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Improvement Test

Sub	ENGINEERING MATHEMATICS IV (REGULAR)							Code	15MAT41
Date	30 / 05 / 2017	Duration	90 mins	Max Marks	50	Sem	IV	Branch	TCE A,B

	Question 1 is compulsory. Answer any SIX questions from the rest.	Marks	OBE	
	$[z_{.05} = 1.96$(two-tail),1.645(one-tail) and $z_{.01} = 2.58$(two-tail),2.33 (one-tail)]		CO	RBT
1.	If $y' = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$ and $y(0.3) = 2.090$, find y(0.4) correct to 4 decimal places using Adams Bashforth method. Use the corrector formula twice.	08	C401.1	L3
2.	Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1, y'(0) = 0$, compute y(0.2) and y'(0.2) using fourth order RK method.	07	C401.1	L3
3.	Use Modified Euler's method to find y(0.2) correct to 4 decimal places solving the equation $y' = x - y^2$, $y(0) = 1$ taking h = 0.1.	07	C401.1	L3
4.	Use Taylor's series method to find y at x = 0.1, 0.2, 0.3 considering terms upto third degree given that $y' = x^2 + y^2$ and y(0) = 1.	07	C401.1	L3
5.	Apply Milne's method to solve y" = 1 + y' from the following table of values. Compute y(0.4) numerically and theoretically.	07	C401.1	L3

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.399
y'	1	1.2103	1.4427	1.699

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6	The joint probability distribution of two discrete random variables X and Y is given by $f(x,y) = k(2x + y)$ where x and y are integers such that $0 \le x \le 2;\ 0 \le y \le 3$. Find (i) the value of k (ii) the marginal probability distributions of X and Y (iii) show that the random variables are dependent. (iv) Find E(X), E(Y), E(XY) and (v) P(X=1; Y= 2) and P(X+Y > 2)	07	C401.6	L3
7.	Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is a regular stochastic matrix. Also find the associated unique fixed probability vector.	07	C401.6	L3
8.	Three boys are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as like to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws (i) A has the ball (ii) B has the ball (iii) C has the ball	07	C401.6	L3
9.	Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis at 5% and 1% levels of significance that the mean height of the population is 66 inches. Given $t_{0.025}$= 2.262 and $t_{0.005}$= 3.250 for 9 degrees of freedom.	07	C401.5	L3

6.	The joint probability distribution of two discrete random variables X and Y is given by $f(x,y) = k(2x + y)$ where x and y are integers such that $0 \le x \le 2;\ 0 \le y \le 3$. Find (i) the value of k (ii) the marginal probability distributions of X and Y (iii) show that the random variables are dependent. (iv) Find E(X), E(Y), E(XY) and (v) P(X=1; Y= 2) and P(X+Y > 2)	07	C401.6	L3
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SCHEME & SOLUTIONS of ENGINEERING MATHEMATICS-IV ①

IMPROVEMENT TEST - 30-05-2017 · TCE A&B 15MAT41.

① Given $y' = \frac{dy}{dx} = f(x,y) = 2e^{x} - y$; $y(0) = 2$; $y(0.1) = 2.01$

$y(0.2) = 2.04$; $y(0.3) = 2.09$.

Let us formulate a table as follows,

x	y	$y' = f(x,y) = 2e^x - y$
$x_0 = 0$	$y_0 = 2$	$y_0' = 2e^{x_0} - y_0 = 0$
$x_1 = 0.1$	$y_1 = 2.01$	$y_1' = 2e^{x_1} - y_1 = 0.2003$
$x_2 = 0.2$	$y_2 = 2.04$	$y_2' = 2e^{x_2} - y_2 = 0.4028$
$x_3 = 0.3$	$y_3 = 2.09$	$y_3' = 2e^{x_3} - y_3 = 0.6097$.
so $x_4 = 0.4$	$y_4 = ?$	

$\longrightarrow$ (2M)

$\Rightarrow h = 0.1$

By Adams Bashforth formula $y_4^{P} = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0')$

$$\Rightarrow y_4^{(P)} = 2.09 + \frac{0.1}{24}\left[55(0.6097) - 59(0.4028) + 37(0.2003) - 9(0)\right].$$

