CMR INSTITUTE OF TECHNOLOGY

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Improvement Test

Sub	ENGINEERING MATHEMATICS IV (REGULAR)							Code	15MAT41
Date	30 / 05 / 2017	Duration	90 mins	Max Marks	50	Sem	IV	Branch	EC D, EE A,B

	Question 1 is compulsory. Answer any SIX questions from the rest.	Marks	OBE	
	[$z_{.05}$ = 1.96(two-tail),1.645(one-tail) and $z_{.01}$ = 2.58(two-tail),2.33 (one-tail)]		CO	RBT
1.	If $y' = 2e^x - y, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040$ and $y(0.3) = 2.090$, find y(0.4) correct to 4 decimal places using Milne's method and Adams Bashforth method. Use the corrector formula twice.	08	C401.1	L3
2.	Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1, y'(0) = 0$, compute y(0.2) and y'(0.2) using fourth order RK method.	07	C401.1	L3
3.	Use Modified Euler's method to find y(0.2) correct to 4 decimal places solving the equation $y' = x - y^2, y(0) = 1$ taking h = 0.1.	07	C401.1	L3
4.	A machine is supposed to produce washers of mean thickness 0.12 cm. A sample of 10 washers was found to have a mean thickness of 0.128 cm and standard deviation 0.008. Test whether the machine is working in proper order at 5% level of significance.($t_{0.05}$ at 9 d.f is 1.26)	07	C401.5	L3

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5.	A certain cubical die was thrown 9000 times and 5 or 6 was obtained 3240 times. On the assumption of certain throwing, do the data indicate an unbiased die? Use 5% level of significance.	07	C401.5	L3
6.	An unbiased coin is thrown n times. It is desired that the relative frequency of the appearance of heads should be between 0.49 and 0.51. Find the smallest value of n that will ensure the result with 95% confidence and 90% confidence.	07	C4015	L3
7.	In an elementary school examination the mean grade of 32 boys was 72 with a standard deviation of 8, while the mean grade of 36 girls was 75 with a standard deviation of 6. Test the hypothesis that the performance of girls is better than boys?	07	C401.5	L3
8.	In experiments on Pea breading, the table gives the frequencies of seeds that were obtained. Theory predicts that the frequencies should be in proportions 9:3:3:1. Examine the correspondence between theory and experiment. ($\chi^2_{0.05} = 7.82$ at 3 $d.f$)	07	C401.5	L3

Round & Yellow	Wrinkled & Yellow	Round & Green	Wrinkled & Green	Total
315	101	108	32	556

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Improvement Test
Engineering Mathematics IV May 2017

1. $y' = 2e^x - y$

x	y	
0	2	$y_0' = 2e^0 - 2 = 0$
0.1	2.010	$y_1' = 2e^{0.1} - 2.01 = 0.2003$
0.2	2.040	$y_2' = 2e^{0.2} - 2.04 = 0.4028$
0.3	2.090	$y_3' = 2e^{0.3} - 2.09 = 0.6097$
0.4	?	

2M

Milne's PC Method

$$y_4^P = y_0 + \frac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right)$$

$$= 2 + \frac{4}{3}(0.1)\left(2(0.2003) - 0.4028 + 2(.6097)\right)$$

$$= 2.1623$$

$$y_4' = 2e^{0.4} - 2.1623 = 0.8213$$

$$y_4^C = y_2 + \frac{h}{3}\left(y_2' + 4y_3' + y_4'\right)$$

$$=$$

$$y_4^C = 2.04 + \frac{0.1}{3}\left[0.4028 + 4(0.6097) + 0.8213\right]$$

$$= 2.1621$$

$$y_4' = 2e^{0.4} - 2.1621 = 0.8215$$

$$y_4^C = 2.04 + \frac{0.1}{3}\left[0.4028 + 4(0.6097) + 0.8215\right]$$

$$= 2.1621$$

(3M)

$$y(0.4) = 2.1621$$

Adams Bashforth PC method

$$y_4^P = y_3 + \frac{h}{24}\left(55y_3' - 59y_2' + 37y_1' - 9y_0'\right)$$

$$= 2.09 + \frac{0.1}{24}\left(55(.6097) - 59(.4028) + 37(.2003) - 9(0)\right)$$

$$= 2.1616$$

$$y_4' = 2e^{0.4} - 2.1616 = 0.822$$

$$y_4^C = y_3 + \frac{h}{24}\left(9y_4' + 19y_3' - 5y_2' + y_1'\right)$$

$$= 2.09 + \frac{0.1}{24}\left(9(0.822) + 19(.6097) - 5(.4028) + 0.2003\right)$$

$$= 2.1615$$

$y_4' = 2e^{0.4} - 2.1615 = 0.82215$

$y_4^C = 2.1615$ (3M)

