.0

Determine (i) marginal distribution of X and Y (ii) COV(X,Y) (iii) ρ(X,Y)

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<u>Solution Manual</u>

<u>IAT-2 (M4) - CS-A&B, IS-A</u>

(2017)

1. Consider $w = z^2$

$u + iv = (x^2 - y^2) + i(2xy)$

$\therefore \quad u = x^2 - y^2, \quad v = 2xy \quad$ — (1)

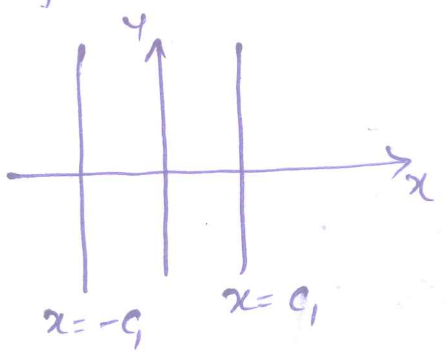
eliminate $y \Rightarrow y = \frac{v}{2x}$

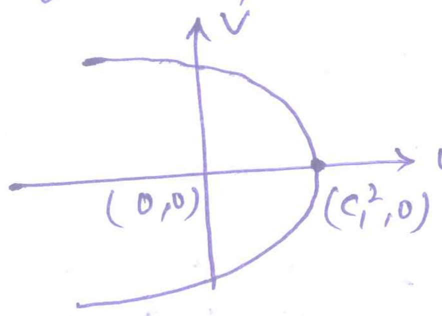
$\therefore \quad u = x^2 - \frac{v^2}{(2x)^2} \Rightarrow v^2 = -4x^2(u - x^2)$

<u>Case-1</u> Consider $x = $ const. (c_1 say)

$\therefore \quad v^2 = -4c_1^2(u - c_1^2)$, which is a parabola with vertex $(c_1^2, 0)$ and focus at origin

If $x = -c_1$ then also we get same parabola





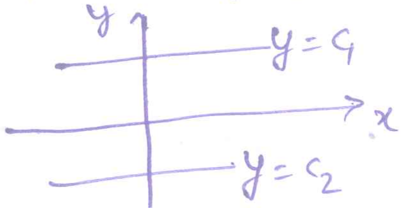
— $(2\frac{1}{2})$

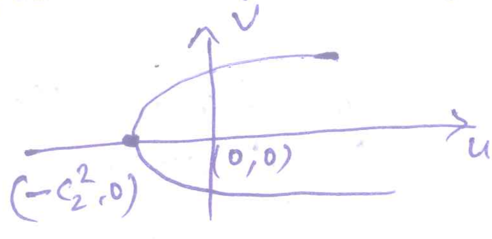
Now eliminate $x \Rightarrow x = \frac{v}{2y}$

we get $v^2 = 4y^2(u + y^2)$

<u>Case-2</u> Consider $y = $ const. (say c_2)

$\therefore \quad v^2 = 4c_2^2(u + c_2^2)$ which is a parabola with vertex $(-c_2^2, 0)$ and focus at origin. for $y = -c_2$ we get the sam





— $(2\frac{1}{2})$

Case-3 Let us consider a circle in z-plane

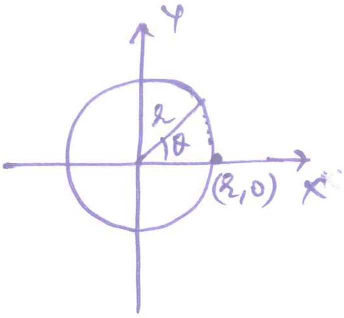
$\therefore \quad |z| = r \Rightarrow z = re^{i\theta}$

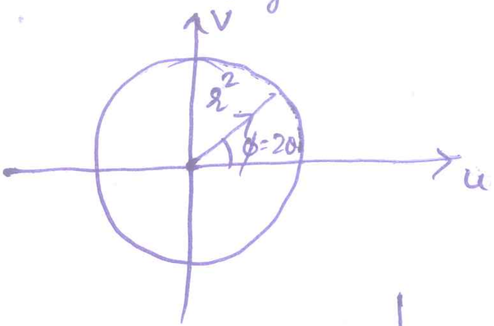
Now for $w = z^2$

$$u + iv = r^2 (e^{i2\theta}) = r^2(\cos 2\theta + i \sin 2\theta)$$

$$= R^2 e^{i\phi}$$

This is also a circle with centre origin and radius $R = r^2$

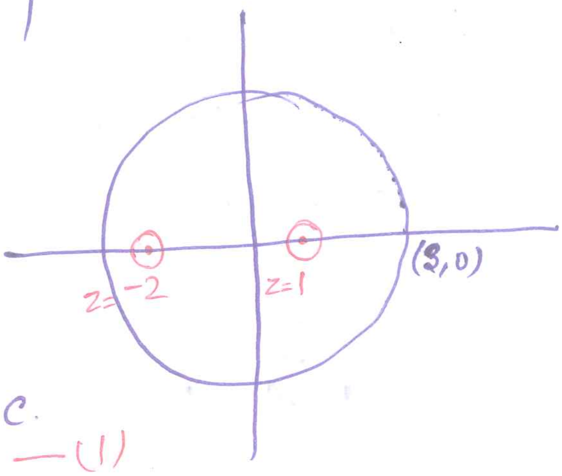




— (2)

