CMR INSTITUTE OF TECHNOLOGY		
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INTERNAL ASSESSMENT TEST 2 – MAY 2017

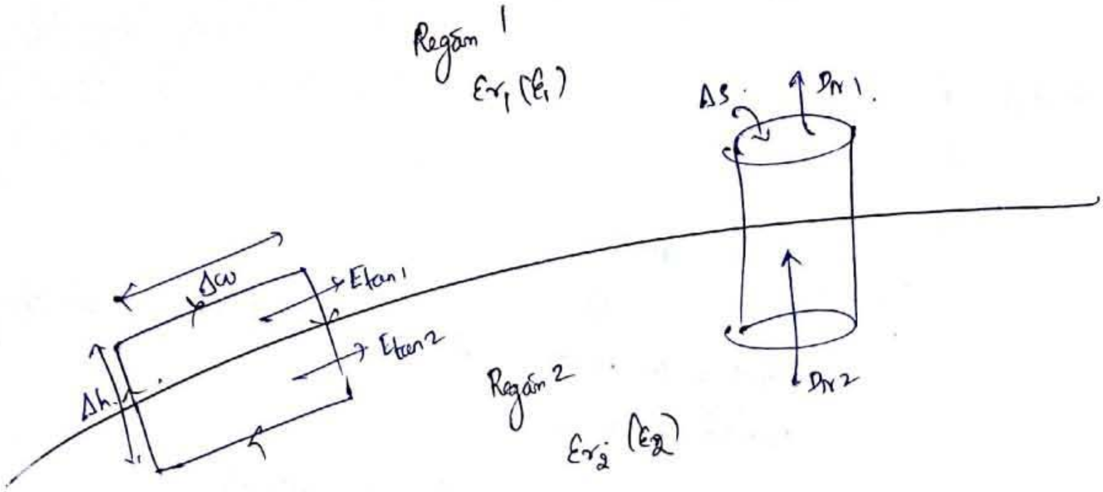
Sub:	ELECTROMAGNETIC FIELD THEORY							Code:	15EE45
Date:	10/05/2017	Duration:	90 mins	Max Marks:	50	Sem:	4	Branch:	EEE
Answer Any FIVE FULL Questions									

		Marks	OBE CO	OBE RBT
1(a)	Obtain the boundary conditions for a dielectric – dielectric boundary.	[05]	CO4	L2
1(b)	Derive the equation of continuity of current.	[05]	CO4	L2
2	Using Laplace's equation for potential function, derive the expression for capacitance of a cylindrical capacitor, $C = \frac{2\pi\varepsilon L}{\ln(b/a)}$, where L is the length of the capacitor, a & b are inner and outer radii of the cylindrical layers.	[10]	CO3	L2
3(a)	Determine whether the potential field $V = \rho^2 + z^2$ satisfies Laplace's equation or not.	[02]	CO3	L3
3(b)	State and Prove Uniqueness theorem.	[08]	CO3	L1, L2
4(a)	Derive the expression for magnetic field intensity due to finite and infinite conductor using Biot – Savart law.	[07]	CO5	L2
4(b)	Given $\boldsymbol{E} = (x + 2y + az)\boldsymbol{a_x} + (bx - 3y - z)\boldsymbol{a_y} + (4x + cy + 2z)\boldsymbol{a_z}\ V/m$. Find a, b and c if **E** is irrotational.	[03]	CO5	L3
5(a)	A charged particle moves with a uniform velocity of $4\ \boldsymbol{a_x}\ m/s$ in a region where $\boldsymbol{E} = 20\ \boldsymbol{a_y}\ \ V/m$ and $\boldsymbol{B} = B_0\ \boldsymbol{a_z}\ \ wb/m^2$. Determine B_0 such that the velocity of the particle remains constant.	[04]	CO5	L3
5(b)	Discuss the concept of scalar and vector magnetic potentials	[06]	CO5	L2
6	Given $\boldsymbol{H} = 20\ \rho^2 \boldsymbol{a_\phi}\ A/m$ in cylindrical coordinates. Determine a. Current density **J.** b. Total current crossing the surface $\rho = 1\ m\ ; 0 \le \phi \le 2\pi\ and\ z = 0$. c. Verify the result using Stokes theorem.	[10]	CO5	L3
7(a)	A horizontal metal wire is carrying an electric current from North to South. Using uniform magnetic field, it is to be prevented from falling under gravity. What should be the direction of this magnetic field? Justify the answer.	[05]	CO5	L4
7(b)	Derive Lorentz force equation and mention the applications of its solutions.	[05]	CO5	L3
8(a)	Derive the expression for magnetic torque due to a rectangular current loop.	[06]	CO5	L3
8(b)	Two infinitely long straight conductors, both carrying current of 10mA each in the same direction are located at x=0; y=0 and x=0; y=10m respectively. Determine the force per unit length between them.	[04]	CO5	L3

SCHEME OF EVALUATION

		Mark Split-Up
1(a)	Diagram Derivation	1 4
(b)	Derivation Final Expression	3 2
2	Diagram Derivation Final Expression	2 7 1
3(a)	Formula Approach & Answer	1 1
3(b)	Statement Proof	2 6
4(a)	Diagram Derivation Final Expression	1 4 2
(b)	Formula Approach Answer	1 1 1
5(a)	Formula Approach Answer	1 2 1
5(b)	Scalar Potential: Explanation Final Expressions Vector Potential: Explanation Final Expressions	 2 1 2 1
6	Current Density Current [RHS of Stokes Theorem] LHS of Stokes theorem and verification	3 2 5
7(a)	Diagram Approach Answer	2 2 1
(b)	Derivation Applications	4 1
8(a)	Diagram Derivation Final Expression	1 4 1
(b)	Formula Approach Answer	1 2 1

1 a)