Ans $y(0.4) = 2.1615$

2. $y' = z \quad y'' = \frac{dz}{dx}$

$\frac{dz}{dx} = xz + y, \quad \frac{dy}{dx} = z,$

$y = 1, \; z = 0, \; x = 0$ (2M)

Let $f(x, y, z) = z, \quad g(x, y, z) = xz + y$

$h = 0.2$

$x_0 = 0, \; y_0 = 1, \; z_0 = 0$

$k_1 = hf(x_0\, y_0\, z_0) = (0.2)(0) = 0$

$l_1 = hg(x_0\, y_0\, z_0) = 0.2(0 \times 0 + 1) = 0.2$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$

$= 0.2\, f(0.1, 1, 0.1) = (0.2)(0.1) = 0.02$

$l_2 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$

$0.2\, g(0.1, 1, 0.1) = 0.2\left[(0.1)(0.1) + 1\right]$

$= 0.202$

$$k_3 = hf\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}, z_0+\frac{l_2}{2}\right)$$

$$0.2\, f(0.1, 1.01, 0.101)$$
$$= 0.2(0.101)$$
$$= 0.0202$$

$$l_3 = 0.2\, g(0.1, 1.01, 0.101)$$
$$= 0.2\left[(0.1)(0.101)+1.01\right] = 0.204$$

$$K_4 = hf(x_0+h\ \ y_0+k_3\ \ z_0+l_3)$$
$$0.2\, f(0.2, 1.0202, 0.204)$$
$$= 0.2(0.204) = 0.0408$$

$$l_4 = h\, g(x_0+h\ \ y_0+k_3\ \ z_0+l_3)$$
$$= 0.2\, g(0.2, 1.0202, 0.204)$$
$$= 0.2\,(0.2(.204)+1.0202)$$
$$= 0.2122$$

$$y(x_0+h) = y_0+\frac{1}{6}(k_1+2k_2+2k_3+K_4)$$
$$= 1.0202$$

$$z(x_0+h) = z_0+\frac{1}{6}(l_1+2l_2+2l_3+l_4)$$

4M

$$= 0.204$$

Ans $y(0.2) = 1.0202, \quad y'(0.2) = 0.204$

1M

3. $y' = x - y^2$

$y(0) = 1$

(1M)

$x_0 = 0 \quad y_0 = 1 \quad h = 0.1$

$y_1^F = y_0 + h f(x_0\ y_0) = 1 + 0.1(0 - 1^2) = 0.9$

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0\ y_0) + f(x_1\ y_1^F)\right]$

$= 1 + \frac{0.1}{2}\left[-1 + 0.1 - (0.9)^2\right] = 0.9145$

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0\ y_0) + f(x_1\ y_1^{(1)})\right]$

$= 1 + \frac{0.1}{2}\left[-1 + 0.1 - (.9145)^2\right] = 0.9132$

(3M)

$y(0.1) = 0.9132$

II stage $\quad x_0 = 0.1 \quad y_0 = 0.9132$

$y_1^F = y_0 + h f(x_0\ y_0) = 0.9132 + 0.1(0.1 - (.9132)^2)$

$= 0.8399$ at $x_1 = x_0 + h = 0.2$

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0\ y_0) + f(x_1, y_1^E)\right]$

$= 0.9132 + \frac{0.1}{2}\left[0.1 - (.9132)^2 + 0.2 - (.8399)^2\right]$

$= 0.8513$

$y_1^{(2)} = 0.9132 + \frac{0.1}{2}\left[0.1 - (.9132)^2 + (0.2) - (.8513)^2\right]$

$= 0.8504$

$y_1^{(2)} = 0.8504$ at $x_1 = 0.2$

$y(0.2) = 0.8504$ (3M)

