SOLUTION AND SCHEME OF EVALUATION · IAT-II

Elements of civil engineering and Engineering mechanics

sub: code: 15CIV23

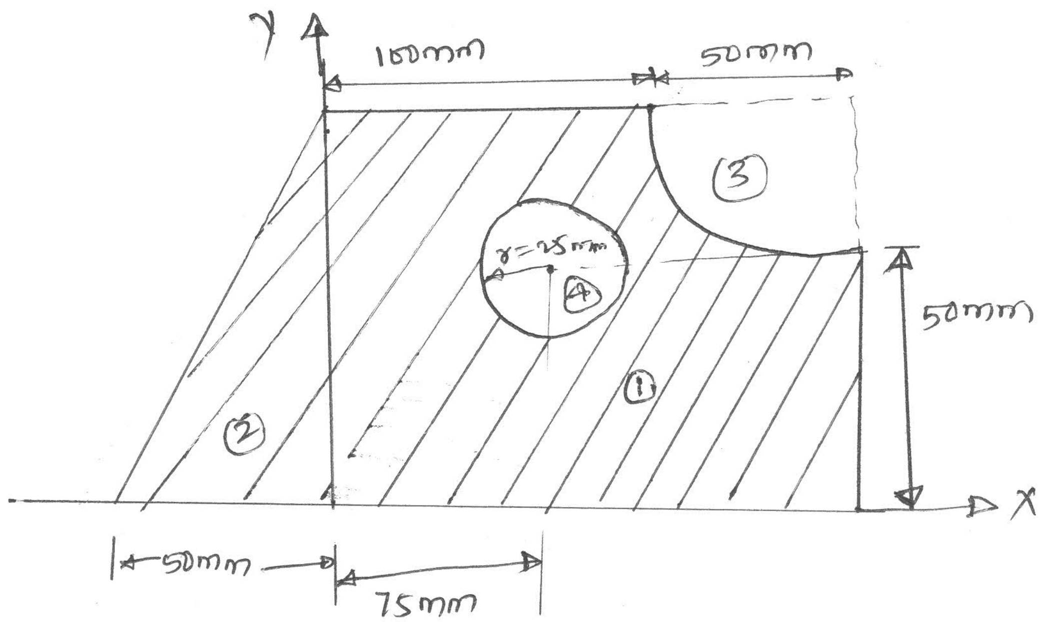
1 (a) Differentiate between centroid and centre of gravity [03]

Ans:

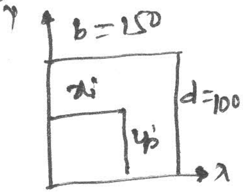
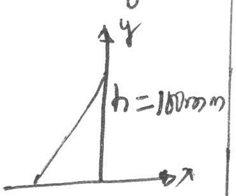
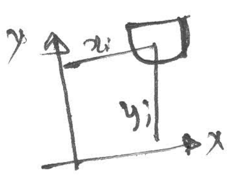
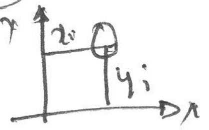
01X03 = 03

centroid	centre of gravity.
① Applicable to plane figures	① Applicable to the bodies which are having mass
② It is the point about which if you consider any axis, moment of area about that axis will be zero.	② It is the point through which resultant weight of the body acts for any orientation.
③ It may lies outside the given area	③ It will ~~be~~ lie within the object.

1 (b) Locate the position of centroid of the shaded area shown in figure. [07]



Soln: 1(b)

component	Area a_i mm^2	x_i mm	y_i mm	a_ix_i mm^3	a_iy_i mm^3
(1) Rectangle 	150×100 $=15000$	$\frac{150}{2}$ $=75$	$\frac{100}{2}$ $=50$	1125000	750000
(2) Triangle 	$\frac{1}{2}\times50\times100$ $=2500$	$-b/3$ $-\frac{50}{3}$ $=-16.66$	$\frac{h}{3}$ $=\frac{100}{3}$ $=33.33$	-416667	83325
(3) Quarter circle 	$-\frac{\pi\times50^2}{4}$ $=-1963.49$	$150-\frac{4\times50}{3\pi}$ $=128.77$	$100-\frac{4\times50}{3\pi}$ $=78.78$	-252838.607	-154682.44
(4) Circle 	$-\pi\times r^2$ $=-\pi\times25^2$ $=-1963.49$	75	50	-147262.15	-98174.77

