

Internal Assesment Test - II

1. Flux linkages due to a single current carrying conductor. Consider a long straight cylindrical conductor of radirus r metres and carrying a current I amperes (r.m.s.) as shown in Fig. 9.4 (i). This current will set up magnetic field. The magnetic lines of force will exist inside the conductor as well as outside the conductor. Both these fluxes will contribute to the inductance of the conductor.

(i) Flux linkages due to internal flux. Refer to Fig. 9.4 (ii) where the X-section of the conductor is shown magnified for clarity. The magnetic field intensity at a point x metres from the centre is given by;

$$
^*H_x = \frac{I_x}{2\pi x}
$$

Assuming a uniform current density,

$$
I_x = \frac{\pi x^2}{\pi r^2} I = \frac{x^2}{r^2} I
$$

$$
H_x = \frac{x^2}{r^2} \times I \times \frac{1}{2\pi x} = \frac{x}{2\pi r^2} I A T / m
$$

z,

If μ (= $\mu_0\mu_r$) is the permeability of the conductor, then flux density at the considered point is given by;

$$
B_x = \mu_0 \mu_r H_x \text{wb/m}^2
$$

= $\frac{\mu_0 \mu_r x}{2 \pi r^2} I = \frac{\mu_0 x I}{2 \pi r^2} \text{wb/m}^2 [\because \mu_r = 1 \text{ for non-magnetic material}]$

Now, flux $d\phi$ through a cylindrical shell of radial thickness dx and axial length 1 m is given by;

$$
d\phi = B_x \times 1 \times dx = \frac{\mu_0 x I}{2 \pi r^2} dx
$$
 weber

This flux links with current $I_x\left(\frac{I \pi x^2}{\pi r^2}\right)$ only. Therefore, flux linkages per metre length of the conductor is

$$
d\psi = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu_0 I x^3}{2\pi r^4} dx
$$
 weber-tums

Total flux linkages from centre upto the conductor surface is

$$
\Psi_{int} = \int_{0}^{r} \frac{\mu_0 I x^3}{2\pi r^4} dx
$$

= $\frac{\mu_0 I}{8\pi}$ weber-turns per metre length

(ii) Flux linkages due to external flux. Now let us calculate the flux linkages of the conductor due to external flux. The external flux extends from the surface of the conductor to infinity. Referring to Fig. 9.5, the field intensity at a distance x metres (from centre) outside the conductor is given by;

$$
H_x = \frac{I}{2 \pi x} A T / m
$$

$$
B_x = \mu_0 H_x = \frac{\mu_0 I}{2 \pi x} w b / m^2
$$

 \sim \sim

Flux density,

A

Now, flux $d\phi$ through a cylindrical shell of thickness dx and axial length 1 metre is

$$
d\phi = B_x dx = \frac{\mu_0 I}{2\pi x} dx
$$
 webers

The flux $d\phi$ links all the current in the conductor once and only once.

 $d\psi = d\phi = \frac{\mu_0 I}{2\pi r} dx$ weber-turns Flux linkages, A.

Total flux linkages of the conductor from surface to infinity,

$$
\Psi_{\text{ext}} = \int_{r}^{\infty} \frac{\mu_0 I}{2\pi x} dx
$$
 weber-turns

Overall flux linkages, $\psi = \psi_{int} + \psi_{ext} = \frac{\mu_0 I}{8\pi} + \int_{r}^{\infty} \frac{\mu_0 I}{2\pi x} dx$ Λ

$$
\Psi = \frac{\mu_0 I}{2\pi} \left[\frac{1}{4} + \int_{r}^{\infty} \frac{dx}{x} \right]
$$
wb-turns/m length

Expression in alternate form. The expression for the inductance of a conductor can be put in a concise form.

$$
L_A = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{d}{r} \right] H/m
$$

= 2 × 10⁻⁷ $\left[\frac{1}{4} + \log_e \frac{d}{r} \right]$
= 2 × 10⁻⁷ $\left[\log_e e^{1/4} + \log_e \frac{d}{r} \right]$

$$
L_A = 2 × 10^{-7} \log_e \frac{d}{re^{-1/4}}
$$

÷

If we put $r e^{-1/4} = r'$, then,

$$
L_A = 2 \times 10^{-7} \log_e \frac{d}{r'} \text{ H/m}
$$
...(iii)

The radius r' is that of a fictitious conductor assumed to have no internal flux but with the same inductance as the actual conductor of radius r. The quantity $e^{-1/4} = 0.7788$ so that

$$
r' = r e^{-1/4} = 0.7788 r
$$

The term r' (= $r e^{-1/4}$) is called *geometric mean radius* (GMR) of the wire. Note that eq. (iii) gives the same value of inductance L_A as eq. (i). The difference is that eq. (iii) omits the term to account for internal flux but compensates for it by using an adjusted value of the radius of the conductor.