4. $n = 10 < 30$

$\bar{x} = 0.128$ $\quad \mu = 0.12 \quad s = 0.008$

$d.f = n - 1 = 8$

H_0: Maching is working

$$|t| = \left| \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \right| = \left| \frac{0.128 - 0.12}{0.008/\sqrt{9}} \right|$$ (3M)

$= 3$

$|t| = 3 > t_{.05}(9) = 1.26$ (3M)

Machine is not working at 5% level of significance (1M)

5. H_0: die is unbiased

H_1: die is biased

$n = 9000$ $\quad x = $ no. of success $= 3240$

$p = \frac{2}{6} = \frac{1}{3}$ $\quad q = 1 - p = \frac{1}{3}$

$$|z| = \left| \frac{\bar{x} - \mu}{\sqrt{npq}} \right|$$

$$= \left| \frac{\frac{3240}{9000} - \frac{1}{3}}{\sqrt{9000\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)}} \right| =$$

(4)

p = Proportion of success in the sample

$= \frac{x}{n} = \frac{3240}{9000} = 0.36$ (3M)

$|z| = \left| \frac{0.36-0.33}{\sqrt{\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{9000}\right)}} \right| = 0.03496$ (3M)

$|z| = 0.03496 < 1.96$ (1M)

H_0 is accepted at 5%.

6. $p - P(H) = \frac{1}{2}$ $q = 1 - p = \frac{1}{2}$

S.E proportion of heads $= \sqrt{pq/n}$ $= \cancel{\frac{1}{2}\sqrt{n}}$

$= \sqrt{\left(\frac{1}{2}\cdot\frac{1}{2}\right)/n} = \frac{1}{2\sqrt{n}}$ (1M)

a) 95% Confidence level $p \pm 1.96\sqrt{pq/n}$

$0.5 \pm 1.96\left(\frac{1}{2\sqrt{n}}\right) = 0.51$ or 0.49

$0.5 + \frac{1.96}{2\sqrt{n}} = 0.51$ and $0.5 - \frac{1.96}{2\sqrt{n}} = 0.49$

$\frac{1.96}{2\sqrt{n}} = 0.01$ or $\sqrt{n} = \frac{1.96}{0.02} = 98$

$\boxed{n = 9604}$ (3M)

b) 90% Confidence limits

$$\frac{1.645}{2\sqrt{n}} = 0.01 \qquad \sqrt{n} = \frac{1.645}{0.02} = 82.25$$

$$\boxed{n = 6765}$$ (3M)

7. $\bar{x}_B = 72 \quad \sigma_B = 8 \quad n_B = 32$

$\bar{x}_G = 75 \quad \sigma_G = 6 \quad n_G = 36$ (2M)

$$Z = \frac{\bar{x}_G - \bar{x}_B}{\sqrt{\frac{\sigma_G^2}{n_G} + \frac{\sigma_B^2}{n_B}}} = \frac{75 - 72}{\sqrt{\frac{36}{36} + \frac{64}{32}}}$$

$$= 1.73$$

$$Z = 1.73 > Z_{0.05} = 1.645$$

$$< Z_{0.01} = 2.33$$ (1M)

The difference is significant at 5%, not at 1%. (1M)

8. Assume that theory fits the expt.

$p(1) = \frac{9}{16}$ $p(2) = \frac{3}{16}$

$p(3) = \frac{3}{16}$ $p(4) = \frac{1}{16}$

$f(1) = \frac{9}{16}(556) = 312.75 \sim 313$

$f(2) = \frac{3}{16}(556) = 104.25 \sim 104$

$f(3) = \frac{3}{16}(556) = 104.25 \sim 104$

$f(4) = \frac{1}{16}(556) = 34.75 \sim 35$

x	O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1	315	313	4	4/313
2	101	104	9	9/104
3	108	104	16	16/104
4	32	35	9	9/35

(3M)

$$\chi^2 = \sum \frac{(O-E)^2}{E} = 0.51$$

$$d.f = n-1 = 4-1 = 3$$

$$\chi^2_{0.05}\ \cancel{at}\ 3\ d.f = 7.82$$

$$\chi^2_{cal} = 0.51 < 7.82$$

(3M)

The value is not significant at 5% level of significance.