Work done around a closed path $= 0$

$$\oint_L \vec{E}\cdot d\vec{l} = 0.$$

$$E_{tan1}\,\Delta w - E_{tan2}\,\Delta w = 0$$

Tangential $\vec{E}$ is continuous across the boundary. $\boxed{E_{tan1} = E_{tan2}}$ (or) $\boxed{\frac{D_{tan1}}{D_{tan2}} = \frac{\epsilon_1}{\epsilon_2}.}$ Tangential $\vec{D}$ is discontinuous across boundary.

$$\boxed{\frac{D_{tan1}}{\epsilon_1} = E_{tan1} = E_{tan2} = \frac{D_{tan2}}{\epsilon_2}}$$

Gauss's law:

$$D_{N1}\cdot\Delta S - D_{N2}\,\Delta S = \Delta Q = \rho_s\,\Delta S.$$

$\boxed{D_{N1} - D_{N2} = \rho_s}$ → no free charge in dielectric → $\rho_s = 0$ (free surface charge density)

Normal $\vec{D}$ is continuous → $\boxed{D_{N1} = D_{N2}}$

$$\boxed{\epsilon_1 E_{N1} = \epsilon_2 E_{N2}.}$$

→ Normal $\vec{E}$ is discontinuous.

1 b) Continuity of Current:

principle of conservation of charge and continuity equation — follow the principle

Charges can neither be created nor be destroyed, although equal amount of positive and negative charge may be simultaneously created, obtained by separation, destroyed or lost by recombination.

(Q)

Consider any region bounded by a closed surface. Current through the closed surface,

$$\boxed{I = \oint_S \vec{J}\cdot d\vec{s}}$$

This outward flow of positive charge must be balanced by a decrease of positive charge within that closed surface.

charge inside closed surface $\rightarrow Q_i \Rightarrow$ rate of decrease of $Q_i = -\frac{dQ_i}{dt}$.

Principle of conservation of charge

$$\boxed{I = \oint_S \vec{J}\cdot d\vec{s} = -\frac{dQ_i}{dt}}$$ Integral form of Continuity equation

Differential form:

Divergence theorem

$$\oint_S \vec{J}\cdot d\vec{s} = \int_V (\nabla\cdot\vec{J})\,dv.$$

& $Q_i = \int_V \rho_v\, dv.$

$$\frac{dQ_i}{dt} = \frac{d}{dt}\int_V \rho_v\, dv.$$

closed surface is constant $\Rightarrow \frac{dQ_i}{dt} = \int_V \frac{\partial \rho_v}{\partial t}\, dv.$

$$\therefore \boxed{I = \int_V (\nabla\cdot\vec{J})\,dv = -\int_V \frac{\partial \rho_v}{\partial t}\, dv.}$$

Since the expression is true for any volume, however small, it is true for an incremental volume Δv.

$$\therefore \boxed{(\nabla\cdot\vec{J})\,\Delta v = -\frac{\partial \rho_v}{\partial t}\,\Delta v}$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$ Point form of Continuity equation.

$\Downarrow$

Current or charge per second diverging from a small volume per unit volume is equal to the time rate of decrease of charge per unit volume at every point.

Eg. $\vec{J} = \frac{1}{r} e^{-t} \vec{a_r}$ A/m^2.

At $t = 1s$, Current at $r = 5m \Rightarrow$ $I = J_r \cdot S = \frac{1}{5} e^{-1} \cdot 4\pi(5)^2$.

$$\boxed{I = 23.1 A}$$

At $r = 6m \Rightarrow$ $I = J_r \cdot S = \frac{1}{6} e^{-1} \, 4\pi(6)^2$

$$\boxed{I = 27.7 A}$$

$$-\frac{\partial \rho_v}{\partial t} = \nabla \cdot \vec{J} = \nabla \cdot \left(\frac{1}{r} e^{-t} \vec{a_r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \cdot \frac{1}{r} e^{-t} \right)$$

$$\nabla \cdot \vec{J} = \frac{1}{r^2} \cdot e^{-t}.$$

$$\rho_v = -\int (\nabla \cdot \vec{J}) \, dt + K(r)$$

$$\rho_v = -\int \frac{1}{r^2} e^{-t} dt + K(r).$$

As $t \to \infty$, $\rho_v \to 0 \Rightarrow K(r) = 0$.

$$\therefore \rho_v = -\frac{1}{r^2} \left[\frac{e^{-t}}{-1} \right]$$

$$\rho_v = \frac{1}{r^2} e^{-t} \; C/m^2$$

$$\vec{J} = \rho_v \vec{v}.$$

$$v_r = \frac{J_r}{\rho_v} = \frac{\frac{1}{r} e^{-t}}{\frac{1}{r^2} e^{-t}} = r \; m/s.$$

$$\boxed{v_r = r} \; m/s$$

2)

$$V = \frac{V_0 x}{d}.$$

$$\vec{E} = -\nabla V$$

$$= -\left[\frac{\partial V}{\partial x}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \frac{\partial V}{\partial z}\vec{a_z}\right]$$

1) $\vec{E} = -\frac{V_0}{d}\vec{a_x}$

2) $\vec{D} = -\frac{\varepsilon V_0}{d}\vec{a_x}.$

$$\vec{D_N} = \vec{D}\Big|_{x=0} = -\frac{\varepsilon V_0}{d}\vec{a_x}.$$

3) $D_N = -\frac{\varepsilon V_0}{d} = \rho_s.$

4) $Q = \int_s \rho_s \, ds$

$$Q = -\frac{\varepsilon V_0}{d} \cdot S$$

5) Capacitance, $C = \frac{|Q|}{V_0} = \frac{\varepsilon S}{d}.$

Cylindrical co-ordinates:

Variations with respect to z are nothing new.

