CMR INSTITUTE OF TECHNOLOGY

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Improvement Test

Sub	ENGINEERING MATHEMATICS IV							Code:	15MAT41
Date:	31 /5/ 2017	Duration	90 mins	Max Marks	50	Sem	IV	Branch	CSE-A,B, ISE-A

First question is compulsary. Answer ANY SIX questions from Q2 toQ8

		Marks	OBE CO	OBE RBT
1.	With usual notation, derive the Rodrigues's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$	[08]	CO2	L3
2.	With usual notation, prove that $J_{-1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)} \cos x$.	[07]	CO2	L3
3.	Express $f(x) = x^3 - 5x^2 + 14x + 5$ in terms of Legendre's Polynomials.	[07]	CO2	L3
4.	Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probabilities that after three throws (i) A has the ball (ii) b has the ball (iii) C has the ball.	[07]	CO5	L3

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5. Ten individuals are chosen at random from a population and their heights in inches are found to be 63,63,66,67,68,69,70,70,71,71. Test the hypothesis that the mean height of the universe is 66 inches. $(t_{0.05} = 2.262 \;\; for \;\; 9\, d.f.)$ [07] CO5 L3

6. When a coin is tossed 4 times find, using Binomial distribution, the probability of getting (i) exactly one head (ii) at most three heads (iii) at least three heads [07] CO4 L3

7. The sales per day in a shop are exponentially distributed with the average sale amounting to Rs. 10000 and net profit 8%. Find the probability that the net profit exceeds Rs. 3000 on two consecutive days. [07] CO4 L3

8. Show that probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$ is regular stochastic matrix [07] CO5 L3

And find the associated unique fixed probability vector.

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Solution - IAT(3) M4 - (CS-A&B, IS-A)

(1.) Prove that $P_n(x) = \frac{1}{2^n . n!} \frac{d^n}{dx^n} (x^2-1)^n$

Let $u = (x^2-1)^n$ —(i) —(1)

First we'll prove that u_n satisfies ~~Leibnitz's theo~~ Legendre's diff. eq^n i.e.

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0$$ —(1)

Diff (i) $u_1 = 2nx(x^2-1)^{n-1} = \frac{2nxu}{(x^2-1)}$

$(x^2-1)u_1 = 2nxu$ —(1)

Diff again $(x^2-1)u_2 + 2xu_1 = 2n(xu_1+u)$

Now apply Leibnitz's rule, we get —(1)

$$(1-x^2)u_{n+2} - 2xu_{n+1} + n(n+1)u_n = 0$$

Here u is a polynomial of degree $2n$ and u_n be the polynomial of degree n. —(1)

Sly $P_n(x)$ is also a polynomial of degree n

Let $P_n(x) = Ku_n = K\frac{d^n}{dx^n}(x^2-1)^n = k\frac{d^n}{dx^n}(x-1)^n(x+1)^n$ —(2)

Apply Leibnitz's rule on RHS

$$P_n(x) = k[(x-1)^n . n! + {}^nC_1\, n(x-1)^{n-1}\{(x+1)^n\}_{n-1} + \cdots + n!(x+1)^n]$$ —(1)

Put $x = 1$, we have $P_n(1) = 1$

$\Rightarrow k = \frac{1}{2^n n!}$, Put in eq^n (2) —(1)

$$P_n(x) = \frac{1}{2^n . n!}\frac{d^n}{dx^n}(x^2-1)^n$$ —(1)

(2) we have $J_n(x) = \sum_0^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{\Gamma(n+r+1)\cdot r!}$ — $(1\frac{1}{2})$

Put $n = -1/2$ — $\left(\frac{1}{2}\right)$

$$J_{-1/2}(x) = \sum_0^{\infty} (-1)^r \left(\frac{x}{2}\right)^{-\frac{1}{2}+2r} \frac{1}{\Gamma\left(r+\frac{1}{2}\right) \cdot r!} \quad \text{— (1)}$$

$$= \sqrt{\frac{2}{x}} \left[\frac{1}{\Gamma\frac{1}{2}} - \left(\frac{x}{2}\right)^2 \frac{1}{\Gamma\frac{3}{2} \cdot 1!} + \left(\frac{x}{2}\right)^4 \frac{1}{\Gamma\frac{5}{2} \cdot 2!} - \cdots\right] \quad \text{— (1)}$$

$$= \sqrt{\frac{2}{x}} \left[\frac{1}{\sqrt{\pi}} - \frac{x^2}{4} \times \frac{2}{\sqrt{\pi}} + \frac{x^4}{16} \times \frac{4}{3\sqrt{\pi} \cdot 2} - \cdots\right] \quad \text{— (1)}$$

$$= \sqrt{\frac{2}{\pi x}} \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right] = \sqrt{\frac{2}{\pi x}} \cos x$$