(ii) Unsymmetrical spacing. When 3-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical. Under such conditions, the flux linkages and inductance of each phase are not the same. A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced. Therefore, the voltage at the receiving end will not be the same for all phases. In order that voltage drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as *transposition*. Fig. 9.9 shows the

transposed line. The phase conductors are designated as A , B and C and the positions occupied are numbered 1, 2 and 3. The effect of transposition is that each conductor has the same average inductance.

Fig. 9.9 shows a 3-phase transposed line having unsymmetrical spacing. Let us assume that each of the three sections is 1 m in length. Let us further assume balanced conditions *i.e.*, $I_A + I_B +$ $I_C = 0$. Let the line currents be :

$$
I_A = I(1+j 0)
$$

\n
$$
I_B = I(-0.5 - j 0.866)
$$

\n
$$
I_C = I(-0.5 + j 0.866)
$$

Transmission lines which carry three phase power are usually configured as either single circuit or double circuit. A single circuit configuration has three conductors for the three phases. While a double circuit configuration has six conductors (three phases for each circuit).

Double Circuits are used where greater reliability is needed. This method of transmission enables the transfer of more power over a particular distance. The transmission is thus cheaper and requires less land and is considered ideal from an ecological and aesthetic point of view. However, running two circuits in close proximity to each other will involve inductive coupling between the conductors. This needs to be taken into account when calculating the fault level and while designing the protection schemes.

Double circuit transmission lines usually contain **bundled conductors** with the conductors placed as far as possible to minimize inductance.

Example 9.4. The three conductors of a 3-phase line are arranged at the corners of a triangle of sides $2 m$, $2.5 m$ and $4.5 m$. Calculate the inductance per km of the line when the conductors are regularly transposed. The diameter of each conductor is 1.24 cm.

Solution. Fig. 9-12 shows three conductors of a 3-phase line placed at the corners of a triangle of sides $D_{12} = 2$ m, $D_{23} = 2.5$ m and $D_{31} = 4.5$ m. The conductor radius $r = 1.24/2 = 0.62$ cm.

Equivalent equilateral spacing, $D_{eq} = \sqrt[3]{D_{12} \times D_{23} \times D_{31}} = \sqrt[3]{2 \times 2 \cdot 5 \times 4 \cdot 5} = 2 \cdot 82 \text{ m} = 282 \text{ cm}$ = $10^{-7} (0.5 + 2 \log_e D_{eq}/r)$ H = $10^{-7} (0.5 + 2 \log_e 282/0.62)$ H Inductance/phase/m $= 12.74 \times 10^{-7}$ H $= 12.74 \times 10^{-7} \times 1000 = 1.274 \times 10^{-3}$ H = 1.274 mH Inductance/phase/km

Q^{3a.} Derive the expression for capacitance of a transposed 3-phase, 3wire line with unsymmetrical spacing.

[5] $|$ C403.1 $|$ L3

(ii) Unsymmetrical spacing. Fig. 9.25 shows a 3-phase transposed line having unsymmetrical spacing. Let us assume balanced conditions *i.e.* $Q_A + Q_B + Q_C = 0$.