The theory fits the experiment. (1M)

4. Two functions $\phi(x)$ and $\psi(x)$ are said ~~Ba~~ to be orthogonal in $[a, b]$ if

$$\int_a^b f(x)\left[\phi(x)\ \psi(x)\right] dx = 0$$

If α and β are the distinct roots ~~&~~ of the equation $J_n(x) = 0$ then

$$\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = 0$$

(or)

$$\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = \begin{cases} 0 & \alpha \neq \beta \\ \frac{1}{2} J_{n+1}^2(\alpha), & \alpha = \beta \end{cases}$$

where α and β are the roots of $J_n(x) = 0$ (2M)

Let $u = J_n(\alpha x)$ $v = J_n(\beta x)$

put $t = \alpha x$ so that $u = J_n(t)$

$$\frac{du}{dx} = \frac{du}{dt}\frac{dt}{dx} = \alpha \frac{d}{dt} J_n(t)$$

$$\frac{d^2u}{dx^2} = \frac{d}{dt}\left(\frac{du}{dx}\right)\frac{dt}{dx} = \alpha^2 \frac{d^2}{dt^2} J_n(t) \quad (1)$$

Since $J_n(t)$ satisfies the Bessel's DE

$$t^2 \frac{d^2}{dt^2} J_n(t) + t\frac{d}{dt} J_n(t) + (t^2 - n^2) J_n(t) = 0 \quad (2)$$

Using $t = \alpha x$

$$\cancel{(\alpha x)^2 \frac{d^2}{dt^2} \left(\frac{1}{\alpha^2}\right.}$$

$$(\alpha x)^2 \left(\frac{1}{\alpha^2}\frac{d^2u}{dx^2}\right) + (\alpha x)\left(\frac{1}{\alpha}\frac{du}{dx}\right) + (\alpha^2 x^2 - n^2)u = 0$$

$$\frac{d^2u}{dx^2} + \frac{1}{x}\frac{du}{dx} + \left(\alpha^2 - \frac{n^2}{x^2}\right)u = 0 \quad (3)$$

|||ly if $v = J_n(\beta x)$

$$\frac{d^2v}{dx^2} + \frac{1}{x}\frac{dv}{dx} + \left(\beta^2 - \frac{n^2}{x^2}\right)v = 0 \quad (4)$$

$(3) \times v - (4) \times u$

$$\left(v\frac{d^2u}{dx^2} - u\frac{d^2v}{dx^2}\right) + \frac{1}{x}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right) + (\alpha^2 - \beta^2)uv = 0$$

$$\frac{d}{dx}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right) + \frac{1}{x}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right) = (\beta^2 - \alpha^2)uv$$

Multiplying by x

$$(\beta^2-\alpha^2)\, x\, u v = x \frac{d}{dx}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right) + \left(v\frac{du}{dx} - u\frac{dv}{dx}\right)$$

$$= \frac{d}{dx}\left[x\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)\right]$$

Intg w.r.t x from 0 to 1

$$(\beta^2-\alpha^2)\int_{x=0}^{1} x\, u v\, dx = \left[x\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)\right]_0^1 \quad (5)$$

$$= \left(v\frac{du}{dx} - u\frac{dv}{dx}\right) \text{ at } x=1$$

$$\frac{du(x)}{dx} = \alpha \frac{d}{dt} J_n(t) = \alpha J_n'(t)$$
$$= \alpha J_n'(\alpha x)$$

$$\frac{du(1)}{dx} = \alpha J_n'(\alpha)$$

|||ly since $v = J_n(\beta x)$ $\frac{d}{dx} v(1) = \beta J_n'(\beta)$

Since $u = u(x) = J_n(\alpha x)$ and $v = v(x) = J_n(\beta x)$

$u(1) = J_n(\alpha)$ $\quad v(1) = J_n(\beta)$

$$(\beta^2-\alpha^2)\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = \left\{\alpha J_n(\beta) J_n'(\alpha) - \beta J_n(\alpha) J_n'(\beta)\right\}$$

$$\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx$$
$$= \frac{1}{\beta^2-\alpha^2}\left[\alpha J_n(\beta) J_n'(\alpha) - \beta J_n(\alpha) J_n'(\beta)\right] \quad (6)$$

If α and β are the distinct roots of $J_n(x)=0$ we have $J_n(\alpha)=0$ and $J_n(\beta)=0$

With $\alpha \neq \beta$, (6) becomes

$$\int_0^1 x J_n(\alpha) J_n(\beta)\, dx = 0 \quad \text{5M}$$

This is known as the orthogonality relation of Bessel's function.