Variation with ρ only.

$$\nabla^2 V = 0$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) = 0$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dV}{d\rho}\right) = 0.$$

Multiply by ρ, $\frac{d}{d\rho}\left(\rho\frac{dV}{d\rho}\right) = 0.$

Integrate $\rho\frac{dV}{d\rho} = A.$ (Constant).

$$\frac{dV}{d\rho} = \frac{A}{\rho}.$$

Integrate,

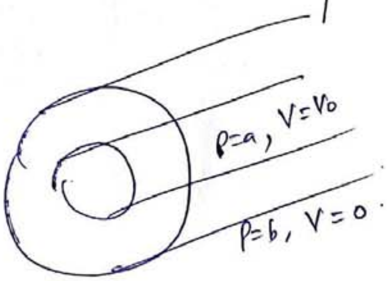
$$\boxed{V = A \ln \rho + B}$$

Boundary conditions

Equipotential surfaces given by ρ = constant are cylinders.

↓

Coaxial capacitor (or) Coaxial transmission line



$V_0 = A \ln a + B$

$0 = A \ln b + B$

$B = -A \ln b$

$V_0 = A \ln a + (-A \ln b)$

$V_0 = A \left[\ln (a/b) \right]$

$\boxed{A = V_0 / \ln (a/b)}$; $\boxed{B = -\dfrac{V_0 \ln b}{\ln (a/b)}}$

$\therefore V = \dfrac{V_0}{\ln (a/b)} \ln \rho - \dfrac{V_0}{\ln (a/b)} \ln b$

$$\boxed{V = V_0 \frac{\ln (b/\rho)}{\ln (b/a)}}$$

$\vec{E} = -\nabla V$

$= -\dfrac{\partial V}{\partial \rho} \vec{a_\rho} = -\dfrac{\partial}{\partial \rho} \left[\dfrac{V_0 \ln (b/\rho)}{\ln (b/a)} \right] \vec{a_\rho}$

$= -\dfrac{\partial}{\partial \rho} \left[\dfrac{V_0 \ln (b)}{\ln (b/a)} - \dfrac{V_0 \ln \rho}{\ln (b/a)} \right] \vec{a_\rho}$

$$\boxed{\vec{E} = \frac{V_0}{\rho} \cdot \frac{1}{\ln (b/a)} \vec{a_\rho}}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{D} = \frac{\varepsilon V_0}{\rho} \frac{1}{\ln(b/a)} \vec{a_\rho}$$

$$D_N = |\vec{D}|_{\rho = a} = \frac{\varepsilon V_0}{a} \frac{1}{\ln(b/a)}$$

$$\rho_S = \frac{\varepsilon V_0}{a} \frac{1}{\ln(b/a)}.$$

$$Q = \int_S \rho_s \, ds = \frac{\varepsilon V_0}{a} \frac{2\pi a L}{\ln(b/a)}$$

$$C = \frac{Q}{V_0} = \frac{\varepsilon V_0}{a V_0} \frac{2\pi a L}{\ln(b/a)}$$

$$\boxed{C = \frac{2\pi\varepsilon L}{\ln(b/a)}}$$

3 a)

Problem:

Determine whether or not the potential field $V = \rho^2 + z^2$ satisfies Laplace's equation.

$V = \rho^2 + z^2$.

Solution

Cylindrical system:

Laplace's equation

$$\nabla^2 V = 0$$

$$\nabla^2 V = \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}\left(\rho \cdot \frac{\partial}{\partial \rho}(\rho^2+z^2)\right)\right) + 0 + \frac{\partial^2}{\partial z^2}(\rho^2+z^2)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cdot 2\rho) + \frac{\partial}{\partial z}(2z)$$

$$= \frac{1}{\rho}(2\times 2\rho) + 2$$

$$= 4+2$$

$$\nabla^2 V = \boxed{\nabla^2 V = 6 \neq 0}$$

V does not satisfy Laplace's equation

3 b) Uniqueness theorem:

Laplace's equation →

wherever volume charge density is zero, every conceivable configuration of electrodes or conductors produce a field for which $\nabla^2 V = 0$

Solution of Laplace's equation subject to certain boundary conditions

Every physical problem contains atleast one conducting boundary (usually has two or more).

Potentials on these boundaries are $V_0, V_1, \ldots$ Equipotential surface.

Uniqueness theorem:

If a solution to Laplace's equation can be found that satisfies the boundary conditions, then the solution is unique

there is only one solution

Theorem applies to any solution of Laplace's and Poisson's equations.

Proof of uniqueness theorem:

Proved by contradiction -

Assume two solutions of Laplace's equation; V_1 and V_2

$\therefore \nabla^2 V_1 = 0$ & $\nabla^2 V_2 = 0$.

$$\boxed{\nabla^2 (V_1 - V_2) = 0}$$

Each solution must satisfy the boundary conditions.

At boundary potential values are given by V_b.

Value of V_1 on boundary → V_{1b}

Value of V_2 on boundary → V_{2b}

} Equipotential conducting boundary.

$\therefore V_{1b} = V_{2b} = V_b$.

$$\therefore \boxed{V_{1b} - V_{2b} = 0}$$

Vector identity,

$$\boxed{\nabla\cdot(V\vec{D}) = V(\nabla\cdot\vec{D}) + \vec{D}\cdot\nabla V}$$ for any scalar V & vector $\vec{D}$.