~~A = 13573.02 mm²~~

$A = 13573.02\ mm^2$ (01)

$\Sigma a_ix_i = 683232.57\ mm^3$ (02)

$\Sigma a_iy_i = 580467.79\ mm^3$ (02)

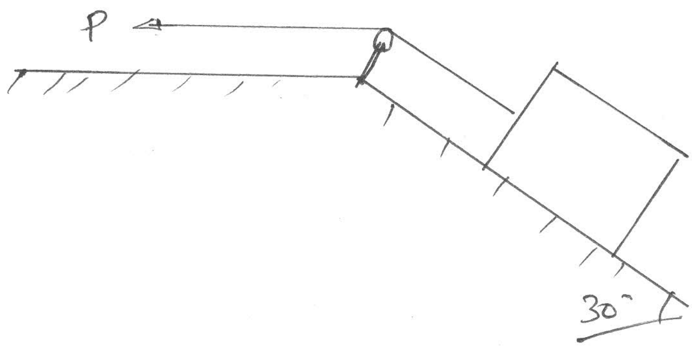
$$\bar{X} = \frac{683232.57}{13573.02} \qquad \bar{Y} = \frac{580467.79}{13573.02}$$

$\bar{X} = 50.33\ mm.$ $\qquad \bar{Y} = 42.766\ mm$

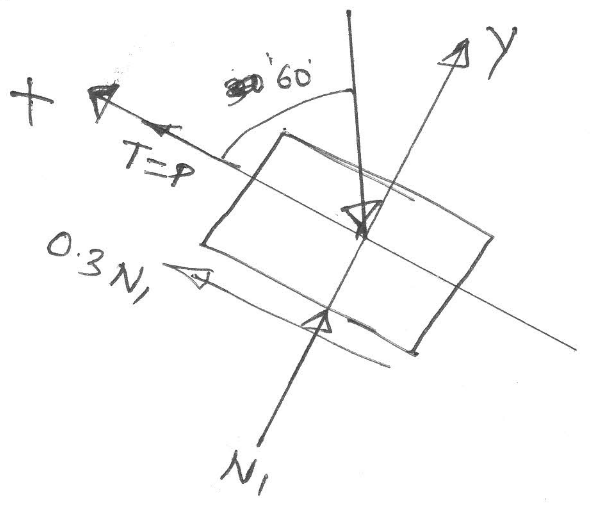
(02)

2(b) A body of mass 80 kg is to be lowered down by rope passing over a smooth pulley and the free end is held by a man as shown in figure. Determine the force he has to exert to prevent the downward motion motion of the block.

Soln:



FBD



(02)

$\Sigma F_y = 0$

$N_1 - 80 \times 9.81 \sin 60 = 0$

$N_1 = 679.656$

(01)

$\Sigma F_x = 0$

$T - 80 \times 9.81 \cos 60° + 0.3 N_1 = 0$

$\therefore T = 80 \times 9.81 \cos 60° - 0.3 \times 679.656$

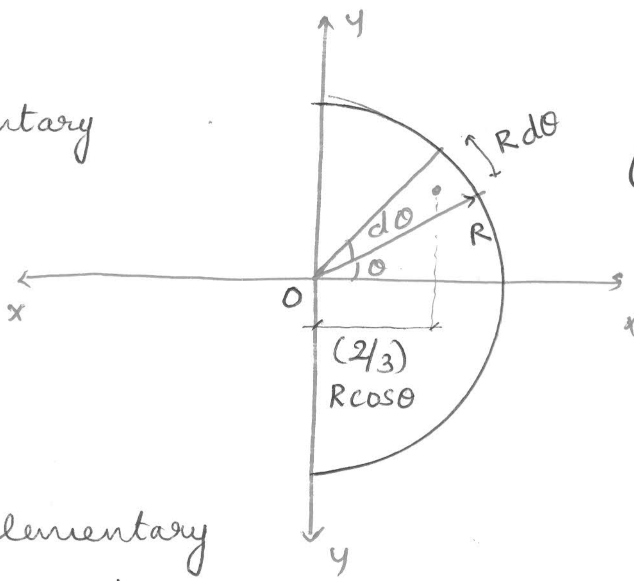
$T = 188.5032$ N

(03)

$\therefore P = 188.5032$ N.