$= 2.1616 \longrightarrow$ (2M)

$$\Rightarrow y_4' = 2e^{x_4} - y_4^{(P)} = 2e^{0.4} - 2.1616 = 0.822$$

Now the corrector $y_4^{(c)} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$

$$\Rightarrow y_4^{(c)} = 2.09 + \frac{0.1}{24}\left[9(0.822) + 19(0.6097) - 5(0.4028) + 0.2003\right]$$

$= 2.1615 \longrightarrow$ (2M)

$$\Rightarrow y_4' = 2e^{0.4} - y_4^{(c)} = 0.82215 \longrightarrow \text{(1M)}$$

So applying the corrector the 2^{nd} time $y_4^{(c)} = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$

$$\Rightarrow y_4^{(c)} = 2.09 + \frac{0.1}{24}\left(9(0.82215) + 19(0.6097) - 5(0.4028) + 0.2003\right]$$

$= 2.1615$

$\therefore \boxed{y_4 = 2.1615}$, $\longrightarrow$ (1M)

2

2. Let $y' = \frac{dy}{dx} = z \Rightarrow y'' - xy' - y = 0$ is $z' - xz - y = 0$

So we've $\frac{dy}{dx} = f(x,y,z) = z$ & $\frac{dz}{dx} = y + xz = g(x,y,z)$ → (1M)

given $y(0)=1$; $y'(0) = z(0) = 0$; To compute $y(0.2)$ take $h = 0.2$ → (1M)

$\Rightarrow x_0 = 0;\ y_0 = 1;\ z_0 = 0;\ \&\ x_1 = 0.2;\ y_1 = y(0.2)$

By classical 4th order RK Method $y_1 = y_0 + k$; $z_1 = y'(0.2) = z_0 + l$

Where $k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$ & $l = \frac{l_1 + 2l_2 + 2l_3 + l_4}{6}$ → (1M)

$k_1 = h f(x_0, y_0, z_0) = 0.2(z_0) = 0$; $l_1 = h g(x_0, y_0, z_0) = 0.2(y_0 + x_0 z_0)$
$= 0.2$

$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$
$= 0.2\left(z_0 + \frac{l_1}{2}\right) = 0.2(0 + 0.1) = 0.02$;

$l_2 = h g\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$
$= 0.2\left[\left(y_0 + \frac{k_1}{2}\right) + \left(x_0 + \frac{h}{2}\right)\left(z_0 + \frac{l_1}{2}\right)\right]$
$= 0.202$.

$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$
$= 0.2\left(z_0 + \frac{l_2}{2}\right) = (0.2)(0 + 0.101) = 0.0202$;

$l_3 = h g\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$
$= 0.2\left[\left(y_0 + \frac{k_2}{2}\right) + \left(x_0 + \frac{h}{2}\right)\left(z_0 + \frac{l_2}{2}\right)\right]$
$= 0.204$

$k_4 = h f(x_0 + h, y_0 + k_3, z_0 + l_3)$
$= 0.2(z_0 + l_3) = (0.2)(0.204) = 0.0408$

$l_4 = h g(x_0 + h, y_0 + k_3, z_0 + l_3)$
$= 0.2\left[(y_0 + k_3) + (x_0 + h)(z_0 + l_3)\right]$
$= 0.2122$

So $k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} = 0.0202$ & $l = \frac{l_1 + 2l_2 + 2l_3 + l_4}{6} = 0.204$

$\Rightarrow y_1 = y(0.2) = 1 + 0.0202 = 1.0202$ → (2M) & $z_1 = y'(0.2) = y_0' + l = 0 + 0.204$
$= 0.204$ → (2M)

$\therefore$ $\boxed{y(0.2) = 1.0202 \ \&\ y'(0.2) = 0.204}$

3. Given $y' = \frac{dy}{dx} = f(x,y) = x - y^2$; $x_0 = 0$; $y_0 = 1$; $h = 0.1$

$\Rightarrow x_1 = 0.1$ & $x_2 = 0.2$. We've to find $y_1 = y(0.1)$ & $y_2 = y(0.2)$. → (1M)

To find $y_1 = y(0.1)$: By Euler's formula $y_1^{(0)} = y_0 + h f(x_0, y_0)$
$= 1 + 0.1(x_0 - y_0^2)$
$= 0.9$

By Modified Euler's formula

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(0)})\right] = 1 + \frac{0.1}{2}\left[(x_0 - y_0^2) + (x_1 - y_1^{(0)^2})\right] = 0.9145$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(1)})\right] = 1 + \frac{0.1}{2}\left[(x_0 - y_0^2) + (x_1 - y_1^{(1)^2})\right] = 0.9132$$