Let V_1-V_2 as the scalar & $\nabla(V_1-V_2)$ as the vector

$$\nabla\cdot\left((V_1-V_2)\,\nabla(V_1-V_2)\right) = V_1-V_2\left(\nabla\cdot\nabla(V_1-V_2)\right) + \nabla(V_1-V_2)\cdot\nabla(V_1-V_2).$$

Integrate throughout the volume enclosed by the boundary surfaces

$$\int_{vol}\nabla\cdot\big(\underbrace{(V_1-V_2)\,\nabla(V_1-V_2)}_{\text{Vector}}\big)\,dv = \int_{vol}\left[V_1-V_2\left(\nabla\cdot\nabla(V_1-V_2)\right) + \nabla(V_1-V_2)\cdot\nabla(V_1-V_2)\right]dv$$

$$= \int_{vol}(V_1-V_2)\left[\nabla\cdot\nabla(V_1-V_2)\right]dv + \int_{vol}\left[\nabla(V_1-V_2)\cdot\nabla(V_1-V_2)\right]dv$$

$$= \int_{vol}(V_1-V_2)\big[\underbrace{\nabla\cdot\nabla(V_1-V_2)}_{\nabla^2(V_1-V_2)}\big]dv + \int_{vol}\left[\nabla(V_1-V_2)\right]^2 dv$$

Apply divergence theorem to L.H.S. of equation

$$\boxed{\int_{vol}\nabla\cdot\left((V_1-V_2)\,\nabla(V_1-V_2)\right)dv = \oint_S (V_1-V_2)\,\nabla(V_1-V_2)\cdot d\vec{s}} = 0.$$

closed surface encloses equipotential boundary

$$V_{1b}-V_{2b} = 0 \Rightarrow V_{1b} = V_{2b}$$

$$\Downarrow$$

$$V_1-V_2 = 0$$

$$\boxed{\int_{vol}\left[\nabla(V_1-V_2)\right]^2 dv = 0}$$

① Integral can be zero ~~when~~ everywhere

② Integrand is positive in some regions & negative in some other regions, their contribution is zero

$\left[\nabla(V_1-V_2)\right]^2 \leftarrow$ cannot be negative.

$$\Downarrow$$

$$\left[\nabla(V_1-V_2)\right]^2 = 0$$

$$\Downarrow$$

$$\nabla(V_1-V_2) = 0$$

Gradient of $V_1 - V_2$ is zero everywhere.

$\Downarrow$

$V_1 - V_2$ is constant with any co-ordinates

$$\boxed{V_1 - V_2 = \text{constant}}$$

If this constant is zero, uniqueness theorem is proved.

If we consider a point on the boundary

$$V_1 - V_2 = V_{1b} - V_{2b} = 0.$$

$$\therefore \boxed{V_1 = V_2}$$

$\Downarrow$

giving two identical solutions.

Uniqueness theorem also applies to Poisson's equation

$$\nabla^2 V_1 = -\frac{\rho_v}{\varepsilon} \quad \text{and} \quad \nabla^2 V_2 = -\frac{\rho_v}{\varepsilon}.$$

$$\nabla^2 (V_1 - V_2) = 0$$

Boundary conditions require that $V_{1b} - V_{2b} = 0$.

4 a)

Biot Savart law:

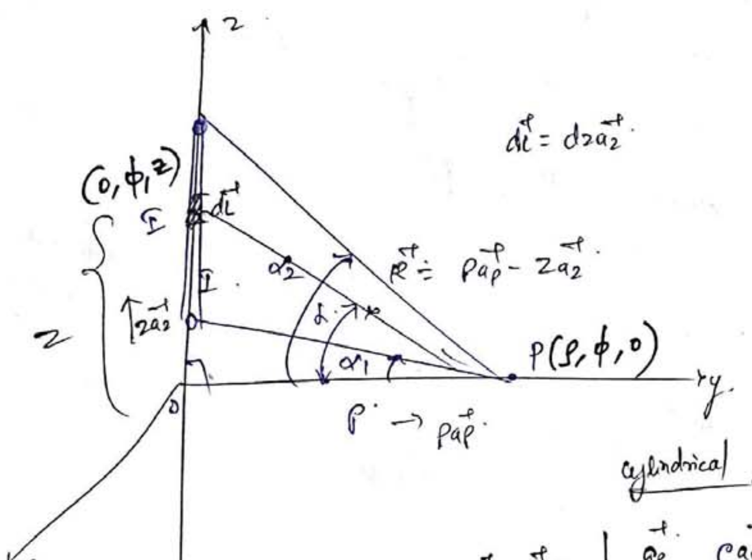
$$\boxed{H = \oint \frac{I\,d\vec{L} \times \vec{a_R}}{4\pi |\vec{R}|^2}}$$

$$H = \int_S \frac{\vec{K}\,ds \times \vec{a_R}}{4\pi |\vec{R}|^2} \qquad H = \int_V \frac{\vec{J}\,dv \times \vec{a_R}}{4\pi |\vec{R}|^2}$$

$$\boxed{H = \int_S \frac{\vec{K} \times \vec{a_R}\,ds}{4\pi |\vec{R}|^2}} \qquad \boxed{H = \int_V \frac{\vec{J} \times \vec{a_R}\,dv}{4\pi |\vec{R}|^2}}$$

Application of Biot-Savart law:

Infinitely long straight filament:



$$\vec{H} = \int_L \frac{I\,d\vec{L} \times \vec{a_R}}{4\pi |\vec{R}|^2}$$

$$\vec{H} = \int_L \frac{I\,d\vec{L} \times \vec{R}}{4\pi |\vec{R}|^3}$$

cylindrical

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \vec{a_\rho} & \rho\vec{a_\phi} & \vec{a_z} \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

