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Internal Assessment Test II

Sub	ENGINEERING MATHEMATICS IV (REG)						Code	15MAT41	
Date	08 / 05 / 2017	Duration	90 mins	Max Marks	50	Sem	IV	Branch	EC D, EE A,B

		Marks	OBE	
	Question 1 is compulsory. Answer any SIX questions from the rest.		CO	RBT
	REGULAR			
1.	Derive the mean and variance of binomial distribution.	08	401.4	L2
2.	The number of telephone lines busy at an instant of time is a binomial variate with probability 0.1. If 10 lines are chosen at random, what is the probability that a) no line is busy b) all lines are busy c) at least one line is busy d) at most 2 lines are busy.	07	401.4	L3
3.	The probability that a newsreader commits no mistake in reading the news is $1/e^3$. Find the probability that, on a particular news broadcast, he commits a) only 2 mistakes b) more than 3 mistakes c) at most 3 mistakes.	07	401.4	L3
4.	In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation of the distribution, given A(0.5)=0.19, A(1.4)=0.42	07	401.4	L3
5.	The pdf of a continuous random variate x is given by $p(x) = ke^{-\|x\|}, -\infty < x < \infty$. Find k. Also find the mean, variance and standard deviation of the distribution.	07	401.4	L3

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6.	The range of a random variable X={1,2,3,...} and the probabilities of X are such that $P(X=k)=\frac{\lambda^k}{k!}, k=1,2,3,.....$ Find the value of λ and $P(0<X<3)$	07	401.4	L3
7.	The average daily turnout in a medical store is Rs.10000 and the net profit is 8%. If the turnout has an exponential distribution, find the probability that the net profit will exceed Rs.3000 each on two consecutive days.	07	401.4	L3
8.	A fair coin is tossed thrice. The random variables X and Y are defined as follows: X= 0 or 1 accordingly as head or tail occurs in the first toss ; Y = Number of heads. a) Determine the marginal distributions of X and Y. b) Determine the joint distribution of X and Y c) Obtain the expectations of X, Y and XY. Also find standard deviations of X and Y. d) Compute covariance and correlation of X and Y.	07	401.4	L3
9.	Each year a man trades his car for a new car in 3 brands of the popular company Maruthi Udyog Limited. If he has a *Standard* he trades it for *Zen*. If he has a *Zen*, he trades for *Esteem*. If has an *Esteem*, he is just likely to trade for a *new Esteem* or for a *Zen* or a *Standard* one. In 1996, he bought his first car which was Esteem. i)Find the probability that he has a) 1998 Esteem b) 1998 Standard c) 1999 Zen d) 1999 Esteem. ii) In the long run, how often will he have Esteem?	07	401.6	L4

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Engineering Maths IV
II Internals

1. Mean $\mu = E(x) = \sum x\, b(n,p,x)$

$$= \sum_{x=0}^{n} x\, nC_x p^x q^{n-x}$$

$$= 0 \cdot nC_0 p^0 q^{n-0} + 1 \cdot nC_1 p q^{n-1} + 2 \cdot nC_2 p^2 q^{n-2}$$
$$+ 3 \cdot nC_3 p^3 q^{n-3} + \dots + n \cdot nC_n p^n$$

$$= npq^{n-1} + 2\frac{n(n-1)}{2} p^2 q^{n-2} + 3\frac{n(n-1)(n-2)}{3!} p^3 q^{n-3}$$
$$+ \dots n \cdot p^n$$

$$= np\left\{ q^{n-1} + (n-1) p q^{n-2} + \frac{(n-1)(n-2)}{2!} p^2 q^{n-3} + \dots p^{n-1} \right\}$$

$$= np(q+p)^{n-1} = np$$

(3M)

$$\boxed{\mu = np}$$

Variance $\sigma^2 = \sum x^2 b(n,p,x) - \mu^2$

Consider $\sum x^2 b(n,p,x) = \sum \{x(x-1) + x\} b(n,p,x)$

$$= \sum x(x-1) b(n,p,x) + \sum x\, b(n,p,x)$$

$$= \sum x(x-1) b(n,p,x) + \mu$$

$$= \sum x(x-1)$$

$$E(x^2) = \left[2\,{}^nC_2\, p^2 q^{n-2} + 3.2\,{}^nC_3\, p^3 q^{n-3} + 4.3\,{}^nC_4\, p^4 q^{n-4} + \dots + n(n-1)p^n\right] + \mu$$

$$= \left[n(n-1)p^2 q^{n-2} + n(n-1)(n-2)p^3 q^{n-3} + \frac{n(n-1)(n-2)(n-3)}{2!} p^4 q^{n-4} + \dots + n(n-1)p^n\right] + \mu$$