3(a) Derive an expression to locate the centroid of a semi-circular lamina with respect to diametric axis.

Consider a semicircular lamina of area $\frac{\pi r^2}{2}$ as shown in fig. Now consider a triangular elementary strip of area $\frac{1}{2} \times R \times R \times d\theta$ at an angle of θ from the x-axis, whose centre of gravity is at a distance of $\frac{2}{3}R$ from O and its projection on 'x' axis $= \left(\frac{2}{3}\right) R\cos\theta$.



Moment of area of elementary strip about 'y' axis

$$= \frac{1}{2} R^2 \, d\theta \left(\frac{2}{3}\right) R\cos\theta$$

$$\frac{R^3}{3} \cos\theta \, d\theta$$

02

Sum of moments of such elementary strips about 'y'-axis $= \int_{-\pi/2}^{\pi/2} \frac{R^3}{3}\cos\theta \, d\theta = \frac{R^3}{3}\left[\sin\theta\right]_{-\pi/2}^{\pi/2}$

$$= \frac{2R^3}{3}$$

Moment of total area about 'y'-axis $= \frac{\pi R^2}{2} \times \bar{x}$

02

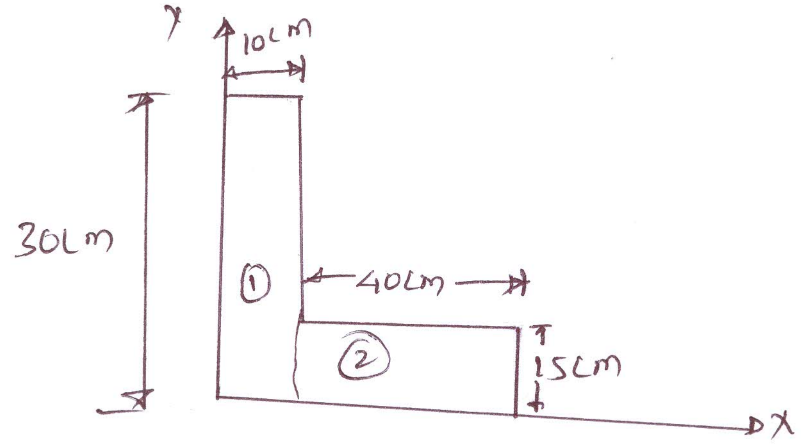
Using principle of moments

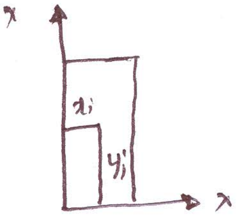
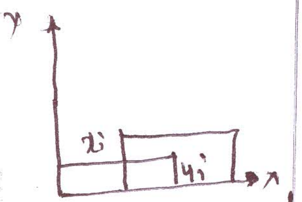
$$\frac{2R^3}{3} = \frac{\pi R^2}{2} \times \bar{x}$$

$$\boxed{\bar{x} = \frac{4R}{3\pi}}$$

02

3 (b) Locate the position of centroid of area shown in figure.



Component	Area a_i	x_i cm	y_i cm	$a_i x_i$ cm^3	$a_i y_i$ cm^3
	10×30 $= 300$	$10/2$ $= 5$	$30/2$ $= 15$	1500	4500
	40×15 600 $= \cancel{450}$	$10 + \frac{40}{2}$ $= 30$	$15/2$ $= 7.5$	18000	4500

$A = 900 mm^2$

$\Sigma a_i x_i = 19500 \text{ cm}^3$ (03)

$\Sigma a_i y_i = 9000 \text{ cm}^3$

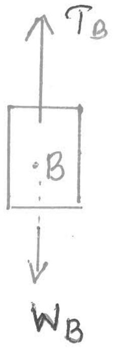
$\bar{x} = \frac{19500}{900}$ $\qquad$ $\bar{x} = \frac{9000}{900}$