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(2)})\right] = 1 + \frac{0.1}{2}\left[(x_0 - y_0^2) + (x_1 - y_1^{(2)^2})\right] = 0.9133$$

$$y_1^{(4)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(3)})\right] = 1 + \frac{0.1}{2}\left[(x_0 - y_0^2) + (x_1 - y_1^{(3)^2})\right] = 0.9133$$

So $\boxed{y_1 = y(0.1) = 0.9133}$. → (3M)

To find $y_2 = y(0.2)$, By Euler's formula $y_2^{(0)} = y_1 + h f(x_1, y_1)$
$= 0.9133 + 0.1(x_1 - y_1^2)$
$= 0.8399$

By Modified Euler's formula:-

$$y_2^{(1)} = y_1 + \frac{h}{2}\left[f(x_1,y_1) + f(x_2, y_2^{(0)})\right] = 0.9133 + \frac{0.1}{2}\left[(x_1 - y_1^2) + (x_2 - y_2^{(0)^2})\right] = 0.8513$$

$$y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1,y_1) + f(x_2, y_2^{(1)})\right] = 0.9133 + \frac{0.1}{2}\left[(x_1 - y_1^2) + (x_2 - y_2^{(1)^2})\right] = 0.8504$$

$$y_2^{(3)} = y_1 + \frac{h}{2}\left[f(x_1,y_1) + f(x_2, y_2^{(2)})\right] = 0.9133 + \frac{0.1}{2}\left[(x_1 - y_1^2) + (x_2 - y_2^{(2)^2})\right] = 0.8504$$ → (3M)

So $\boxed{y_2 = y(0.2) = 0.8504}$.

4. We know Taylor's series expansion is

$$y(x) = y(x_0) + (x - x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \ldots$$ → (1)

Given $x_0 = 0$; $y_0 = 1$; $\frac{dy}{dx} = y' = f(x,y) = x^2 + y^2$

$y_0'(0) = x_0^2 + y_0^2 = 0 + 1 = 1$; $y'' = 2x + 2yy' \Rightarrow y_0'' = 2x_0 + 2y_0 y_0' = 2$

$y''' = 2+2\left(yy''+(y')^2\right) \Rightarrow y_0''' = 2+2\left(y_0 y_0'' + (y_0')^2\right) = 2+2\left(1(2)+1^2\right) = 8 \longrightarrow$ 2M

$\Rightarrow y(x) = 1 + (x-0)\cdot 1 + \frac{(x-0)^2}{2}\cdot 2 + \frac{(x-0)^3}{6}\cdot 8$ (considering terms upto 3rd degree)

$= 1+x+x^2+\frac{4}{3}x^3 \longrightarrow$ 1M

$\Rightarrow y(0.1) = 1+(0.1)+(0.1)^2+\frac{4}{3}(0.1)^3 = 1.1113$

$y(0.2) = 1+(0.2)+(0.2)^2+\frac{4}{3}(0.2)^3 = 1.2507$

$y(0.3) = 1+(0.3)+(0.3)^2+\frac{4}{3}(0.3)^3 = 1.4260$ $\longrightarrow$ 3M

5. Let $\frac{dy}{dx} = z \Rightarrow \frac{d^2y}{dx^2} = 1+\frac{dy}{dx}$ is same as $\frac{dz}{dx} = 1+z$

So we've $\frac{dy}{dx} = f(x,y,z) = z$; $\frac{dz}{dx} = g(x,y,z) = 1+z$. $\longrightarrow$ 1M

So tabulating x, y, y' & z' values we've .

x	y	$y'=z$	z'
$x_0 = 0$	$y_0 = 1$	$z_0 = y_0' = 1$	$z_0' = 1+z_0 = 2$
$x_1 = 0.1$	$y_1 = 1.1103$	$z_1 = y_1' = 1.2103$	$z_1' = 1+z_1 = 2.2103$
$x_2 = 0.2$	$y_2 = 1.2427$	$z_2 = y_2' = 1.4427$	$z_2' = 1+z_2 = 2.4427$
$x_3 = 0.3$	$y_3 = 1.399$	$z_3 = y_3' = 1.699$	$z_3' = 1+z_3 = 2.699$
$x_4 = 0.4$	$y_4 = ?$.		