$$= \left[n(n-1)p^2\left[q^{n-2} + (n-2)p q^{n-3} + \frac{(n-2)(n-3)}{2!}p^2 q^{n-4} + \dots p^{n-2}\right]\right] + \mu$$

$$= n(n-1)p^2 (q+p)^{n-2} + \mu = n(n-1)p^2 + \mu$$

$$\sigma^2 = E(x^2) - \mu^2 = n(n-1)p^2 - (np)^2 + np$$

$$= \cancel{n^2p^2} - np^2 - \cancel{n^2p^2} + np$$

$$= np(1-p) = npq$$

$$\sigma^2 = npq \Rightarrow \sigma = \sqrt{npq}$$ 5M

2. The chance that a telephone line is busy is given by $p = 0.1 = 1/10$.

Prob that x lines out of 10 lines are busy is given by

$b(n, p, x) = nC_x p^x q^{n-x}$

$b(10, 0.1, x) = 10C_x (0.1)^x (0.9)^{10-x}$ (3M)

1) $b(0) = 10C_0 (0.1)^0 (0.9)^{10-0} = (0.9)^{10}$
$= 0.3487$

2) $b(10) = 10C_{10} (0.1)^{10} (0.9)^{10-10} = (0.1)^{10}$

3) $b(x \geq 1) = 1 - P(x < 1)$
$= 1 - b(x=0) = 1 - (0.9)^{10}$
$= 0.6513$

4) $b(x \leq 2) = b(0) + b(1) + b(2)$
$= (0.9)^{10} + 10C_1 (0.1)(0.9)^9$
$+ 10C_2 (0.1)^2 (0.9)^8$ (4M)
$= 0.9298$

3. Consider the Poisson distribution

$P(x) = \frac{m^x e^{-m}}{x!}$ x denotes committing a mistake

$P(x=0) = e^{-3}$.

$e^{-m} = e^{-3} \Rightarrow \boxed{m = 3}$

$P(x) = \frac{e^{-3} 3^x}{x!}$ (4M)

i) $P(2) = \frac{e^{-3}3^2}{2!} = 0.22404$

ii) $P(x > 3) = 1 - P(x \leq 3)$

$= 1 - [P(0) + P(1) + P(2) + P(3)]$

$= 1 - \left[e^{-3} + 3e^{-3} + \frac{3^2 e^{-3}}{2!} + \frac{3^3 e^{-3}}{3!}\right]$

$= 1 - e^{-3}(13) = 0.3528$

iii) $P(x \leq 3) = 1 - P(x > 3)$

$= 1 - 0.3528 = 0.6472$

or $P(0) + P(1) + P(2) + P(3)$

$= e^{-3}(13) = 0.6472$ (3M)

4. Let μ and σ be the mean and S.D of the normal distrubution

$P(x < 45) = 0.31$

$P(x > 64) = 0.08$

SNV $Z = \frac{x - \mu}{\sigma}$

$x = 45 \quad Z = \frac{45 - \mu}{\sigma} = z_1$ (say)

$x = 64 \quad Z = \frac{64 - \mu}{\sigma} = z_2$ (say)

$$2K \int_0^{\infty} e^{-x} dx = 1$$

$$2K - (e^{-x})_0^{\infty} = 1$$

$$-2K(e^{-\infty} - e^{0}) = 1$$

$$2K = 1 \Rightarrow \boxed{K = \frac{1}{2}}$$ (3M)

Mean $\mu = \int_{-\infty}^{\infty} x \, p(x) \, dx$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{2} e^{-|x|} dx$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \underbrace{x}_{\text{odd}} \underbrace{e^{-|x|}}_{\text{even}} dx = \text{odd}$$

$$= 0 \qquad \boxed{\mu = 0}$$

Variance $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 p(x) \, dx$

$$= \int_{-\infty}^{\infty} x^2 \frac{1}{2} e^{-|x|} dx = \frac{1}{2} \int_{-\infty}^{\infty} \underbrace{x^2 e^{-|x|}}_{\text{even}} dx$$

$$= \frac{1}{2} \, 2 \int_0^{\infty} x^2 e^{-x} dx$$

$P(z < z_1) = 0.31 \quad P(z > z_2) = 0.08$

$0.5 + \phi(z_1) = 0.31 \quad 0.5 - \phi(z_2) = 0.08$

$\phi(z_1) = -0.19, \quad \phi(z_2) = 0.42$ (1M)