In spherical

$$\nabla \times \vec{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \vec{a_r} & r\vec{a_\theta} & r\sin\theta\,\vec{a_\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

$$d\vec{L} \times \vec{R} = \frac{1}{\rho}\begin{vmatrix} \vec{a_\rho} & \rho\vec{a_\phi} & \vec{a_z} \\ 0 & 0 & dz \\ \rho & 0 & -z \end{vmatrix}$$

$$= \frac{1}{\rho}\left[-\rho\vec{a_\phi}\,(0 - \rho\,dz)\right]$$

$$= \frac{\rho^2}{\rho}\,dz\,\vec{a_\phi}$$

$$\boxed{d\vec{L} \times \vec{R} = \rho\,dz\,\vec{a_\phi}}$$

$$\vec{H} = \int_{z=z_1}^{z_2} \frac{I \ \rho dz \ \vec{a_\phi}}{4\pi \left[\rho^2 + z^2\right]^{3/2}}$$

$$\tan\alpha = \frac{z}{\rho}$$

$$z = \rho \tan\alpha$$

z	z_1	z_2
α	α_1	α_2

$$dz = \rho \sec^2\alpha \ d\alpha$$

$$\vec{H} = \frac{I}{4\pi} \int_{\alpha=\alpha_1}^{\alpha_2} \frac{\rho^2 \sec^2\alpha \ d\alpha \ \vec{a_\phi}}{\left[\rho^2 + \rho^2 \tan^2\alpha\right]^{3/2}}$$

$$= \frac{I}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \sec^2\alpha \ d\alpha \ \vec{a_\phi}}{(\rho^2 \sec^2\alpha)^{3/2}}$$

$$= \frac{I}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \cos\alpha \ d\alpha \ \vec{a_\phi}$$

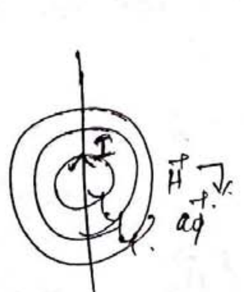
Finite conductor. $\boxed{\vec{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right] \vec{a_\phi}}$ A/m

$z_1 \rightarrow -\infty$ & $z_2 \rightarrow \infty$.

$\alpha_1 = -\pi/2$. & $\alpha_2 = \pi/2$.

$$\vec{H} = \frac{I}{4\pi\rho}\left[1 - (-1)\right] \vec{a_\phi}$$

Infinite conductor. $\boxed{\vec{H} = \frac{I}{2\pi\rho} \vec{a_\phi}}$ A/m



4 b)

Solution:

$\nabla \times \vec{E} = 0$ (Irrotational)

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = \vec{a_x}\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] - \vec{a_y}\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] + \vec{a_z}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$$

$$= \vec{a_x}\left[\frac{\partial}{\partial y}(4x+cy+2z) - \frac{\partial}{\partial z}(bx-3y-z)\right]$$

$$-\vec{a_y}\left[\frac{\partial}{\partial x}(4x+cy+2z) - \frac{\partial}{\partial z}(x+2y+az)\right]$$

$$+\vec{a_z}\left[\frac{\partial}{\partial x}(bx-3y-z) - \frac{\partial}{\partial y}(x+2y+az)\right]$$

$$\Rightarrow \vec{a_x}[c+1] - \vec{a_y}[4-a] + \vec{a_z}[b-2] = 0\vec{a_x} + 0\vec{a_y} + 0\vec{a_z}$$

$\boxed{c = -1}$

$-4 + a = 0$

$\boxed{a = 4}$

$\boxed{b = 2}$

5 a)

Solution:

If a particle moves with constant velocity, its acceleration is zero.

$$F = ma = 0 = Q(\vec{E} + \vec{v} \times \vec{B})$$

$$0 = Q(20\vec{a_y} + 4\vec{a_x} \times B_0\vec{a_z})$$

$$0 = Q(20\vec{a_y} - B_0 4\vec{a_y})$$

$Q \neq 0$

$$\therefore 20\vec{a_y} = 4B_0\vec{a_y}$$

$$\boxed{B_0 = 5}$$

5 b)

Scalar & Vector Magnetic Potentials:

electrostatic potential V → greatly simplified electrostatic field problems.

Can a scalar magnetic potential be defined?

Let us assume the existence of scalar magnetic potential, V_m.

whose negative gradient gives magnetic field intensity.

Let, $\vec{H} = -\nabla V_m$.

w.k.t.

$\nabla \times \vec{H} = \vec{J}$

$\nabla \times \vec{H} = \vec{J} = \nabla \times (-\nabla V_m)$

$\nabla \times (-\nabla V_m) = 0 \Rightarrow \vec{J} = 0$.

Vector Identity:

$\nabla \times \nabla V_m = 0$.

Curl of gradient of a scalar is zero.

∴ If a scalar magnetic potential is defined for a region, then current density must be zero through out the region.

∴ $\boxed{\vec{H} = -\nabla V_m \quad (\vec{J} = 0)}$.

Scalar magnetic potential is useful in magnetic problems involving geometries in which current carrying conductors occupy a relatively small fraction of total region of interest & also in case of permanent magnets

V_m is in amperes.

This scalar potential also satisfies Laplace's equation.

In free space,

$$\vec{B} = \mu_0 \vec{H}$$

$$\nabla \cdot \vec{B} = \nabla \cdot \mu_0 \vec{H} = 0 . \quad (\text{From } \nabla \cdot \vec{B} = 0).$$

$$\mu_0 (\nabla \cdot \vec{H}) = 0$$

$$\mu_0 (\nabla \cdot (-\nabla V_m)) = 0$$

$$\boxed{\nabla^2 V_m = 0 \quad \text{for } \vec{J} = 0}$$ (In homogeneous magnetic materials).