2. ~~Case~~

$$\oint_C \frac{z^2}{(z-1)^2(z+2)} dz$$



Here $z=1$ is a double pole and $z=-2$ is a simple pole } in C. — (1)

$$\underset{z=1}{\text{Res}} f(z) = \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz}\left[\cancel{(z-1)^2} \frac{z^2}{\cancel{(z-1)^2}(z+2)}\right]$$

$$= \lim_{z \to 1} \frac{d}{dz}\left(\frac{z^2}{(z+2)}\right) = \lim_{z \to 1}\left[\frac{(z+2) \times 2z - z^2}{(z+2)^2}\right]$$

$$= \frac{3 \times 2 - 1}{(3)^2} = \frac{5}{9} \qquad \text{— (2)}$$

Sly $\underset{z=-2}{\text{Res.}} f(z) = \lim_{z \to -2}\left[\cancel{(z+2)} \frac{z^2}{(z-1)^2 \cancel{(z+2)}}\right]$

$$= \frac{4}{9} \qquad \text{— (2)}$$

$\therefore$ By Cauchy's Residue theorem

$$\oint_C f(z) dz = 2\pi i(\text{Sum of Residues}) = 2\pi i \qquad \text{— (2)}$$

(3.) Prob. function $p(x)$ for Bino. Dist. is given by - ${}^nC_x p^x q^{n-x}$

$$\text{Mean } (\mu) = \sum_{x=0}^{n} x\, P(x)$$

$$= \sum_{0}^{n} x \cdot {}^nC_x\, p^x q^{n-x}$$

$$= \sum_{0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^x q^{n-x} = \sum_{x=1}^{n} \frac{n!}{(x-1)!\,(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n\,(n-1)!}{(x-1)!\,(n-1-(x-1))!} p \cdot p^{x-1} q^{(n-1)-(x-1)}$$

$$\mu = np\,(p+q)^{n-1} = np \qquad (p+q) = 1 \quad \text{— (3)}$$

$$\text{Variance } (V) = \sum_{x=0}^{n} x^2 P(x) - \mu^2$$

$$= \sum_{0}^{n} [x(x-1) + x] P(x) - \mu^2$$

$$= \sum_{x=0}^{n} x(x-1)\, {}^nC_x\, p^x q^{n-x} + \sum_{0}^{n} x\, P(x) - \mu^2$$

$$= \sum_{x=2}^{n} \frac{n(n-1)(n-2)!}{(x-2)!\,((n-2)-(x-2))!} p^2 p^{x-2} q^{(n-2)-(x-2)} + \mu - \mu^2$$

$$= n(n-1)p^2 (p+q)^{n-2} + \mu - \mu^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2} \qquad (\because p+q=1 \;\&\; \mu = np)$$

$$= np(1-p) = npq$$

$$\therefore \text{S.D. } \sigma = \sqrt{npq} \qquad \text{— (4)}$$

(4) Given $p = 0.002$

In a pkt of 10, the mean is $m = np = 10 \times 0.002 = 0.02$ — (1)

P.D. is $P(x) = \dfrac{m^x e^{-m}}{x!} = \dfrac{e^{-0.02} \times (0.02)^x}{x!}$ — (1)

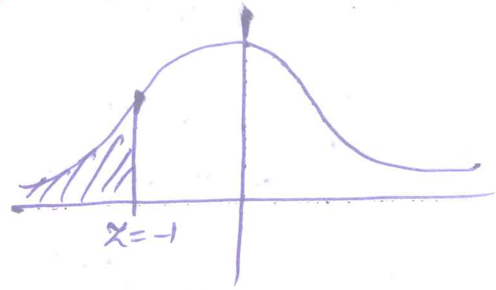
$f(x) = 10000\, P(x) = 10000 \times \dfrac{e^{-0.02}(0.02)^x}{x!}$ — (2)

(i) $f(x=0) = 9802$ —(1) (ii) $f(x=1) = 196$ —(1) (iii) $f(x=2) = 2$ —(1)