$\longrightarrow$ 2M

$\Rightarrow h = 0.1$

Now $y_4^{(P)}$ by Milne's method is $y_4^{(P)} = y_0 + \frac{4h}{3}\left(2z_1 - z_2 + 2z_3\right)$

$= 1 + \frac{4(0.1)}{3}\left[2(1.2103) - 1.4427 + 2(1.699)\right]$

$= 1.5835 \longrightarrow$ 1M

also $z_4^{(P)} = z_0 + \frac{4h}{3}\left(2z_1' - z_2' + 2z_3'\right) = 1 + \frac{4(0.1)}{3}\left[2(2.2103) - 2.4427 + 2(2.699)\right]$

$= 1.9835$.

Using the Milne's corrector formula we've $y_4^{(C)} = y_2 + \frac{h}{3}\left(z_2 + 4z_3 + z_4\right)$

$= 1.2427 + \frac{0.1}{3}\left[1.4427 + 4(1.699) + 1.9835\right] = 1.58344 \longrightarrow$ 1M

Theoretical Solution: $y'' = 1 + y' \Rightarrow y'' - y' = 1 \Rightarrow (D^2 - D)y = 1$

So Auxiliary eqn is $m^2 - m = 0 \Rightarrow m(m-1) = 0 \Rightarrow m = 0, 1$

So CF is $y = c_1 e^0 + c_2 e^x = c_1 + c_2 e^x \Rightarrow y' = 0 + c_2 e^x$

$PI = \frac{1}{D^2 - D} \cdot 1 = \frac{1}{D^2 - D} e^0 = \frac{1}{0}$ (Undefined) So $PI = x \cdot \frac{1}{2D-1} e^{0 \cdot x} = \frac{x}{-1} = -x$

So CS is $y = CF + PI \Rightarrow y = c_1 + c_2 e^x - x; \Rightarrow y' = 0 + c_2 e^x - 1$

$y(0) = c_1 + c_2 e^0 - 0 = 1 \; (\because y_0 = 1) \Rightarrow c_1 + c_2 = 1$

$y'(0) = c_2 e^0 - 1 = 1 \Rightarrow \boxed{c_2 = 2 \quad \text{So } c_1 = -1}$

$\therefore \boxed{y = -1 + 2e^x - x} = 2e^x - 1 - x.$

$y(0.4) = 2e^{0.4} - 1 - 0.4 = \underline{1.58365}$ → 2M

(6) $\because$ $0 \le x \le 2;\ 0 \le y \le 3$, we've the joint prob. distribution table

X \ Y	0	1	2	3	Sum
0	0	k	2k	3k	6k
1	2k	3k	4k	5k	14k
2	4k	5k	6k	7k	22k
Sum	6k	9k	12k	15k	42k

$\Rightarrow 42k = 1$ So $\boxed{k = 1/42}$ → 1m

Marginal Prob. Distributions are

$X = x_i$	0	1	2
$P(X = x_i) = f(x_i)$	$6k = \frac{6}{42}$	$14k = \frac{14}{42}$	$22k = \frac{22}{42}$

$Y = y_i$	0	1	2	3
$P(Y = y_i) = g(y_i)$	$6k = \frac{6}{42}$	$9k = \frac{9}{42}$	$12k = \frac{12}{42}$	$15k = \frac{15}{42}$

→ 2M

$E(X) = \sum_i x_i f(x_i) = 0\left(\frac{6}{42}\right) + 1\left(\frac{14}{42}\right) + 2\left(\frac{22}{42}\right) = 58/42 = \frac{29}{21}$

$E(Y) = \sum_i y_i g(y_i) = 0\left(\frac{6}{42}\right) + 1\left(\frac{9}{42}\right) + 2\left(\frac{12}{42}\right) + 3\left(\frac{15}{42}\right) = \frac{78}{42} = \frac{13}{7}$

$E(XY) = 0 + \left(0 + \frac{3}{42} + \frac{8}{42} + \frac{15}{42}\right) + \left(0 + \frac{10}{42} + \frac{24}{42} + \frac{42}{42}\right) = \frac{102}{42} = \frac{51}{21}$

$\because$ $E(X)\,E(Y) \neq E(XY)$, the Variables are dependent. $\longrightarrow$ (2M)

v) $P(X=1;\ Y=2) = 4K$ (from table) $= \frac{4}{42} = \frac{2}{21}$,

$P(X+Y>2) = 3k + 4k + 5k + 5k + 6k + 7k = 30k = \cancel{30} = \frac{30}{42} = \frac{15}{21} = \frac{5}{7}$ $\longrightarrow$ (2M)

(7) $P^2 = P \cdot P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$.