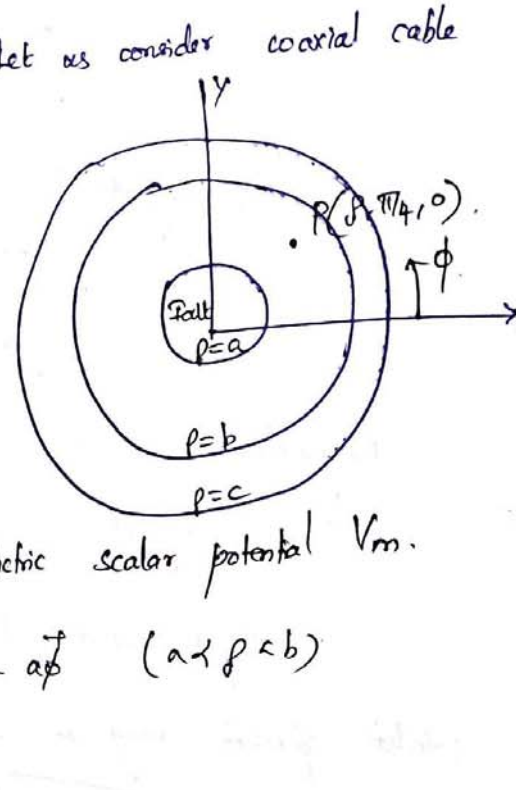
One difference between electric scalar potential (V) & magnetic scalar potential (V_m)

V.
↓
It is single valued
(only one value of V associated with each point in space if zero reference is assigned).

$$\boxed{V_{ab} = -\int_b^a \vec{E} \cdot d\vec{L}} \rightarrow \text{independant of the path.}$$

V_m.
↓
not single valued function of position.

Let us consider coaxial cable



Let $\vec{J} = 0$ in $a < \rho < b$
↓
we may establish electric scalar potential V_m.

$$\vec{H} = \frac{I}{2\pi\rho} \vec{a_\phi} \quad (a < \rho < b)$$

$$\vec{H} = -\nabla V_m .$$

$$\frac{I}{2\pi\rho} \vec{a_\phi} = -\nabla V_m$$

$$-\frac{1}{\rho}\frac{\partial V_m}{\partial \phi} = \frac{I}{2\pi\rho}$$

$$\frac{\partial V_m}{\partial \phi} = \frac{-I}{2\pi}$$

$$\boxed{V_m = \frac{-I}{2\pi}\phi}$$ where constant of integration is set to zero.

If $\phi = 0 \Rightarrow V_m = 0$
$\phi = 2\pi \Rightarrow V_m = -I$.
$\phi = 4\pi \Rightarrow V_m = -2I$.
} Represents a same point but with different values of V_m. (multivaluedness).

At P, $(\rho, \pi/4, 0)$ $V_{mp} = \frac{-I}{2\pi}\left(\pi/4 - 2n\pi\right)$.

$$= \frac{I}{2\pi}\left(2n - \tfrac{1}{4}\right)\cdot\pi$$

$$\boxed{V_{mp} = I\left(n - \tfrac{1}{8}\right)}$$ where $n = 0, \pm 1, \pm 2, \ldots$

In magnetostatic case,

$\nabla \times \vec{H} = 0$ when $\vec{J} = 0$

$\oint \vec{H}\cdot d\vec{L} = I$ even if $\vec{J} = 0$ along the path of integration

Every time when we take another lap around I, the integration increases by I.

If no current is enclosed by the path, then a single valued potential function may be defined.

In general, $$\boxed{V_{m\,a,b} = -\int_b^a \vec{H}\cdot d\vec{L} \quad \text{(specified path)}}$$

Electrostatic potential (V) is a conservative field.

Magnetostatic potential (V_m) is not a conservative field.

Vector Magnetic Potential:

extremely useful in studying radiation from antennas, radiation leakage from transmission lines, waveguides & microwave ovens.

used in regions where current density is zero or nonzero.
& extended to time varying case.

w.k.t.

$$\boxed{\nabla \cdot \vec{B} = 0}$$

Divergence of curl of a vector field is zero.

Let $\boxed{\nabla \cdot (\nabla \times \vec{A}) = 0.}$

Therefore we get

Useful Definition of $\vec{A}$. $\Rightarrow \therefore \boxed{\vec{B} = \nabla \times \vec{A}}$ where $\vec{A}$ signifies Vector magnetic potential

$$\therefore \vec{H} = \frac{1}{\mu_0} (\nabla \times \vec{A}).$$

$$\boxed{\nabla \times \vec{H} = \vec{J} = \frac{1}{\mu_0} \underbrace{(\nabla \times \nabla \times \vec{A})}_{\text{taking curl twice}}}$$

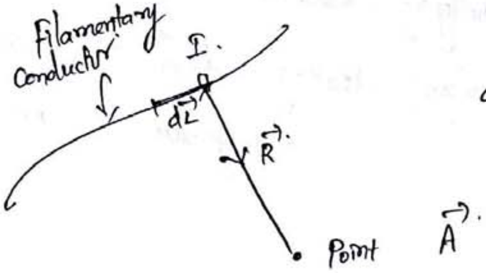
Vector Identity

$$\boxed{\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}}$$

$\boxed{\text{X. } \vec{A} \text{ is in } Wb/m.}$

Magnetic Vector potential $\vec{A}$, may be determined from differential current elements

as $$\boxed{\vec{A} = \oint_L \frac{\mu_0 I \, d\vec{L}}{4\pi |\vec{R}|}}$$



$$d\vec{A} = \frac{\mu_0 I \, d\vec{L}}{4\pi |\vec{R}|}.$$

(no significance of this expression without considering any closed path in which current flows).

6) Problem:

Given $\vec{H} = 20\rho^2 \vec{a}_\phi$ A/m. Determine current density $\vec{J}$.

Also determine total current crossing the surface $\rho = 1$ m, $0<\phi<2\pi$ & $z = 0$.

