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Improvement Test

Sub:	FINITE ELEMENT ANALYSIS						Code:	10CV841
Date:	/ 05 / 2017	Duration:	90 mins	Max Marks:	50	Sem: VIII	Branch:	CIVIL

Note: Attempt any two full questions from Q1 to Q3 and any one from Q4 &Q5

		Marks	OBE CO	OBE RBT
1(a)	Differentiate between Isoparametric, Superparametric and Subparametric elements	[05]	CIV810.3	L2
(b)	Determine the shape functions for cubic serendipity family elements.	[10]	CIV810.3	L3
2	Determine the shape function for a two node beam element using first order Hermitian polynomial.	[15]	CIV810.3	L3
3	Using Lagrange functions derive shape functions for the nine node rectangular element.	[15]	CIV810.3	L3
4	Using Lagrange Polynomial find shape function for 3 node bar element and plot its variation	[10]	CIV810.3	L3
5	Determine the Cartesian coordinates of the point P(ξ =0.8, η=0.9) for the 4 node element shown in figure below	[10]	CIV810.3	L3

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SOLUTION FOR IMPROVEMENT TEST

FINITE ELEMENT ANALYSIS - 10CV841

Q1

An element in the finite element model is characterized by its two principle features, the degree and order of approximating function describing the dependent function in the element(order of shape function), and the order of geometric description of the element. An element is called isoparametric when the order of approximating shape function and the order of geometricϖ description of the element is equal. An element is called subparametric when the geometric order is less than the order of the approximating shapeϖ function of the element. An element is called Superparametric when the geometric order is larger than the order of approximating shapeϖ function. Consider, for example, a one dimensional element. Let u describing the displacement of the element. The nodal values for the displacement are denoted by Ui and the shape function u and element geometric description x are

$u = \sum_{i=1}^{m} NiUi$ $, s =$ $\sum_{i=1}^{m} Ni' \, Xi$……………………………………………………(1) Where Ni and Ni' are the shape functions describing the dependent function u and the geometric configuration s of the element in one dimensional space. The two expressions of equation (1) for u and s give the values of u and s within the element in terms of the nodal values Ui and Xi .

The types of element are defined as follows:

1. An element is isoparametric when m = n

2. An element is subparametric when m > n

3. An element is isoparametric when m < n

Q2

Solution: Typical element is shown in Fig. 5.24. Shape functions for corner nodes:

$N_1 = 0$ for all nodes except 1 and is 1 for node 1.

$N_1 = 0$ is satisfied for nodes 2, 7, 8, 3 if $1 - \xi = 0$

$N_1 = 0$ is satisfied for nodes 3, 9, 10, 4 if $1 - \eta = 0$

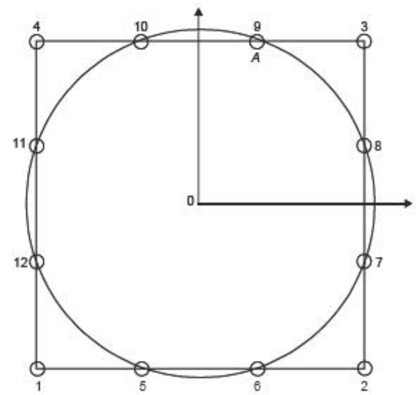


Fig. 5.24 *Cubic serendipity family element*

The points 5, 6, 7, 8, 9, 10, 11, 12 lie on the circle shown in Figure. The radius of this circle $= OA$

$$= \sqrt{\left(\frac{1}{3}\right)^2 + 1} \text{ since } A\left(\frac{1}{3}, 1\right) \text{ and } O = (0, 0) = \sqrt{\frac{10}{3}}$$

$\therefore$ The equation of the circle is

$$\xi^2 + \eta^2 = \frac{10}{3}$$

$$\xi^2 + \eta^2 - \frac{10}{3} = 0$$

Satisfies $N_1 = 0$ for nodes 5 to 12.

$$N_1 = C(1-\xi)(1-\eta)\left(\xi^2 + \eta^2 - \frac{10}{9}\right) \text{ Satisfies } N_1 = 0 \text{ for all nodes except for node 1.}$$

For node 1 $N_1 = 1$.

$$\therefore 1 = (1-\xi_1)(1-\eta_1)\left(\xi_1^2 + \eta_1^2 - \frac{10}{9}\right)$$