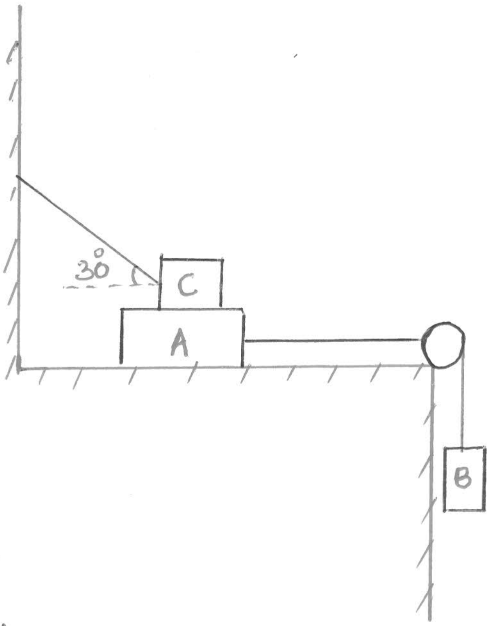
$\bar{x} = 21.66 CM$ $\qquad$ $\bar{y} = 10 cm$ (01)

(4 9) (10M)

F.B.D of Body B

Applying equations of equilibrium





$\Sigma F_y = 0 \Rightarrow \quad T_B - W_B = 0 \quad ; \quad T_B = W_B$

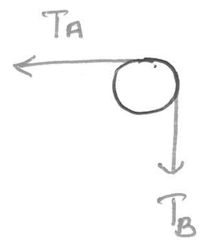
As block A moves towards right, frictional forces act on it towards left on both of its faces. Hence the frictional force acting on the block C must be in opposite direction ie towards the right.

$T_B = W_B = T_B = 10 \times 9.81 = \underline{\underline{98.1 N}}$

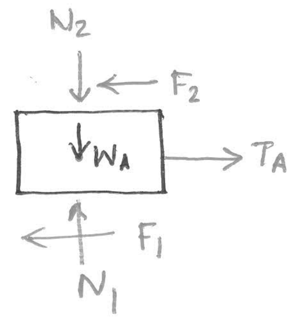
Since pulley is frictionless, tensions at both ends of string passing over it are equal.

$T_A = T_B = \underline{\underline{98.1 N}}$

01



Block A



$\Sigma F_x = 0 \quad T_A - F_1 - F_2 = 0$

$T_A = F_1 + F_2$

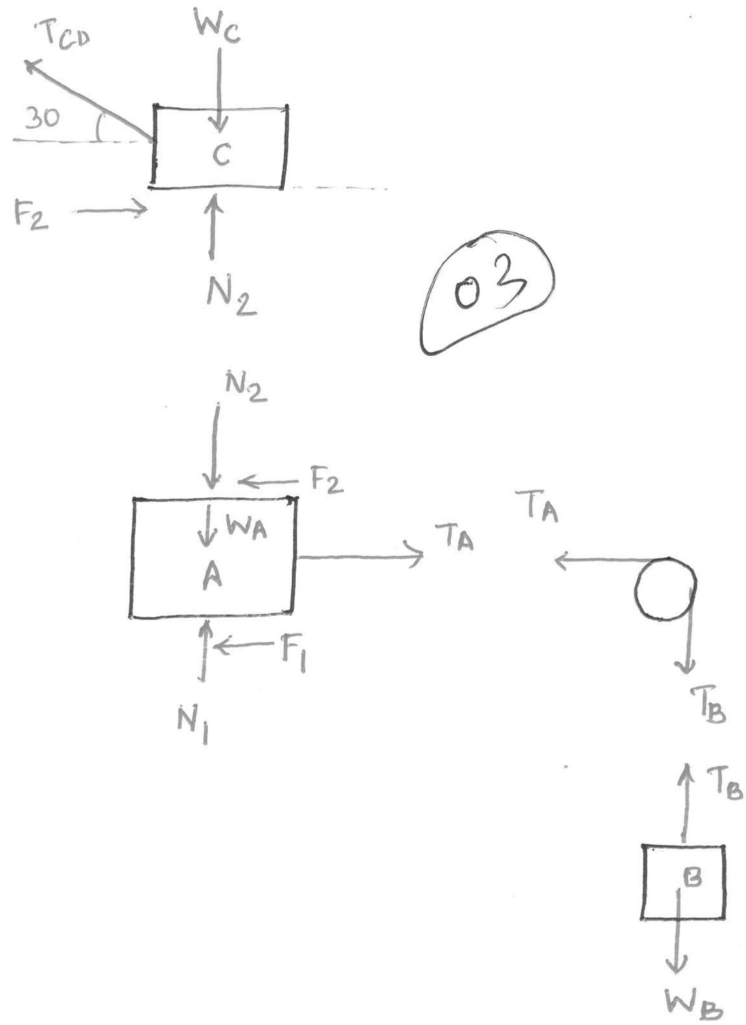
$F_1 = \mu_1 N_1 \quad F_2 = \mu_2 N_2$ at point of impending motion