Fig. 9.25

Considering all the three sections of the transposed line for phase A ,

Potential of 1st position, $V_1 = \frac{1}{2\pi\epsilon_0} \left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_2} + Q_C \log_e \frac{1}{d_2}\right)$ Potential of 2nd position, $V_2 = \frac{1}{2\pi\epsilon_0} \left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_C \log_e \frac{1}{d_2}\right)$ Potential of 3rd position, $V_3 = \frac{1}{2\pi\epsilon_0} \left(Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_2} + Q_C \log_e \frac{1}{d_1}\right)$

Average voltage on condutor Λ is

$$
V_A = \frac{1}{3} (V_1 + V_2 + V_3)
$$

= $\frac{1}{3 \times 2\pi\varepsilon_0} * \left[Q_A \log_e \frac{1}{r^3} + (Q_B + Q_C) \log_e \frac{1}{d_1 d_2 d_3} \right]$

As $Q_A + Q_B + Q_C = 0$, therefore, $Q_B + Q_C = -Q_A$

$$
V_A = \frac{1}{6\pi\varepsilon_0} \left[Q_A \log_{\varepsilon} \frac{1}{r^3} - Q_A \log_{\varepsilon} \frac{1}{d_1 d_2 d_3} \right]
$$

$$
= \frac{Q_A}{6\pi\varepsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3}
$$

$$
= \frac{1}{3} \times \frac{Q_A}{2\pi\varepsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3}
$$

$$
= \frac{Q_A}{2\pi\varepsilon_0} \log_e \left(\frac{d_1 d_2 d_3}{r^3}\right)^{1/3}
$$

$$
= \frac{Q_A}{2\pi\varepsilon_0} \log_e \frac{(d_1 d_2 d_3)^{1/3}}{r}
$$

 \therefore Capacitance from conductor to neutral is

$$
C_A = \frac{Q_A}{V_A} = \frac{2 \pi \varepsilon_0}{\log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r}} F/m
$$

Q^{3b.} Fig 3.1 shows a stranded conductor having 7 identical strands each of radius "r". Find geometrical mean radius of the conductor
and ratio of GMR to overall conductor radius. Comment on the results.

Ć

Fig 3.1

Solution. The distances from strand 1 to the other strands are as follows (ref. Fig E2.6)

$$
D_{12} = D_{16} = D_{17} = 2r; \quad D_{14} = 4r
$$

$$
D_{13} = \sqrt{D_{14}^2 - D_{34}^2} = \sqrt{(4r)^2 - (2r)^2}
$$

$$
D_{13} = 2r\sqrt{3}, \quad D_{15} = D_{13}
$$

The geometric mean radius of the seven strand conductor is the 49th root of the product of 4 distances

$$
D_s = \sqrt[49]{(\frac{r}{r})^7 (D_{12}^2 D_{13}^2 D_{14}^2 D_{17}^2)^6 (2r)^6}
$$

where r' is the GMR of each strand. It is raised to the seventh power to account for seven strands. The tern $(D_{12}^2 D_{13}^2 D_{14} D_{17})$ represents the product of distance from one outside strand to every other strand. Thi term is raised to the sixth power to account for si: outside strands. The term $(2r)^6$ accounts for the product of the distances from the central strand to every outside strand

$$
D_s = \sqrt[49]{(0.7788r)^7[(2r)^2(2\sqrt{3}r)^2.(4r)(2r)](2r)^6}
$$

= 2.177 r

The overall conductor radius is 3r. Therefore the ratio of GMR to overall conductor radius is $2.177 r/3r$: 0.7257. As the number of strands increases, this ratio approaches 0.7788 which is that for a solic monducto

Q^{4a.} A 3-phase, 50 Hz transmission line, 100km long, delivers 20MW at 0.9 power factor lagging and at 110kV. The resistance and reactance of the line per phase per km are 0.2Ω and 0.4Ω respectively, while the capacitive admittance is $2.5X10^{-6}$ mho per phase per km. calculate $[10]$ $C403.4$ L₃ Voltage and current at the sending end. i. ii. Efficiency of the transmission line. Use nominal T method. $3-4$, $4=504$, $1=100km$, $P_{3-4,8}=20km$, $P_{3-6,9}=0.9$ log
 $V_{16}=110hV.$ $8=0.13/1cm$ $y=2.5\times10^{6}27/1cm$ Nowand T $\sqrt{127}$
 $8 = 8.1 = 202$ (3) VL = ? $\frac{1}{2}$ = ? $\frac{v_{x1}}{v_{x2}}$

$$
V_{r} = \frac{V_{r1}}{\Omega} = 63.5 \text{ k} \frac{10^{6} \text{V}}{\Omega}
$$

$$
= \frac{3663.5 \text{ N} \frac{6}{3} \text{V}}{363.5 \text{ N} \frac{6}{3} \text{V}} = 116.65 \text{ A}
$$

