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Improvement test

Sub: Engineering Mathematics II				Code: 15MAT21
Date: 01/06/2017	Duration: 90 mins	Max Marks: 50	Sem: 2	Sections: ALL

Answer **Q1** and ANY SIX from **Q2** to **Q8**.

		Marks	OBE CO	OBE RBT
Q1.	Solve by Laplace Transform method: $y''' + 2y'' - y' - 2y = 0$ given that $y(0) = y'(0) = 0$ *and* $y''(0) = 6$.	[8]	CO201.6	L3
Q2.	Find $L\left[\dfrac{2\sin t\sin 5t}{t}\right]$.	[7]	CO201.6	L3
Q3.	Express $f(t) = \begin{cases} \cos t, & 0 < t \le \pi \\ 1, & \pi < t \le 2\pi \\ \sin t, & t > 2\pi \end{cases}$ in terms of unit step function and find its Laplace Transform.	[7]	CO201.6	L3

Question	Marks	CO	Level
Q4. If a periodic function of period 2a is defined by $f(t)=\begin{cases} t & if \quad 0\le t\le a \\ 2a-t & if \quad a\le t\le 2a \end{cases}$ then show that $L\{f(t)\}=\frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$.	[7]	CO201.6	L1
Q5. Find the Laplace Transform of $te^{-2t}\sin 3t$.	[7]	CO201.6	L3
Q6. Find $L^{-1}\left\{\frac{s+3}{s^2-4s+13}\right\}$.	[7]	CO201.6	L3
Q7. Find $L^{-1}\left\{\frac{1}{(s+1)(s^2+9)}\right\}$ using convolution theorem.	[7]	CO201.6	L3
Q8. Find $L^{-1}\left\{\log\left(\frac{1}{s^2}-1\right)+\tan^{-1}\left(\frac{s}{a}\right)\right\}$.	[7]	CO201.6	L3

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Q1. $y''' + 2y'' - y' - 2y = 0$

Taking LT on both sides of the eq^n

$$L[y'''(t)] + 2L[y''(t)] - L[y'(t)] - 2L[y(t)] = L[0]$$

$$\Rightarrow \{s^3 L[y(t)] - s^2 y(0) - s y'(0) - y''(0)\} + 2\{s^2 L[y(t)] - s y(0) - y'(0)\} - \{s L[y(t)] - y(0)\} - 2L[y(t)] = 0$$

$$\Rightarrow L[y(t)]\{s^3 + 2s^2 - s - 2\} - 6 = 0$$

$$\Rightarrow L[y(t)]\{(s+2)(s^2-1)\} = 6$$

$$\Rightarrow L[y(t)] = \frac{6}{(s+2)(s-1)(s+1)} = \frac{A}{s+2} + \frac{B}{s-1} + \frac{C}{s+1}$$

$A = 2, \quad B = 1, \quad C = -3$

$$\Rightarrow y(t) = L^{-1}\left\{\frac{6}{(s+2)(s-1)(s+1)}\right\}$$

$$= L^{-1}\left\{\frac{2}{s+2} + \frac{1}{s-1} - \frac{3}{s+1}\right\}$$

$$= 2e^{-2t} + e^{t} - 3e^{-t}$$

Q2. Let $f(t) = 2 \sin t \sin 5t$

$$= 2 \cdot \frac{1}{2}\left[\cos(-4t) - \cos(6t)\right]$$

$$= \cos 4t - \cos 6t$$

$$\therefore F(s) = \frac{s}{s^2+16} - \frac{s}{s^2+36}$$

Hence $L\left[\frac{f(t)}{t}\right] = \int_s^\infty \left(\frac{s}{s^2+16} - \frac{s}{s^2+36}\right) ds$

$$= \frac{1}{2}\left[\log(s^2+16) - \log(s^2+36)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+16}{s^2+36}\right)\right]_s^\infty$$

$$= \lim_{s\to\infty} \frac{1}{2}\log\left[\frac{s^2(1+16/s^2)}{s^2(1+36/s^2)}\right] - \frac{1}{2}\log\left(\frac{s^2+16}{s^2+36}\right)$$

$$= \frac{1}{2}\left\{\log 1 - \log\left(\frac{s^2+16}{s^2+36}\right)\right\}$$

$$= \log\sqrt{\frac{s^2+36}{s^2+16}}$$

Q3. $f(t) = \begin{cases} \cos t, & 0 < t \le \pi \\ 1, & \pi < t \le 2\pi \\ \sin t, & t > 2\pi \end{cases}$

$= \cos t + (1-\cos t)\, u(t-\pi) + (\sin t - 1)\, u(t-2\pi)$

$\mathcal{L}[f(t)] = \mathcal{L}[\cos(t)] + \mathcal{L}[(1-\cos t)\, u(t-\pi)] + \mathcal{L}[(\sin t - 1)\, u(t-2\pi)]$

$F(t-\pi) = 1-\cos t$ $\qquad$ $G(t-2\pi) = \sin t - 1$

$\therefore F(t) = 1+\cos t$ $\qquad$ $G(t) = \sin t - 1$

$\mathcal{L}[F(t)] = \frac{1}{s} + \frac{s}{s^2+1}$ $\qquad$ $\mathcal{L}[G(t)] = \frac{1}{s^2+1} - \frac{1}{s}$

$\therefore \mathcal{L}[f(t)] = \frac{s}{s^2+1} + e^{-\pi s}\left(\frac{1}{s} + \frac{s}{s^2+1}\right) + e^{-2\pi s}\left(\frac{1}{s^2+1} - \frac{1}{s}\right)$

Q4. We have $T = 2a$.