01

$T_A = \mu N_1 + \mu N_2$

$N_1 + N_2 = T_A/\mu = 98.1/0.25 = \underline{\underline{392.4 N}}$

Free Body Diagram of force system



03

$\Sigma F_y = 0$

$$N_1 - N_2 - W_A = 0$$

$$N_1 - N_2 = W_A = 15 \times 9.81 = \underline{147.15N}$$

Solving N_1 and N_2, we get

$$N_1 = \underline{269.78N}$$

$$N_2 = \underline{122.62N}$$

02

Block C

$\Sigma F_x = 0 \Rightarrow F_2 - T_{CD} \cos 30^\circ = 0$

$$T_{CD} = \frac{0.25 \times 122.62}{\cos 30} = \underline{35.4N}$$

$\Sigma F_y = 0 \Rightarrow N_2 + T_{CD} \sin 30 - W_C = 0$

$$W_C = N_2 + T_{CD} \sin 30$$

$$= 122.62 + 35.4 \sin 30$$

$$= \underline{140.32N}$$

03

Therefore, mass of the block C is obtained as

$$m_C = W_C / g$$

$$= 140.32 / 9.81 = 14.3 \text{ kg}$$

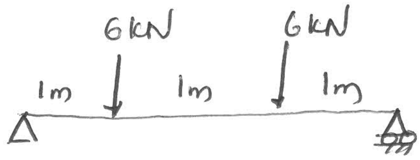
$$\boxed{m_C = 14.3 \text{ kg}}$$

01

5(a) Explain different types of loads and briefly explain the procedure to find the equivalent point loads for UDL, UVL & Trapezodial loads.

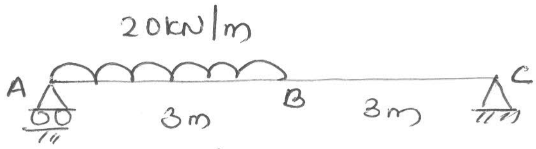
Types of loads

- Concentrated loads
- Load which is concentrated at a point in beam.

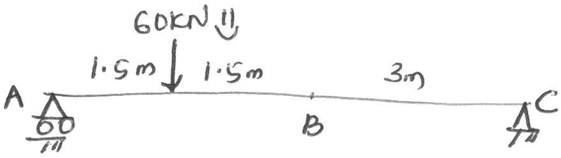


- Uniformly distributed load

A load which is distributed uniformly along entire length of beam. eg:~ 20kN/m – To convert into point load acting at centre of span say 3m we proceed as follows.