$$
= 116.65 \text{ A}
$$

$$
V_{21} = 2608.37137.59^{\circ}V
$$

\n
$$
V_{c} = V_{8} + V_{21} = 65.58811.39^{\circ}V
$$

\n
$$
T_{c} = V_{c} + V_{21} = 16.39119.39^{\circ}A
$$

\n
$$
T_{c} = V_{c} + T_{s}
$$

\n
$$
T_{c} = 110.1161.8.23^{\circ}A
$$

\n
$$
V_{22} = T_{c} + T_{s}
$$

\n
$$
V_{32} = T_{2} + T_{s}
$$

\n
$$
V_{32} = T_{2} + T_{s}
$$

\n
$$
V_{33} = T_{3} + T_{s} = 2.462111.82^{\circ}V
$$

\n
$$
V_{33} = T_{3} + T_{s} = 2.462111.82^{\circ}V
$$

\n
$$
V_{33} = T_{3} + T_{s}
$$

\n
$$
V_{34} = 110.11611.812.29^{\circ}V
$$

\n
$$
V_{34} = 2.462111.29^{\circ}V
$$

\n
$$
V_{
$$

L \perp $\mathsf{Q}5\mathsf{a}.$ Find the capacitance of a 1-phase line $40\mathrm{km}$ long consisting of two parallel wires each 5mm in dia. Determine the capacitance of the same line taking in to account the effect of ground. The height
of the conductors above the ground is 7m comment on the results [10] obtained.

Solution	21.60 m	1.244 MP
200 = 3600 J	1.244 MP	
201 = 0.1744 MP	1.2450 J	
201 = 0.1737 MP	1.2450 J	
200 = $\frac{1}{36(10)}$ MP	1.2450 J	
201 = 0.1737 MP		
200 = 0.1737 MP		
201 = 0.1737 MP		
201 = 0.1737 MP		
201 = 0.1737 MP		
202 = 0.1737 MP		
203 = 0.1737 MP		
204 = 0.173 P		
205 = 0.173 P		
206 = 0.173 P		
207 = 0.174 P		
208 = 0.175 P		
209 = 0.175 P		
201 = 0.174 P		
201 = 0.174 P		

 $Q6a.$ Explain the phenomenon of corona in over head transmission lines.

 $\begin{bmatrix} 5 \end{bmatrix}$ $\begin{bmatrix} C403.1 \end{bmatrix}$ L2

When an alternating potential difference is applied across two conductors whose spacing is large as compared to their diameters, there is no apparent change in the condition of atmospheric air surrounding the wires if the applied voltage is low. However, when the applied voltage exceeds a certain value, called *critical disruptive voltage*, the conductors are surrounded by a faint violet glow called corona.

The phenomenon of corona is accompanied by a hissing sound, production of ozone, power loss and radio interference. The higher the voltage is raised, the larger and higher the luminous envelope becomes, and greater are the sound, the power loss and the radio noise. If the applied voltage is increased to breakdown value, a flash-over will occur between the conductors due to the breakdown of air insulation.

The phenomenon of violet glow, hissing noise and production of ozone gas in an overhead transmission line is known as corona.

If the conductors are polished and smooth, the corona glow will be uniform throughout the length of the conductors, otherwise the rough points will appear brighter. With d.c. voltage, there is

difference in the appearance of the two wires. The positive wire has uniform glow about it, while the negative conductor has spotty glow.

Theory of corona formation. Some ionisation is always present in air due to cosmic rays, ultraviolet radiations and radioactivity. Therefore, under normal conditions, the air around the conductors contains some ionised particles (i.e., free electrons and +ve ions) and neutral molecules. When p.d. is applied between the conductors, potential gradient is set up in the air which will have maximum value at the conductor surfaces. Under the influence of potential gradient, the existing free electrons acquire greater velocities. The greater the applied voltage, the greater the potential gradient and more is the velocity of free electrons.

When the potential gradient at the conductor surface reaches about 30 kV per cm (max. value), the velocity acquired by the free electrons is sufficient to strike a neutral molecule with enough force to dislodge one or more electrons from it. This produces another ion and one or more free electrons, which is turn are accelerated until they collide with other neutral molecules, thus producing other ions. Thus, the process of ionisation is cummulative. The result of this ionisation is that either corona is formed or spark takes place between the conductors.