$\mathcal{L}[f(t)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$

$= \frac{1}{1-e^{-s2a}} \int_0^{2a} e^{-st} f(t)\, dt$

$= \frac{1}{1-e^{-2as}} \left\{ \int_0^a e^{-st} \cdot t\, dt + \int_a^{2a} e^{-st} (2a-t)\, dt \right.$

$= \frac{1}{1-e^{-2as}} \left\{ \left[t\frac{e^{-st}}{-s} - (1)\frac{e^{-st}}{s^2} \right]_0^a + \left[(2a-t)\frac{e^{-st}}{-s} - (-1)\frac{e^{-st}}{s^2} \right]_a^{2a} \right.$

$$= \frac{1}{s^2(1-e^{-2as})}\left(-2e^{-as}+1+e^{-2as}\right)$$

$$= \frac{(1-e^{-as})^2}{s^2(1-e^{-as})(1+e^{-as})}$$

$$= \frac{e^{as/2}-e^{-as/2}}{s^2\left(e^{as/2}+e^{-as/2}\right)} = \frac{2\sinh(as/2)}{s^2 \cdot 2\cosh(as/2)}$$

$$= \frac{1}{s^2}\tanh\left(\frac{as}{2}\right)$$

Q5. $L[\sin 3t] = \frac{3}{s^2+9}$

$$L[t\sin 3t] = (-1)^1 \frac{d}{ds}\frac{3}{s^2+9}$$

$$= -\frac{-3}{(s^2+9)^2}\cdot 2s$$

$$= \frac{6s}{(s^2+9)^2}$$

$$\therefore L[e^{-2t}(t\sin 3t)] = \frac{6(s+2)}{[(s+2)^2+9]^2}$$

$$= \frac{6(s+2)}{(s^2+4s+13)^2}$$

Q6. $L^{-1}\left\{\frac{s+3}{s^2-4s+13}\right\}$

$$= L^{-1}\left\{\frac{s+3}{(s-2)^2+13-4}\right\}$$

$$= L^{-1}\left\{\frac{s\cancel{-2}+2+3}{(s-2)^2+3^2}\right\}$$

$$= L^{-1}\left\{\frac{s-2}{(s-2)^2+3^2}\right\} + L^{-1}\left\{\frac{5}{(s-2)^2+3^2}\right\}$$

$$= e^{2t} L^{-1}\left\{\frac{s}{s^2+3^2}\right\} + 5e^{2t} L^{-1}\left\{\frac{1}{s^2+3^2}\right\}$$

$$= e^{2t}\cos 3t + 5e^{2t}\frac{\sin 3t}{3}$$

Q7. Let $F(s) = \frac{1}{s+1}$, $G(s) = \frac{1}{s^2+9}$

$\Rightarrow f(t) = e^{-t}$, $g(t) = \frac{\sin 3t}{3}$

$$L^{-1}[F(s)\cdot G(s)] = \int_0^t f(u)\,g(t-u)\,du$$

$$= \frac{1}{3}\int_0^t e^{-u}\sin 3(t-u)\,du$$

$$= \frac{1}{3}\left[\frac{e^{-u}}{(-1)^2+(-3)^2}\left[-\sin(3t-3u)+3\cos(3t-3u)\right]\right]_0^t$$

$$= \frac{1}{30}\left[\bar{e}^{t}(0+3) - 1(\sin 3t + 3\cos 3t)\right]$$

$$= \frac{1}{30}\left[3\bar{e}^{t} - \sin 3t - 3\cos 3t\right]$$

Q8. wkt $L^{-1}[F'(s)] = -t\,f(t)$

Let $F(s) = \log\left(\frac{1}{s^2} - 1\right) = \log\left(\frac{1-s^2}{s^2}\right)$

$$= \log(1-s^2) - 2\log s$$

$$F'(s) = \frac{(-2s)}{1-s^2} - \frac{2}{s}$$

$$L^{-1}[-F'(s)] = L^{-1}\left[\frac{2s}{1-s^2}\right] + L^{-1}\left[\frac{2}{s}\right]$$

$$t\,f(t) = -2\cos t + 2$$

$$f(t) = \frac{-2\cos t + 2}{t}$$

Let $G(s) = \tan^{-1}(s/a)$

$$G'(s) = \frac{1}{1+(s/a)^2}(1/a) = \frac{a}{s^2+a^2}$$

$$L^{-1}[-G'(s)] = L^{-1}\left[\frac{-a}{s^2+a^2}\right] = -\sin at$$

$$t\,g(t) = -\sin at \Rightarrow g(t) = \frac{-\sin at}{t}$$

$$\therefore L^{-1}\left\{\log\left(\frac{1}{s^2} - 1\right) + \tan^{-1}(s/a)\right\} = \frac{-2\cos t + 2}{t} - \frac{\sin at}{t}$$