Solution:

(i) $$\vec{J} = \nabla \times \vec{H} = \frac{1}{\rho}\begin{vmatrix} \vec{a}_\rho & \rho\vec{a}_\phi & \vec{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$$

$$= \frac{1}{\rho}\begin{vmatrix} \vec{a}_\rho & \rho\vec{a}_\phi & \vec{a}_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ 0 & 20\rho^3 & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\cdot\vec{a}_z\left[\frac{\partial(20\rho^3)}{\partial\rho}\right]$$

$$= \frac{60\rho^2\,\vec{a}_z}{\rho}$$

$$\boxed{\vec{J} = 60\rho\,\vec{a}_z} \ \text{A/m}^2$$

(ii) $$I = \int_S \vec{J}\cdot\vec{ds} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{1} 60\rho\ \rho\,d\rho\,d\phi = 60\left[\frac{\rho^3}{3}\right]_0^1\left[\phi\right]_0^{2\pi} = 20\times 2\pi$$

$$I = \int_S (\nabla\times\vec{H})\cdot\vec{ds}$$

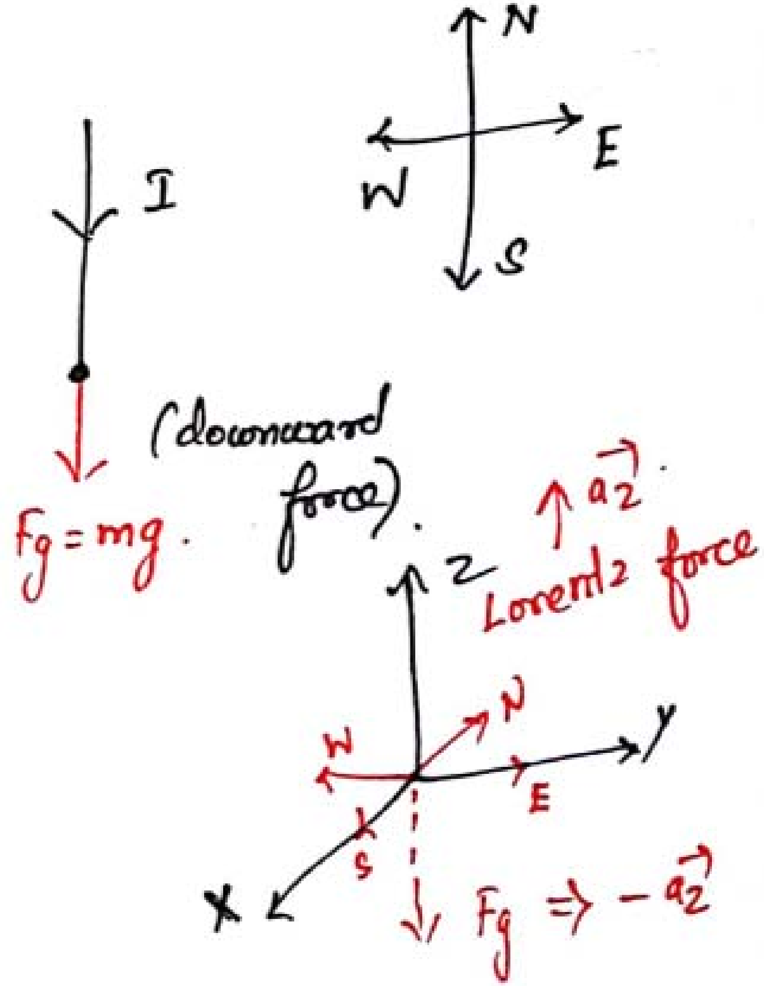
$$\boxed{I = 40\pi} \ \text{A}$$

Verify this using Stokes theorem.

$$\oint_L \vec{H}\cdot\vec{dL} = \int_{\phi=0}^{2\pi} 20\rho^2\cdot\rho\,d\phi\,\Big|_{\rho=1} = 10\times4\pi \ \text{A}.$$

$$\boxed{I = 40\pi} \ \text{A}$$

7 a)



Lorentz force should be upward force.

$\therefore$ If current flows in $\vec{a_x}$.

$$\boxed{\vec{F_m} = I\,dx\,\vec{a_x} \times \vec{B}}$$

is required to be in $\vec{a_z}$

$\therefore$ $\vec{B}$ is required to be in $\vec{a_y}$

(ie) Uniform magnetic field should be directed from <u>West</u> <u>to</u> <u>East</u> to prevent the metal wire from falling under gravity.

7 b) Electric field causes a force to be exerted on a stationary or moving charge

Steady magnetic field → exerts force only on a moving charge
(produced by moving charges)

Force on a moving charge:

Electric force on a charged particle, $\boxed{\vec{F} = Q\vec{E}}$ → (1)

Same dxn as $\vec{E}$ for a positive charge.

If the charge is in motion, the above equation gives the force at any point in its trajectory.

Force on a Charged particle is in motion in a magnetic field of flux density, $\vec{B}$

$\boxed{\vec{F} = Q\vec{v} \times \vec{B}}$ → (2) direction of force is ⊥r to both $\vec{v}$ and $\vec{B}$

$\vec{F}$ applied ⊥r to the dxn in which charge is moving.

↓

can never change its velocity.

Acceleration vector is always normal to velocity vector

Kinetic energy of particle remains unchanged.

Steady magnetic field is incapable of transferring energy to a moving charge

Electric field → exerts force on particle which is independent of the dxn of progressing charge

& effects an energy transfer between field and particle in general.

Force on a moving particle arising from combined electric & magnetic field.

(by superposition)

$$\boxed{\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})} \rightarrow \text{Lorentz force equation} \quad (3)$$

Solution is required in determining

1) electron orbits in magnetron
2) Proton paths in cyclotron
3) plasma characteristics in a magneto hydro dynamic (MHD) generator.