But we know $\xi_1 = \eta_1 = -1$

$$\therefore 1 = 2 \times 2 \times \left(1 + 1 - \frac{10}{9}\right) C = \frac{32}{9} C$$

r $$C = \frac{9}{32}$$

$$\therefore\ N_1 = \frac{9}{32}(1-\xi)(1-\eta)\left(\xi^2+\eta^2-\frac{10}{9}\right)$$

$$= \frac{1}{32}(1-\xi)(1-\eta)\left\{9\left(\xi^2+\eta^2\right)-10\right\}$$

Similarly it may be shown that

$$N_2 = \frac{1}{32}(1+\xi)(1-\eta)\left\{9\left(\xi^2+\eta^2\right)-10\right\}$$

$$N_3 = \frac{1}{32}(1+\xi)(1+\eta)\left\{9\left(\xi^2+\eta^2\right)-10\right\}$$

$$N_4 = \frac{1}{32}(1-\xi)(1+\eta)\left\{9\left(\xi^2+\eta^2\right)-10\right\}$$

For mid side node 5,

$1-\xi=0$ ensures $N_5=0$ at nodes 2, 7, 8, 3

$1-\eta=0$ ensures $N_5=0$ at nodes 3, 9, 10, 4

$1+\xi=0$ ensures $N_5=0$ at nodes 4, 11, 12, 1.

$1-3\xi=0$ ensures $N_5=0$ at node 6.

$\therefore$ Let $N_5 = C(1-\xi)(1-\eta)(1+\xi)(1-3\xi)$

At node 5, $\xi = -\frac{1}{3}$, $\eta=-1$ and $N_5=1$

$$\therefore 1 = C \times \frac{4}{3} \times 2 \times \frac{2}{3} \times 2$$

i.e.,
$$C = \frac{9}{32}$$

$$\therefore\ N_5 = \frac{9}{32}(1-\xi)(1-\eta)(1+\xi)(1-3\xi)$$

$$= \frac{9}{32}\left(1-\xi^2\right)(1-\eta)(1-3\xi)$$

Similarly,
$$N_6 = \frac{9}{32}\left(1-\xi^2\right)(1-\eta)(1+3\xi)$$

$$N_7 = \frac{9}{32}\left(1-\eta^2\right)(1+\xi)(1-3\eta)$$

$$N_8 = \frac{9}{32}\left(1-\eta^2\right)(1+\xi)(1+3\eta)$$

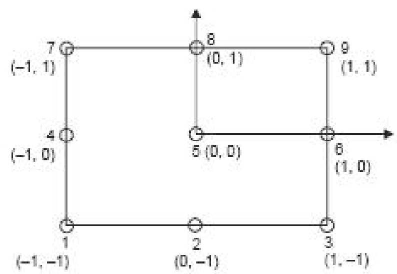
$$N_9 = \frac{9}{32}\left(1-\xi^2\right)(1+\eta)(1+3\xi)$$

$$N_{10} = \frac{9}{32}\left(1-\xi^2\right)(1+\eta)(1-3\xi)$$

$$N_{11} = \frac{9}{32}\left(1-\eta^2\right)(1-\xi)(1+3\eta)$$

$$N_{12} = \frac{9}{32}\left(1-\eta^2\right)(1-\xi)(1-3\eta)$$

Q3



Solution: The natural coordinates of various nodes are as shown in the figure. For the C° continuity element in two dimensions,

$$N_i = L_i(\xi)\, L_i(\eta)$$

where L_i refers to Langrangian function at node *i*. In this case there are 3 nodes in each direction. Hence n = 3 in Lagrange function

$$N_1 = \frac{(\xi - \xi_2)(\xi - \xi_3)}{(\xi_1 - \xi_2)(\xi_1 - \xi_3)} \frac{(\eta - \eta_4)(\eta - \eta_7)}{(\eta_1 - \eta_4)(\eta_1 - \eta_7)}$$

$$= \frac{(\xi - 0)(\xi - 1)}{(-1 - 0)(-1 - 1)} \frac{(\eta - 0)(\eta - 1)}{(-1 - 0)(-1 - 1)} = \frac{\xi(\xi - 1)\eta(\eta - 1)}{4}$$

$$N_2 = \frac{(\xi - \xi_1)(\xi - \xi_3)}{(\xi_2 - \xi_1)(\xi_2 - \xi_3)} \frac{(\eta - \eta_5)(\eta - \eta_8)}{(\eta_2 - \eta_5)(\eta_2 - \eta_8)}$$

$$= \frac{(\xi + 1)(\xi - 1)}{(0 + 1)(0 - 1)} \frac{(\eta - 0)(\eta - 1)}{(-1 - 0)(-1 - 1)} = \frac{(\xi + 1)(\xi - 1)\eta(\eta - 1)}{(-2)}$$

$$N_3 = \frac{(\xi - \xi_1)(\xi - \xi_2)}{(\xi_3 - \xi_1)(\xi_3 - \xi_2)} \frac{(\eta - \eta_6)(\eta - \eta_9)}{(\eta_3 - \eta_6)(\eta_3 - \eta_9)}$$