We have $\phi(0.5) = 0.19 ; \phi(1.4) = 0.42$

$\phi(z_1) = -\phi(0.5) ; \quad \phi(z_2) = \phi(1.4)$

$z_1 = -0.5 ; \quad z_2 = 1.4$

$\frac{45 - \mu}{\sigma} = -0.5 ; \quad \frac{64 - \mu}{\sigma} = 1.4$

$\mu - 0.5\sigma = 45 ; \quad \mu + 1.4\sigma = 64$ (2M)

solving $\mu = 50, \quad \sigma = 10$ (1M)

5. $p(x) = ke^{-|x|}, \quad -\infty < x < \infty$

We k.T $\int_{-\infty}^{\infty} p(x)\, dx = 1$

$\int_{-\infty}^{\infty} ke^{-|x|}\, dx = 1$

$k \int_{-\infty}^{\infty} e^{-|x|} dx = 1$

even fn

$2k \int_{0}^{\infty} e^{-|x|} dx = 1$

$$= \int_0^\infty x^2 e^{-x}\,dx$$

$$= \left\{x^2(-e^{-x}) - (2x)\,e^{-x} + 2(-e^{-x})\right\}_0^\infty$$

$$= \left[-e^{-x}(x^2+2x+2)\right]_0^\infty = -(0-2) = 2$$

(4M)

6. $P(X=k) = \frac{\lambda^k}{k!}$, $k = 1, 2, 3, \ldots$

We have $\sum P(x) = 1$

$$\sum \frac{\lambda^k}{k!} = 1$$

$$\lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots = 1$$

$$e^\lambda - 1 = 1 \qquad \lambda = \log_e 2 \quad (5M)$$

$$P(0 < X < 3) = P(X=1) + P(X=2)$$

$$= \frac{\lambda}{1!} + \frac{\lambda^2}{2!} = \log_e 2 + \frac{1}{2}(\log_e 2)^2$$

(2M)

7. Let x be the random variable of the sale in the shop. Since x is an exponential variate pdf $f(x) = \alpha e^{-\alpha x}$, $x > 0$

$$\text{Mean} = \frac{1}{\alpha} = 10000$$

$$\alpha = \frac{1}{10000} = 10^{-4}$$

$$f(x) = 10^{-4} e^{-10^{-4} x}, \quad x > 0$$

Let A be the amount for which the profit is 8%.

$$A \cdot \frac{8}{100} = 3000 \quad \boxed{A = 37500} \quad (1M)$$

$$P(\text{profit exceeding } 3000) = 1 - P(x \leq 30)$$

$$= 1 - P(\text{sales} \leq 375)$$

$$= 1 - \int_0^{37500} 10^{-4} e^{10^{-4} x}$$

$$= 1 - 10^{-4} \left(\frac{e^{-10^{-4} x}}{-10^{-4}} \right)_0^{37500}$$

$$= 1 + e^{-\frac{37500}{10000}} - 1 = e^{-3.75}$$

(2M)

$$P(\text{profit exceeding } 3000 \text{ for } 2 \text{ days})$$

$$= e^{-3.75} e^{-3.75} = e^{-7.5}$$

$$= 0.00055$$

(1M)

8. $S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$

$P(\text{each outcome}) = 1/8$

X	0	1
$P(X)$	4/8	4/8

Y	0	1	2	3
$P(Y)$	1/8	3/8	3/8	1/8

$P(X = x_i, Y = y_j) = p_{ij}$ $\quad x_i = 0, 1$
$\quad y_j = 0, 1, 2, 3$

$P(X = 0, Y = 3) = 1/8$

$P(X = 0 \; Y = 2) = 2/8$ $\quad P(X = 0 \; Y = 0) = 0$

$P(X = 0, Y = 1) = 1/8$

$P(X = 1 \; Y = 0) = 1/8$ $\quad P(X = 1 \; Y = 2) = 1/8$

$P(X = 1 \; Y = 1) = 2/8$

$P(X = 1 \; Y = 3) = 0$

X/Y	0	1	2	3
0	0	1/8	2/8	1/8
1	1/8	2/8	1/8	0

(3M)

$$E(X) = \Sigma x\, P(x)$$
$$= 0 \times \frac{4}{8} + 1 \times \frac{4}{8} = 1/2$$

$$E(Y) = \Sigma y\, P(y) = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = 3/2$$

$$E(XY) = \Sigma \Sigma\, p_{ij}\, x_i y_j$$
$$= 0 \cdot p_{11} + p_{12} \cdot 0 + p_{13} \cdot 0(2) + p_{14}(0)(3)$$
$$+ p_{21}(0) + p_{22}(1) + p_{23}(1)(2)$$
$$+ p_{24}(3)(1)$$

$$= 1 \cdot 2/8 + 2 \cdot 1/8 = 1/2$$

$$\sigma_X^2 = E(X^2) - \mu_X^2$$
$$= \Sigma x^2 P(x) - (1/2)^2$$
$$\left(0^2 \times \frac{1}{2} + 1^2 \times \frac{1}{2}\right) - (1/2)^2 = 1/4$$