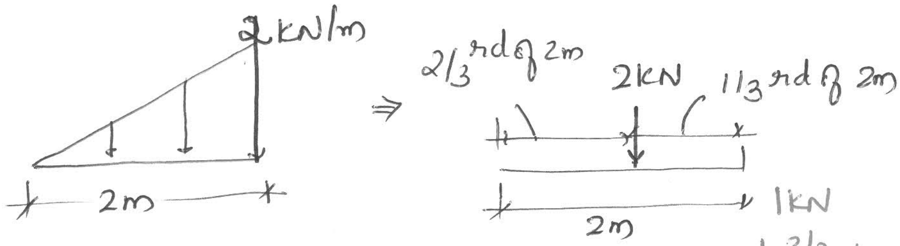


Magnitude of point load = 20kN/m × 3
= 60kN acting @ 1.5m

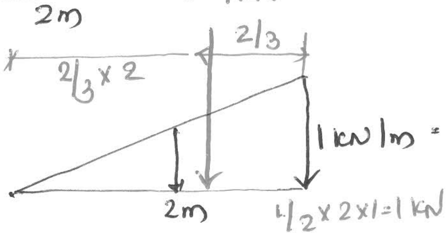


- Uniformly varying load
- load which varies along beam length. eg:~

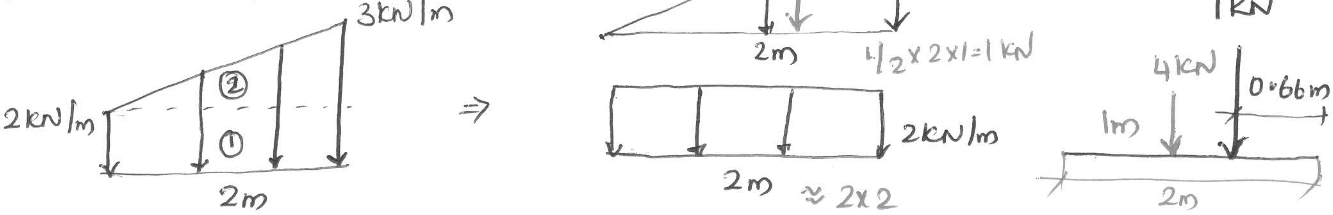
01×03=03



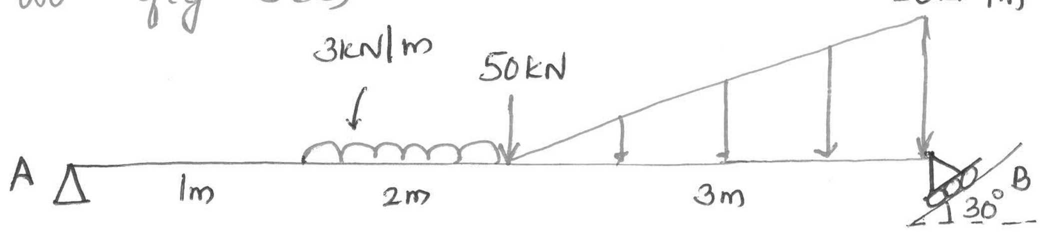
Magnitude ≈ $\frac{1}{2} \times 2 \times 2 = 2kN$



- Trapezodial load -



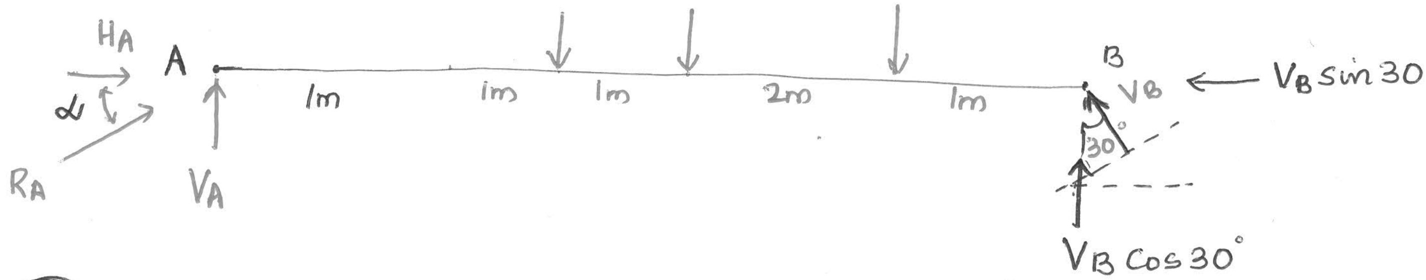
5(b) Determine the support reactions for the beam shown in fig 5(b).



Equations of equilibrium.

$\Sigma F_x = 0$ $\quad \Sigma F_y = 0$ $\quad \Sigma M = 0$.

Replacing by equivalent force system / point loads.



01 $\quad \Sigma F_y = 0 \quad V_A + V_B \cos 30 - 6 - 50 - 30 = 0 \quad -①$

01 $\quad \Sigma F_x = 0 \quad H_A - V_B \sin 30 = 0. \quad -②$

$\Sigma M_B = 0 \Rightarrow V_A \times 6 - 6 \times 4 - 50 \times 3 - 30 \times 1 = 0 \quad -③$

