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CMR

Internal Assessment Test II

Sub	ENGINEERING MATHEMATICS IV							Code:	15MAT41
Date:	8 /5/ 2017	Duration	90 mins	Max Marks	50	Sem	IV	Branch	MECH A,B

First question is compulsary Answer ANY SIX questions from Q2 toQ8

Question	Marks	OBE CO	OBE RBT
1. Derive Mean and standard deviation of Binomial distribution	[08]	CO4	L3
2. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $\lvert z \rvert = 3$, using Cauchy's residue theorem.	[07]	CO3	L3
3. Find the Bilinear transformation that maps $z = \infty, i, 0$ *into* $w = -1, -i, 1$, Also find the fixed points of this transformation.	[07]	CO3	L3
4. Discuss Joukowski's transformation $w = z + \frac{1}{z}$	[07]	CO3	L3
5. In a quiz contest with 'Yes' or 'No' as the 2 possible answers, what is the probability of guessing at least 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options given	[07]	CO4	L3
6. 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains a) No defective fuses. b)3 or more defective fuses. c)At most 3 fuses by Poisson's distribution.	[07]	CO4	L3
7. The marks of 1000 students in an examination follows a normal distribution with mean70 and standard deviation 5. Find the number of students whose marks will be a) Less than 65b) more than 75 c) between 65 and 75. Given $\phi(1)$ =0.3413.	[07]	CO4	L3
8. A fair coin is tossed thrice. The random variables X and Y are defined as follows X= 0 or 1 according as head or tail occurs on first toss. Y= number of heads a) Determine the distribution of X and Y b) Joint distribution of X and Y c) Show that the random variables are dependent.	[07]	CO6	L3

Solution

IAT - II Engineering Maths - IV

1. $$\text{Mean}(\mu) = \sum_{x=0}^{n} x\,p(x)$$

$$\mu = \sum_{x=0}^{n} x\, nC_x\, p^x q^{n-x} \quad \text{——— } (1)$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n(n-1)!}{(x-1)!(n-x)!} p \cdot p^{x-1} q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,[(n-1)-(x-1)]!} p^{x-1} q^{n-1-(x-1)} \quad - (2)$$

$$\mu = np \sum_{x=1}^{n} (n-1)C_{x-1}\, p^{x-1} q^{n-1-(x-1)}$$

$$\mu = np(q+p)^{n-1} = np. \quad \text{——— } (1)$$

$$\text{variance} = \sum_{x=0}^{n} x^2 p(x) - \mu^2$$

$$\sum x^2 p(x) = \sum_{x=0}^{n} [x(x-1) + x]\, p(x) \quad \text{——— } (1)$$

$$= \sum_{x=0}^{n} x(x-1)\, p(x) + \sum x\, p(x)$$

$$= \sum_{x=0}^{n} x(x-1)\, nC_x\, p^x q^{n-x} + np$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x} + np \quad -(2)$$

$$= n(n-1)p^2 \sum_{x=2}^{n} (n-2)C_{x-2}\, p^{x-2} q^{n-2-(x-2)} + np$$

$\therefore \sum x^2 p(x) = n(n-1)p^2 + np$ ——— (1)

Variance $= npq$.

2. $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$

Let $f(z) = \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$

$z=1$ is a pole of order 2 and $z=2$ is a pole of order 1. Both of them lies within the circle $|z|=3$. ——— (1)

Residue at $z=1$, be denoted by R_1

$$R_1 = \lim_{z \to 1} \frac{1}{(2-1)!} \frac{d}{dz} \left\{ (z-1)^2 \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} \right\}$$

$$= \lim_{z \to 1} \frac{d}{dz} \left\{ (\sin \pi z^2 + \cos \pi z^2) \frac{1}{z-2} \right\}$$

$$R_1 = (1+2\pi) \quad \because \sin \pi = 0,\ \cos \pi = -1$$ — (3)

$$R_2 = \lim_{z \to 2} (z-2) \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$$

——— (2)

$\therefore R_2 = 1$.