$\sigma_x = 1/2$

$\sigma_y^2 = \Sigma y^2 P(y) - \mu_y^2$

$= \left\{0^2\left(\frac{1}{8}\right) + 1^2\left(\frac{3}{8}\right) + 2^2\left(\frac{3}{8}\right) + 3^2\left(\frac{1}{8}\right)\right\} - (3/2)^2$

$= \frac{24}{8} - (3/2)^2 = 3 - \frac{9}{4} = 3/4$

$\sigma_y = \sqrt{3}/2$

$\text{cov}(xy) = E(xy) - E(x)E(y)$

$= \frac{1}{2} - (1/2)(3/2) = \frac{1}{2} - \frac{3}{4}$

$= -1/4$

$$\rho(xy) = \frac{\text{cov}(xy)}{\sigma_x \sigma_y} = \frac{-1/4}{1/2 \quad 3/2} = -1/3$$

(4M)

9. tpm

$$P = \begin{array}{c|c} & \begin{array}{ccc} S & Z & E \end{array} \\ \begin{array}{c} S \\ Z \\ E \end{array} & \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \end{array}$$

The initial probability vector is

$p^{(0)} = (0 \quad 0 \quad 1)$

After 2 years $p^{(2)} = p^{(0)} P^2$

$$p^{(2)} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

$$\cancel{= \begin{pmatrix} 1/3 & 1/3 & 1/3 \end{pmatrix}}$$

$$= \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{pmatrix}$$

$$= \begin{array}{c} \begin{pmatrix} \frac{1}{9} & \frac{4}{9} & \frac{4}{9} \end{pmatrix} \\ P_S \quad P_Z \quad P_E \end{array}$$

$P(1998 \text{ Esteem}) = 4/9$

$P(1998 \text{ Stol}) = 1/9$

$P(1999 \text{ Zen}) = p^{(3)} = p^0 p^3$

$$= p^{(3)} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \\ 4/27 & 7/27 & 16/27 \end{pmatrix}$$

$$= \begin{array}{c} \begin{pmatrix} \frac{4}{27} & \frac{7}{27} & \frac{16}{27} \end{pmatrix} \\ P_S \quad P_Z \quad P_E \end{array}$$

$P(1999 \text{ Zen}) = 7/27$ $\quad P(1999 \text{ Esteem}) = \frac{16}{27}$

(AM)

To find the happening for the long run, let $V = (x, y, z)$, $VP = V$

$x + y + z = 1$

$$(x \ y \ z)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} = (x \ y \ z)$$

$$\left(\tfrac{1}{3}z \quad x + \tfrac{1}{3}z \quad y + \tfrac{1}{3}z\right) = (x \ y \ z)$$

$\frac{1}{3}z = x$; $x + \frac{1}{3}z = y$

$y + \frac{1}{3}z = z$

$x + y + z = 1$

$z = 3x$; $3x + z = 3y$; $3y + z = 3z$

$y = 1 - x - z$

$3(1 - x - z) + z = 3z$

$3 - 3x - 2z = 3z$

$3 - 3x - 5z = 0$

$3x + 5z = 3$

$3x + 5z = 3$

$3(3z) + 5z = 3$

$z = 3/14$ $\quad x = 9/14$

$y =$

$z = 3x, \quad 3x + z = 3y, \quad 3y + z = 3z$

$y = 1 - x - z$

$3(1 - x - z) + z = 3z$

$\cancel{3 - 3x = 3z}$ $\qquad$ $3 - 3x - 2z = 3z$

$\cancel{1 - x = z}$ $\qquad$ $3 - 3x = 5z$

$\cancel{1 - x = 3x}$ $\qquad$ $3 - 3x = 5(3x)$

$3 = 18x \Rightarrow \boxed{x = \frac{1}{6}}$

$z = 3/6 = 1/2$

$y = 1 - x - z = 1 - \frac{1}{6} - \frac{1}{2}$

$= 1 - \frac{4}{6}$

$= 1/3$

$$v = (x \; y \; z) = \begin{pmatrix} \frac{1}{6} & \frac{1}{3} & 1/2 \end{pmatrix}$$
$\qquad\qquad P_S \quad P_Z \quad P_E$

~~P(1999 Esteem)~~

P(Esteem in the long run) $= 1/2$

3M