— (1) — (1)

(3) $f(x) = x^3 - 5x^2 + 14x + 5$

we have $P_3(x) = \frac{1}{2}(5x^3 - 3x) \Rightarrow x^3 = \frac{2}{5} P_3(x) + \frac{3}{5} P_1(x)$ — (2)

$P_2(x) = \frac{1}{2}(3x^2 - 1) \Rightarrow x^2 = \frac{2}{3} P_2(x) + \frac{1}{3} P_0(x)$ — (2)

$P_1(x) = x$ & $P_0(x) = 1$ — (2)

$$\therefore f(x) = \left[\frac{2}{5} P_3(x) + \frac{3}{5} P_1(x)\right] - 5\left[\frac{2}{3} P_2(x) + \frac{1}{3} P_0(x)\right] + 14 P_1(x) + 5 P_0(x)$$

$$= \frac{2}{5} P_3(x) + \frac{73}{5} P_1(x) - \frac{10}{3} P_2(x) + \frac{10}{3} P_0(x) \quad \text{— (1)}$$

(4) State Space $= \{A, B, C\}$

t.p.m

$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array} \quad \text{— (1)}$$

$p^{(3)} = p^{(0)} P^3$ — (1), where $p^{(0)} = (0, 0, 1)$ — (1)

$$P^3 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} \quad \text{— (1)}$$

$$p^{(3)} = [0, 0, 1] \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} = \left[\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}\right] \quad \text{— }(1\tfrac{1}{2})$$

$$= \left[p_A^{(3)}, p_B^{(3)}, p_C^{(3)}\right]$$

Prob. that after 3 throws

(i) A has the ball = 1/4 —(1/2)

(ii) B has the ball = 1/4 —(1/2)

(iii) C has the ball = 1/2 —(1/2)

5. $\mu = 66$, $n = 10$ — (1)

$\bar{x} = \frac{\Sigma x}{n} = \frac{678}{10} = 67.8$ — $(1\frac{1}{2})$

$s^2 = \frac{1}{n-1} \Sigma (x - \bar{x})^2 = 9.067 \Rightarrow s = 3.011$ — (2)

$t = \frac{\bar{x} - \mu}{s} \sqrt{n} = 1.89 < 2.262$ — $(1\frac{1}{2})$ ∴ Hypothesis is accepted at 5% level of significance —(1)

6. $p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 4$ — (1)

B.D. ${}^nC_x p^x q^{n-x}$ — (1)

(i) $P(x=1) = {}^4C_1 (0.5)^1 (.5)^3 = 0.25$ — (1)

(ii) $P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$
$= 0.9375$ —(2)

(iii) $P(x \geq 3) = 1 - [P(x=0) + P(x=1)] = 0.6875$ —(2)

7. X : Sale in the shop — (1)

p.d.f $f(x) = \begin{cases} \alpha e^{-\alpha x}, & x > 0 \\ 0, & x \le 0 \end{cases}$ — (1)

Mean $\frac{1}{\alpha} = 10000 \quad \therefore \alpha = \frac{1}{10000} = .0001$ — (1)

Let A be the amount for which profit is 8%.

$A \times 8\% = 3000 \Rightarrow A = 37500$ — (1)

$$\text{Prob}(\text{profit} > 3000) = \text{Prob}(\text{Sale} > 37500)$$

$$= \int_{37500}^{\infty} (.0001)\, e^{-(.0001)x}\, dx \quad \text{— (1)}$$

$$= (.0001)\frac{e^{-(.0001)x}}{-(.0001)}\Bigg|_{375}^{\infty} = e^{-.0375} \quad \text{— (1)}$$

On consecutive days,

$$= \left(e^{-.0375}\right)\left(e^{-.0375}\right) = e^{-0.075} \quad \text{— (1)}$$

8. $$P = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix} = \begin{bmatrix} 1/6 & 1/2 & 1/3 \\ 1/12 & 23/36 & 5/18 \\ 1/9 & 5/9 & 1/3 \end{bmatrix}$$

$\because$ All the entries are +ive $\therefore$ P is regular S.M.

F.P.V $\quad vP = v$, where $v = (a, b, c)$ & $a+b+c = 1$ — (1)

$$\begin{bmatrix} a & b & c \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix} = [a, b, c].$$

we get $\frac{b}{6} = a$ — (2)

$a + \frac{b}{2} + \frac{2c}{3} = b$ — (3)

$\frac{b}{3} + \frac{c}{3} = c$ — (4)

From (1) to (4) we get

$a = \frac{1}{10}$, $b = \frac{6}{10}$, $c = \frac{3}{10}$, $\therefore \left[\frac{1}{10}, \frac{6}{10}, \frac{3}{10}\right]$ is fixed prob. vector