8.13 Advantages and Disadvantages of Corona

Corona has many advantages and disadvantages. In the correct design of a high voltage overhead line, a balance should be struck between the advantages and disadvantages.

Advantages

- (i) Due to corona formation, the air surrounding the conductor becomes conducting and hence virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.
- (ii) Corona reduces the effects of transients produced by surges.

Disadvantages

- (i) Corona is accompanied by a loss of energy. This affects the transmission efficiency of the line.
- (ii) Ozone is produced by corona and may cause corrosion of the conductor due to chemical action.
- (iii) The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidal voltage drop occurs in the line. This may cause inductive interference with neighbouring communication lines.
- Q7a. What is meant by grading of cables? Explain inter-sheath grading $\begin{bmatrix} 5 \end{bmatrix}$ C403.3 $\begin{bmatrix} 12 \end{bmatrix}$ in detail.

Grading of Cables 11.11

The process of achieving uniform electrostatic stress in the dielectric of cables is known as grading of cables.

It has already been shown that electrostatic stress in a single core cable has a maximum value (g_{max}) at the conductor surface and goes on decreasing as we move towards the sheath. The maximum voltage that can be safely applied to a cable depends upon g_{max} *i.e.*, electrostatic stress at the conductor surface. For safe working of a cable having homogeneous dielectric, the strength of di-

electric must be more than g_{max} . If a dielectric of high strength is used for a cable, it is useful only near the conductor where stress is maximum. But as we move away from the conductor, the electrostatic stress decreases, so the dielectric will be unnecessarily overstrong.

The unequal stress distribution in a cable is undesirable for two reasons. Firstly, insulation of greater thickness is required which increases the cable size. Secondly, it may lead to the breakdown of insulation. In order to overcome above disadvantages, it is necessary to have a uniform stress distribution in cables. This can be achieved by distributing the stress in such a way that its value is increased in the outer layers of dielectric. This is known as grading of cables. The following are the two main methods of grading of cables :

 (i) Capacitance grading (ii) Intersheath grading

11.13 Intersheath Grading

In this method of cable grading, a homogeneous dielectric is used, but it is divided into various layers by placing metallic intersheaths between the core and lead sheath. The intersheaths are held at suitable potentials which are inbetween the core potential and earth potential. This arrangement improves voltage distribution in the dielectric of the cable and consequently more uniform potential gradient is obtained. Lead

Consider a cable of core diameter d and outer lead sheath of diameter D . Suppose that two intersheaths of diameters d_1 and d_2 are inserted into the homogeneous dielectric and maintained at some fixed potentials. Let V_1 , Inter- V_2 and V_3 respectively be the voltage between core and intersheath 1, between intersheath 1 and 2 and between intersheath 2 and outer lead sheath. As there is a definite potential difference between the inner and outer layers of Inter each intersheath, therefore, each sheath can be treated like sheath 2 a homogeneous single core cable. As proved in Art. 11.9,

Maximum stress between core and intersheath 1 is

Similarly,

z.

$$
g_{3max} = \frac{\frac{d_1}{2} \log_e \frac{d_2}{d_1}}{\frac{d_2}{2} \log_e \frac{D}{d_2}}
$$

 V_2

 $g_{1max} = \frac{V_1}{\frac{d}{2} \log_e \frac{d_1}{\lambda}}$

Since the dielectric is homogeneous, the maximum stress in each layer is the same *i.e.*,

$$
g_{1max} = g_{2max} = g_{3max} = g_{max} \text{ (say)}
$$

$$
\frac{V_1}{\frac{d}{2}\log_e\frac{d_1}{d}} = \frac{V_2}{\frac{d_1}{2}\log_e\frac{d_2}{d_1}} = \frac{V_3}{\frac{d_2}{2}\log_e\frac{D}{d_2}}
$$

As the cable behaves like three capacitors in series, therefore, all the potentials are in phase *i.e.* Voltage between conductor and earthed lead sheath is

$$
V = V_1 + V_2 + V_3
$$

Intersheath grading has three principal disadvantages. Firstly, there are complications in fixing the sheath potentials. Secondly, the intersheaths are likely to be damaged during transportation and installation which might result in local concentrations of potential gradient. Thirdly, there are considerable losses in the intersheaths due to charging currents. For these reasons, intersheath grading is rarely used.