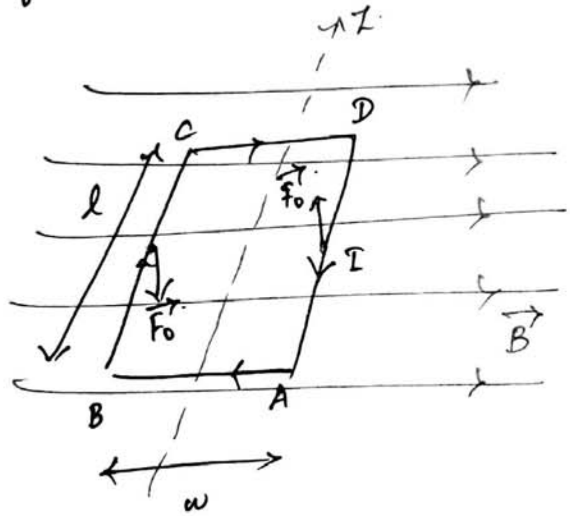
In general charged particle motion in combined electric and magnetic fields

8 a) Magnetic Torque and Moment :

Torque (or mechanical moment of force) on the loop is the vector product of force $\vec{F}$ and the moment arm $\vec{r}$

$$\boxed{\vec{T} = \vec{r} \times \vec{F}} \quad N\text{-}m.$$

Rectangular loop of length 'l' and width 'w' placed in a uniform magnetic field "$\vec{B}$".



$$\text{Force} = \oint_L I \, \vec{dL} \times \vec{B}$$

$$= \int_{AB} \cancel{I \, \vec{dL} \times \vec{B}}_{0} + \int_{BC} I \, \vec{dL} \times \vec{B} + \int_{CD} \cancel{I \, \vec{dL} \times \vec{B}}_{0} + \int_{DA} I \, \vec{dL} \times \vec{B}$$

since $\vec{B}$ & $\vec{dL}$ are parallel

$$\text{Force } \vec{F} = I \int_{BC} \vec{dL} \times \vec{B} + I \int_{DA} \vec{dL} \times \vec{B}$$

$$= I \int_0^l dz \, \vec{a_z} \times \vec{B} + I \int_l^0 dz \, \vec{a_z} \times \vec{B}$$

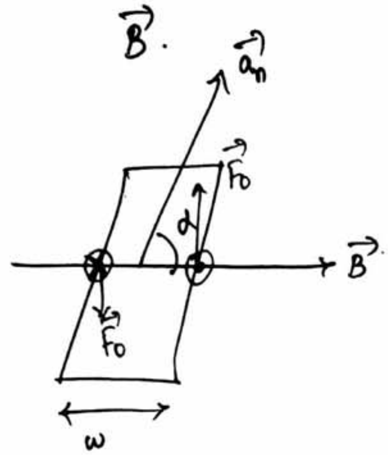
$$= \vec{F_0} - \vec{F_0} \qquad \text{where } |\vec{F_0}| = |\vec{B}| I l \text{ because } \vec{B} \text{ is uniform}$$

$$\vec{F} = 0$$

$\therefore$ No force on the loop as a whole when $\vec{B}$ is uniform

However, $\vec{F_0}$ and $-\vec{F_0}$ act at different points on the loop, thereby creating a couple.

Let Normal to the plane of the loop makes an angle α with $\vec{B}$.



Torque (moment of couple) on the loop

$$\vec{T} = \vec{r} \times \vec{F}$$

$$|\vec{T}| = |\vec{F_0}| \, \omega \sin \alpha$$

$$|\vec{T}| = BIl\omega \sin \alpha$$

Area of the loop $\boxed{S = l\omega}$

$$\boxed{|\vec{T}| = BIS \sin\alpha}$$

Magnetic dipole moment,

$$\boxed{\vec{m} = IS\vec{a_n}} \quad Am^2$$

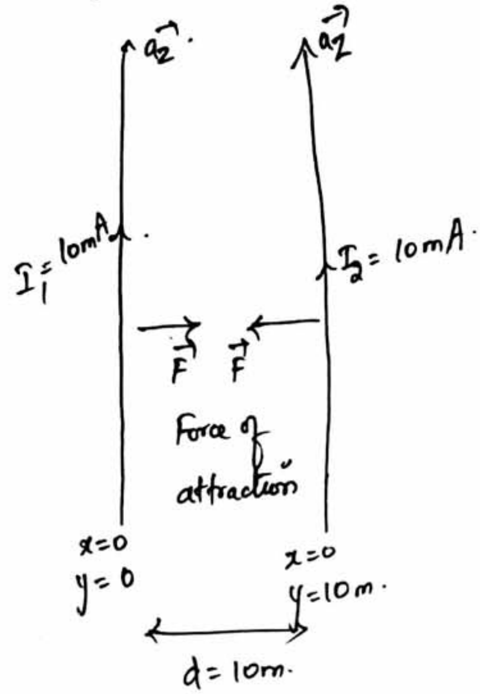
→ product of current and area of the loop.
→ its direction normal to the loop.

$$\boxed{\vec{T} = \vec{m} \times \vec{B}}$$

Torque is in the direction of axis of rotation.
It is directed with the aim of reducing α so that $\vec{m}$ and $\vec{B}$ are in same direction (ie at equilibrium).
If loop is perpendicular to magnetic field,
torque & sum of forces on the loop will be zero.

8 b) **Problem:**

(1) Two infinitely long straight conductors are located at $x=0$; $y=0$ and $x=0$; $y=10$ m. Both carry current of 10 mA in positive $\vec{a_z}$ direction. Determine a force experienced (per meter) between them.



Force per unit length of two infinitely long parallel current carrying conductors $= F/L = \dfrac{\mu_0 I^2}{2\pi d}$

$$= \frac{\overset{2}{\cancel{4}}\cancel{\pi}\times10^{-7} \times (10\times10^{-3})^2}{\cancel{2\pi} \times \underset{5}{\cancel{10}}}$$

$$= 20\times10^{-7}\times10^{-6}$$

$$= 20\times10^{-13}$$

$$F/L = 2\times10^{-12} \text{ N/m}$$

$$\boxed{F/L = 2} \text{ p N/m}$$