Hence by Cauchy's Residue theorem

$$\int_C f(z)dz = 2\pi i (R_1 + R_2)$$ ——— (1)

$$= 4\pi i (1+\pi)$$

3) let $w = \dfrac{az+b}{cz+d}$ be the required bilinear

transformation.

$z = \infty, \quad w = -1$

$$w = \frac{z\,[a+(b/z)]}{z\,[c+(d/z)]} = \frac{a+b/z}{c+d/z} \quad \text{—} \quad ①$$

$$-1 = \frac{a+0}{c+0}$$

$a + c = 0$ — (1)

$z = i, \quad w = -i$

$ai + b - c + di = 0$ — (2)

$z = 0, \quad w = 1,$

Add(1) + (2) $\quad b - d = 0$ — (3)

$(1+i)\,a + b + id = 0$ — (4)

— ④

solve (3) & (4)

$0a + 1b - 1d = 0$

$(1+i)\,a + 1b + id = 0$

$a = 1, \; b = -1, \; d = -1, \; c = -1$ — ②

$$\boxed{w = \frac{1-z}{1+z}}$$

(4) $w = z + \frac{1}{z}$, $z \neq 0$ $\quad$ -(1)

$f'(z) = 1 - \frac{1}{z^2}$ ——— 1

Clearly $f(z)$ is conformal for all the values of z except $x = \pm 1$

Let $z = re^{i\theta}$

(1) reduces to

$$w = u + iv = re^{i\theta} + \frac{1}{re^{i\theta}}$$

$$u + iv = r(\cos\theta + i\sin\theta) + \frac{1}{r}(\cos\theta - i\sin\theta) \quad \text{———} \ 2$$

$$u + iv = \left(r + \frac{1}{r}\right)\cos\theta + i\left(r - \frac{1}{r}\right)\sin\theta$$

$$u = \left(r + \frac{1}{r}\right)\cos\theta \qquad v = \left(r - \frac{1}{r}\right)\sin\theta \qquad -(2)$$

Case (i) Let $r = b$, be a circle with centre at the origin and radius b in z-plane

from (2) $\frac{u}{\left(b + \frac{1}{b}\right)} = \cos\theta$, $\frac{v}{b - \frac{1}{b}} = \sin\theta$

$$\frac{u^2}{\left(b + \frac{1}{b}\right)^2} + \frac{v^2}{\left(b - \frac{1}{b}\right)^2} = 1 \quad \text{———} \ 2$$

$\therefore$ $w = z + \frac{1}{z}$ transforms a circle in z plane to an ellipse in w-plane.

case ii   Let $\theta = c$, be any line in z plane

$$u = \left(r + \frac{1}{r}\right)\cos c, \qquad v = \left(r - \frac{1}{r}\right)\sin c$$

$$\frac{u^2}{\cos^2 c} - \frac{v^2}{\sin^2 c} = 4 \quad \text{———} \quad (2)$$

$$\frac{u^2}{(2\cos c)^2} - \frac{v^2}{(2\sin c)^2} = 1$$

$\therefore$ $w = z + \frac{1}{2}$ transforms a st. line passing through origin in z plane to a hyperbola in w-plane.

5) Let x denote the correct answer and we have in the first case,

$$p = \frac{1}{2}, \quad q = \frac{1}{2}$$

$$P(x) = n_{C_x} p^x q^{n-x} = 10_{C_x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{10-x}$$

$$= 10_{C_x}\left(\frac{1}{2}\right)^{10} \quad \text{———} \quad (1)$$

$$P(x \geq 6) = \frac{1}{2^{10}}\left[10_{C_6} + 10_{C_7} + 10_{C_8} + 10_{C_9} + 10_{C_{10}}\right]$$

$$= \underline{\underline{0.377}} \quad \text{———} \quad (3)$$

In the second case when there are 4 options

$$p = 1/4, \quad q = 3/4, \quad n = 10$$

$$P(x) = 10_{C_x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{10-x} \quad \text{———} \quad (3)$$

$$P(x \geq 6) = P(6) + P(7) + P(8) + P(9) + P(10)$$

$$= 0.01972 \simeq \underline{\underline{0.02}}.$$

6) p = probability of a defective fuse

$= 2/100 = 0.02$

$\therefore$ mean no. of defectives

$\mu = m = np = 200 \times 0.02$ — (1)