$P^3 = P \cdot P^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$

$P^4 = P \cdot P^3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$

$P^5 = P \cdot P^4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \end{bmatrix} = \begin{bmatrix} 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \\ 1/8 & 3/8 & 4/8 \end{bmatrix}$

$\longrightarrow$ (4M)

$\because$ P^5 has no zeros in it, P is a regular stochastic matrix.
The unique fixed probability vector v is given by $vP = v$
Let $v = (x, y, z)$

then $vP = v \Rightarrow (x, y, z)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix} = (x, y, z) \Rightarrow \left(\frac{z}{2},\ x + \frac{z}{2},\ y\right)$
$= (x, y, z)$.

$\Rightarrow$ $\frac{z}{2} = x$; $x + \frac{z}{2} = y$ & $y = z$

$\Rightarrow z = 2x$ So $z = 2x$; $y = 2x$.

but we know $x + y + z = 1$ $\Rightarrow$ $x + 2x + 2x = 1$ $\Rightarrow$ $x = \frac{1}{5}$

So $y = \frac{2}{5}$ & $z = \frac{2}{5}$

So the required unique fixed prob. vector is $(1/5, 2/5, 2/5)$ $\longrightarrow$ (3M)

(8) The state space can be taken as $\{A, B, C\}$.
and the associated transition probability matrix is

$$P = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

$\longrightarrow$ (2M)

If C has the ball initially, the associated initial probability vector is $p^{(0)} = (0, 0, 1)$.

Since we need probabilities after three throws, we need to find $p^{(3)} = p^{(0)} \cdot P^3$ → (2M)

But $P^3 = P \cdot P^2$ where $P^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{pmatrix}$

so $P^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ \cancel{1/2} & 1/2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{pmatrix}$

$\therefore p^{(3)} = p^{(0)} P^3 = (0 \quad 0 \quad 1)\begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{pmatrix} = (1/4 \quad 1/4 \quad 1/2)$ → (2M)

So after three throws, Prob. that A has the ball is 1/4
Prob " B " " is 1/4
Prob " C " " 1/2. → (1M)

9. From the data $\bar{x} = \frac{63+63+ - - - +71}{10} = 67.8$; $n=10$; $\mu = 66$.

$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{1}{9}\left[(63-67.8)^2 + - - - + (71-67.8)^2\right] = 9.067$ → (3M)

$\Rightarrow s = 3.011$.

Step1
$H_0 : \mu = 66$.
$H_1 : \mu \neq 66$

Step2 :- Level of Significance $\alpha = 0.05$
$\alpha/2 = 0.025$

Step3 :- Criterion: Reject Ho if $t < -t_{\alpha/2}$ or $t > t_{\alpha/2}$

ie Reject Ho if $t < -2.262$ or $t > 2.262$

Step4 :- Calculation $t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{67.8 - 66}{3.011/\sqrt{10}} = 1.89$

Step5: Conclusion : $\because 1.89 = t \not< -2.262 = t_{\frac{\alpha}{2}}$ & $t \not> t_{\frac{\alpha}{2}}$, Ho can't be rejected

$\therefore$ "Mean height of popln is 66"" is accepted (2M)

Step1:- $H_0 : \mu = 66$
$H_1 : \mu \neq 66$.

Step2 : Level of Significance $\alpha = 1\% = 0.01$
$\Rightarrow \frac{\alpha}{2} = 0.005$

Step3:- Criterion: Reject Ho if $t < -t_{\frac{\alpha}{2}}$ or $t > t_{\frac{\alpha}{2}}$

ie Reject Ho if $t < -3.250$ or $t > 3.250$

Step4: Calculation: $t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{67.8 - 66}{3.011/\sqrt{10}} = 1.89$

Step5: Conclusion:- $\because t \not< -t_{\frac{\alpha}{2}}$ & $t \not> t_{\frac{\alpha}{2}}$

Ho can't be rejected
So the hypothesis that "Mean height of population is 66inches" is accepted. → (1M)