$V_A = \underline{34\,kN}$

03 $\quad$ put V_A in ①, $\quad V_B = \underline{60.04\,kN}$

$H_A = \underline{30.02\,kN}$

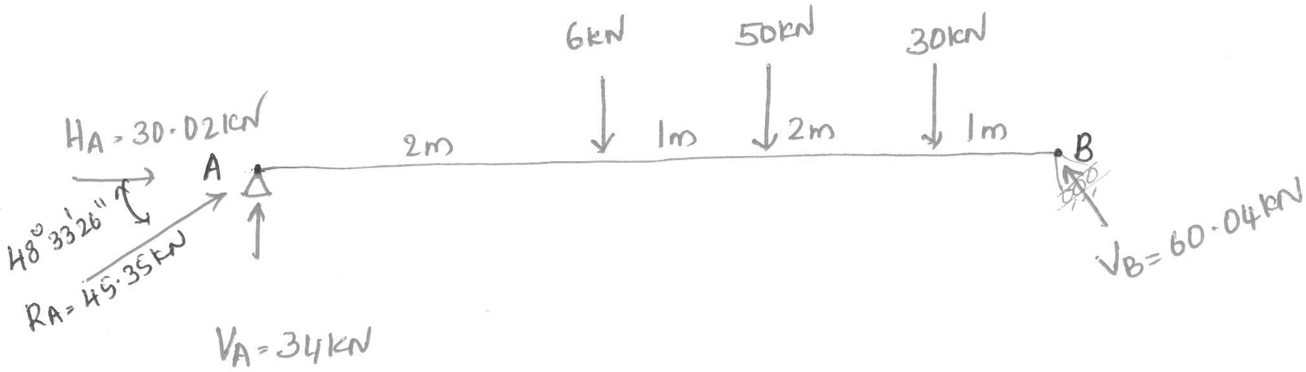
$R_A = \sqrt{H_A^2 + V_A^2}$

$= \sqrt{30.02^2 + 34^2}$

$R_A = \underline{45.35\,kN}$ 01

$\tan\alpha = \frac{V_A}{H_A} = \frac{34}{30.02}$

$\alpha = \underline{48^\circ 33' 26.6''}$ 01



Ans

$V_A = 34kN$

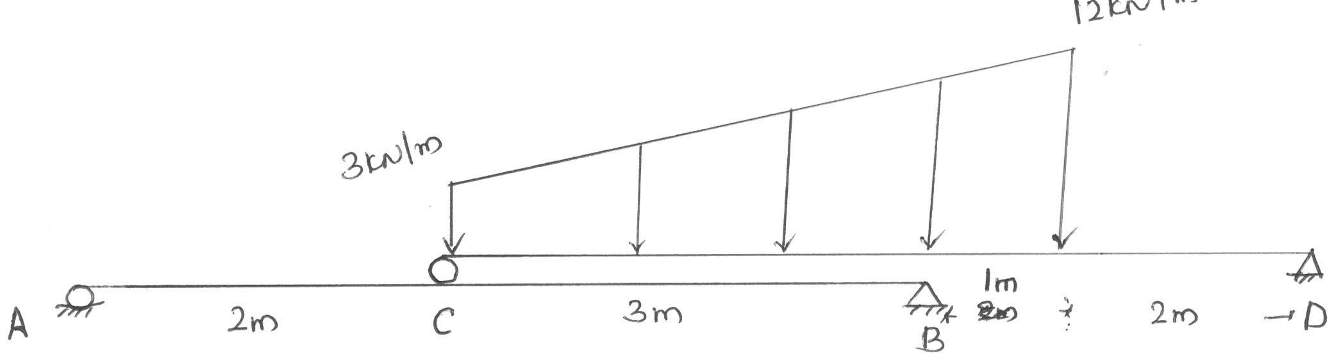
$H_A = 30.02kN$

$R_A = 45.35kN$

$\alpha = 48^\circ 33' 26.6''$

$V_B = 60.04 kN$

6(a). Determine the reactions at the supports for compound beam shown in Fig. 6.a



Conditions of equilibrium

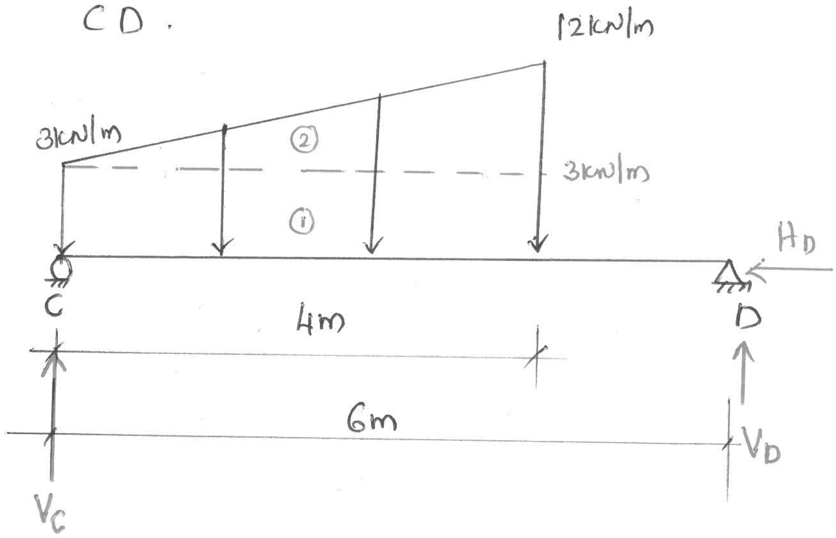
$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

Considering the beam CD.