$= 4$

$\therefore$ The Poisson distribution is given by

$$P(x) = \frac{m^x e^{-m}}{x!}$$

$$P(x) = \frac{4^x e^{-4}}{x!} \quad \text{— (1)}$$

$$P(x) = 0.0183 \frac{4^x}{x!}$$

(i) Prob. of no defective fuse $= P(0) = 0.0183$ — (1)

(ii) Prob. of 3 or more defective fuses

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - 0.0183\left[1 + \frac{4^1}{1!} + \frac{4^2}{2!}\right] = 0.7621$ — (2)

(iii) Prob. of atmost 3 fuses

$P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= 0.0183 \frac{4^0}{0!} + 0.0183 \frac{4^1}{1!} + 0.0183 \frac{4^2}{2!}$

$+ 0.0183 \frac{4^3}{3!}$ — (2)

$= 0.0183 [1 + 4 + 8 + 16]$

$= 0.0183 [29] = 0.5307$

7) Let x represent the marks of students

$\mu = 70, \quad \sigma = 5$

Stand normal variable $z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$ — (1)

(i) if $x = 65$, $z = -1$

$P(z < -1) = P(z > 1)$

$= 0.5 - \phi(1) = 0.1587$

∴ no. of students scoring less than 65 marks

$1000 \times 0.1587 = 158.7 \simeq 159$ — (2)

(ii) if $x = 75$, $z = 1$

$P(z > 1) = 0.1587$

$1000 \times 0.1587 = 158.7 \simeq 159$ — (2)

(iii) we have find $P(-1 < z < 1)$

$= 2P(0 < z < 1)$

$= 2\phi(1) = 0.6826$ — (2)

no. of students scoring marks between 65 and 75

$1000 \times 0.6826 = 682.6 \simeq 683.$

(8)

S	HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
X	0	0	0	0	1	1	1	1
Y	3	2	2	1	2	1	1	0

— (1)

(a) The prob. distribution of X and Y is found as follows

$X = \{0, 1\} = \{x_i\}$

$Y = \{y_i\} = \{0, 1, 2, 3\}$ —— 2

$P(X=0) = 1/2$ $P(X=1) = 1/2$

$P(Y=0) = 1/8$, $P(Y=1) = 3/8$, $P(Y=2) = 3/8$, $P(Y=3) = 1/8$

prob. distribution of X and Y

x_i	0	1
$f(x_i)$	1/2	1/2

y_j	0	1	2	3
$g(y_j)$	1/8	3/8	3/8	1/8

(b) The joint distribution of X and Y is found by computing —— 3

$J_{ij} = P(X = x_i, Y = y_j)$

$x_1 = 0$, $x_2 = 1$ $y_1 = 0$, $y_2 = 1$, $y_3 = 2$, $y_4 = 3$

$J_{11} = P(X=0, Y=0) = 0$

$J_{12} = P(X=0, Y=1) = 1/8$

$J_{13} = P(X=0, Y=2) = 1/4$

$J_{14} = P(X=0, Y=3) = 1/8$

$J_{21} = P(X=1, Y=0) = 1/8$

$J_{22} = P(X=1, Y=1) = 1/4$

$J_{23} = P(X=1, Y=2) = 1/8$

$J_{24} = P(X=1, Y=3) = 0$.

Joint prob. distribution of X and Y

X\Y	0	1	2	3	Sum
0	0	1/8	1/4	1/8	1/2
1	1/8	1/4	1/8	0	1/2
Sum	1/8	3/8	3/8	1/8	=1

(c) It can be easily seen that —— 1

$f(x_i)\, g(y_j) \neq J_{ij}$ hence random variables are dependent.