(5) X : Marks of Students

Given $\mu = 70$, $\sigma = 5$

$\therefore$ s.n.v. $Z = \dfrac{x-70}{5}$ —(1)



(i) If $x = 65$ then $Z = -1$

$\therefore P(x < 65) = P(Z < -1)$

$= P(Z > 1)$ (symmetry)

$= 0.5 - P(0 < Z < 1) = 0.5 - \phi(1)$

$= 0.5 - 0.3413 = 0.1587$ —(2)

$\therefore$ No of Students $= 1000 \times .1587 = 158.7 \approx 159$

(ii) If $x = 75$, then $Z = 1$

$P(x > 75) = P(Z > 1)$ Same as above —(2)

(iii) $P(65 < x < 75) = P(-1 < Z < 1)$

$= 2P(0 < Z < 1) = 2\phi(1)$

$= 2 \times .3413 = 0.6826$ —(2)

$\therefore$ No of Students $= 1000 \times .6826 = 682.6 \approx 683$

(6) X = Sale in the shop

Given mean $\dfrac{1}{\alpha} = 100 \Rightarrow \alpha = 0.01$ —(1)

$\therefore$ prob func. is

$$f(x) = \begin{cases} 0.01 e^{-0.01x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$ —(1)

Let A be the amount for which profit is 8%.

$\therefore A \times 8\% = 30 \Rightarrow A = \dfrac{100 \times 30}{8} = 375$ —(2)

$\therefore$ Prob (Prof. > 30) = Prob. (Sale > 375) —(1)

$\therefore P(x>375) = \cancel{\int_0^{37}} \int_{375}^{\infty} (0.01)\, e^{-0.01x}\, dx = e^{-3.75}$ (Single day)

— (1)

Prob that it repeats on the following day is

$e^{-3.75} \times e^{-3.75} = e^{-7.5} = 0.00055$ — (1)

(7)

S	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
X	0	0	0	0	1	1	1	1
Y	3	2	2	1	2	1	1	0

— (1)

(i) Marginal Dist. of X & Y

x_i	0	1
$f(x_i)$	1/2	1/2

— (1)

y_j	0	1	2	3
$g(y_j)$	1/8	3/8	3/8	1/8

— (1)

(ii) Joint Prob. dist. of X & Y

Y \ X	0	1	
0	0	1/8 (HTT)	1/8
1	1/8 (TTT)	2/8 (THT, TTH)	3/8
2			3/8
3			1/8
	1/2	1/2	1

X \ Y	0	1	2	3	
0	0	$\frac{1}{8}$ (HTT)	$\frac{2}{8}$ (HHT, HTH)	$\frac{1}{8}$ (HHH)	1/2
1	$\frac{1}{8}$ (TTT)	$\frac{2}{8}$ (THT, TTH)	$\frac{1}{8}$ (THH)	0	1/2
	1/8	3/8	3/8	1/8	1

— (4)

(8) Given

X \ Y	−2	−1	4	5
1	0.1	0.2	0.0	0.3
2	0.2	0.1	0.1	0.0

Marginal Dis. of X & Y.

x_i	1	2
$f(x_i)$	0.6	0.4

y_j	-2	-1	4	5
$g(y_j)$	0.3	0.3	0.1	0.3

$E(X) = \sum x_i f(x_i)$

$= 0.6 + 0.8 = 1.4$ —(1)

$E(Y) = \sum y_j\, g(y_j)$

$= -0.6 - 0.3 + 0.4 + 1.5$

$= 1$ —(1)

$E(XY) = \sum x_i y_j J_{ij} = -0.2 - 0.2 + 0 + 1.5 - 0.\cancel{8} - 0.2 + 0.\cancel{8} + 0$

$= 0.9$

(i) $\operatorname{cov}(X,Y) = E(XY) - E(X).E(Y)$

$= 0.9 - 1.4 \times 1 = 0.5$ —(1)

(ii) $\rho(X,Y) = \dfrac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$ —(1)

for $\sigma_X^2 = \sum x_i^2 f(x_i) - (E(X))^2$

$= (0.6 + 1.6) - (1.4)^2 = 2.2 - 1.96 = 0.24$ —$(1\frac{1}{2})$

$\Rightarrow \sigma_x = .49$

$\sigma_Y^2 = \sum y_j^2\, g(y_j) - (E(Y))^2$

$= (1.2 + 0.3 + 1.6 + 7.5) - (1)^2 = 10.6 - 1 = 9.6$

$\sigma_Y^2 = 3.1$

$\therefore \rho(X,Y) = \dfrac{-0.5}{9.6 \times 3.1} = \cancel{\dfrac{0.5}{29.76}} = \cancel{0.017}$ —$(1\frac{1}{2})$

$= -0.6725$