$\Sigma F_x = 0 \quad H_D = 0$. 01



Converting Trapezodial load to equivalent load system.

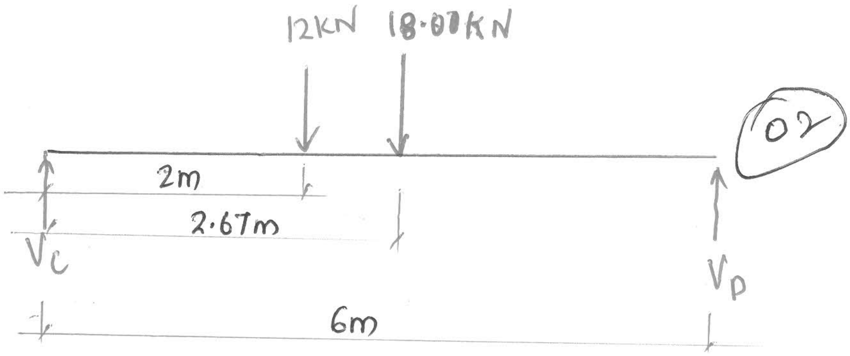
Area of □le ①

$= 3 \times 4 = 12\,kN$ @ $2m$

Area of Δle ②

$= 1/2 \times 9 \times 4 = 18\,kN$ @ $2/3 \times 4$

$= 2.67\,m$



Taking moment about C; $\Sigma M_C = 0$.

$- V_D \times 6 + 18 \times 2.67 + 12 \times 2 = 0$, $\boxed{V_D = 12.01\,kN}$

$$\Sigma F_y = 0.$$

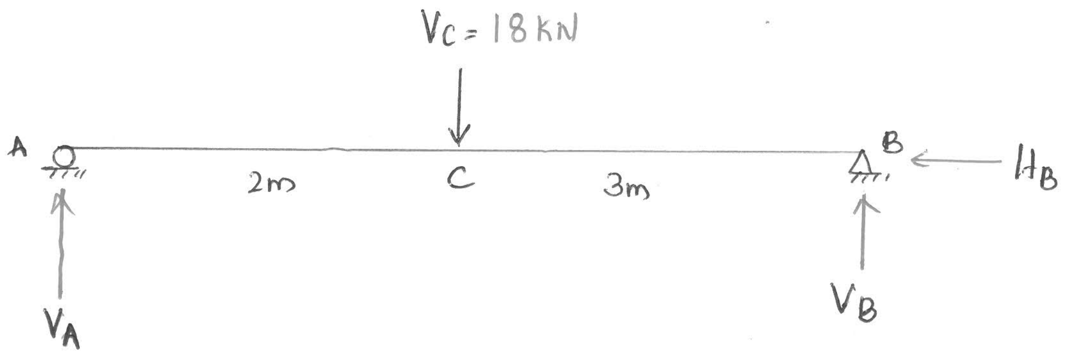
$$V_C + V_D = 12 + 18$$

$$V_C = 12 + 18 - 12.01$$

$$\boxed{V_C = 18\,kN}$$ 02

Consider beam AB

Here reaction from C will be acting as load on beam AB with a reversal in direction so as to keep beams in equilibrium.



$$\Sigma F_x = 0 \Rightarrow H_B = 0$$ 02

$$\Sigma F_y = 0 \Rightarrow V_A + V_B - 18 = 0$$

$$\Sigma M_A = 0 \Rightarrow 18 \times 2 - V_B \times 5 = 0$$

$$\boxed{V_B = 7.2\,kN}$$ 03

$$V_A + 7.2 - 18 = 0 \Rightarrow \boxed{V_A = 10.8\,kN}$$

Ans:~

$$\boxed{\begin{aligned} V_A &= 10.8\,kN \\ V_B &= 7.2\,kN \\ V_C &= 18\,kN \\ V_D &= 12.01\,kN \end{aligned}}$$