$$= \frac{(\xi + 1)(\xi - 0)}{(1 + 1)(1 - 0)} \frac{(\eta - 0)(\eta - 1)}{(-1 - 0)(-1 - 1)} = \frac{(\xi + 1)\xi \quad \eta(\eta - 1)}{4}$$

$$N_4 = \frac{(\xi - \xi_5)(\xi - \xi_6)}{(\xi_4 - \xi_5)(\xi_4 - \xi_6)} \frac{(\eta - \eta_1)(\eta - \eta_7)}{(\eta_4 - \eta_1)(\eta_4 - \eta_7)}$$

$$= \frac{(\xi - 0)(\xi - 1)}{(-1 - 0)(-1 - 1)} \frac{(\eta + 1)(\eta - 1)}{(0 + 1)(0 - 1)} = \frac{\xi(\xi - 1)(\eta + 1)(\eta - 1)}{-2}$$

$$N_5 = \frac{(\xi - \xi_4)(\xi - \xi_6)}{(\xi_5 - \xi_4)(\xi_5 - \xi_6)} \frac{(\eta - \eta_2)(\eta - \eta_8)}{(\eta_5 - \eta_2)(\eta_5 - \eta_8)}$$

$$= \frac{(\xi + 1)(\xi - 1)}{(0 + 1)(0 - 1)} \frac{(\eta + 1)(\eta - 1)}{(0 + 1)(0 - 1)} = \frac{(\xi + 1)(\xi - 1)(\eta + 1)(\eta - 1)}{1}$$

$$N_6 = \frac{(\xi - \xi_4)(\xi - \xi_5)}{(\xi_6 - \xi_4)(\xi_6 - \xi_5)} \frac{(\eta - \eta_3)(\eta - \eta_9)}{(\eta_6 - \eta_3)(\eta_6 - \eta_9)}$$

$$= \frac{(\xi + 1)(\xi - 0)}{(1 + 1)(1 - 0)} \frac{(\eta + 1)(\eta - 1)}{(0 + 1)(0 - 1)} = \frac{(\xi + 1)\xi \quad (\eta + 1)(\eta - 1)}{-2}$$

$$N_7 = \frac{(\xi - \xi_8)(\xi - \xi_9)}{(\xi_7 - \xi_8)(\xi_7 - \xi_9)} \frac{(\eta - \eta_1)(\eta - \eta_4)}{(\eta_7 - \eta_1)(\eta_7 - \eta_4)}$$

$$= \frac{(\xi - 0)(\xi - 1)}{(-1 - 0)(-1 - 1)} \frac{(\eta + 0)(\eta - 0)}{(1 + 1)(1 - 0)} = \frac{\xi(\xi - 1)(\eta + 1)\eta}{4}$$

$$N_8 = \frac{(\xi - \xi_7)(\xi - \xi_9)}{(\xi_8 - \xi_7)(\xi_8 - \xi_9)} \frac{(\eta - \eta_2)(\eta - \eta_5)}{(\eta_8 - \eta_2)(\eta_8 - \eta_5)}$$

$$= \frac{(\xi + 1)(\xi - 1)}{(0 + 1)(0 - 1)} \frac{(\eta + 1)(\eta - 0)}{(1 + 1)(1 - 0)} = \frac{(\xi + 1)(\xi - 1)(\eta + 1)(\eta)}{-2}$$

$$N_9 = \frac{(\xi - \xi_7)(\xi - \xi_8)}{(\xi_9 - \xi_7)(\xi_9 - \xi_8)} \frac{(\eta - \eta_3)(\eta - \eta_6)}{(\eta_9 - \eta_3)(\eta_9 - \eta_6)}$$

$$= \frac{(\xi + 1)(\xi - 0)}{(1 + 1)(1 - 0)} \frac{(\eta + 1)(\eta - 0)}{(1 + 1)(1 - 0)} = \frac{(\xi + 1)\xi\,(\eta + 1)(\eta)}{4}$$

Thus in this case, for corner nodes,

$$N_i = \frac{1}{4}\xi\eta(\xi + \xi_i)(\eta + \eta_i)$$

For nodes 2 and 8 where $\xi = 0$;

$$N_i = \frac{(\xi + 1)(\xi - 1) \quad \eta(\eta + \eta_i)}{-2}$$

For nodes 4 and 6, where $\eta_i = 0$

$$N_i = \frac{\xi(\xi + \xi_i)\ (\eta - 1)(\eta + 1)}{-2}$$

and for central node,

$$N_5 = \frac{(\xi + 1)(\xi - 1)\ (\eta + 1)(\eta - 1)}{4}$$

Q 4

(ii) For Three Noded Element

$n = 3$. Hence when $k = 1$,

$$N_1 = L_1 = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)}$$

When $k = 2$

$$N_2 = L_2 = \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)}$$

and $$N_3 = L_3 = \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)}$$

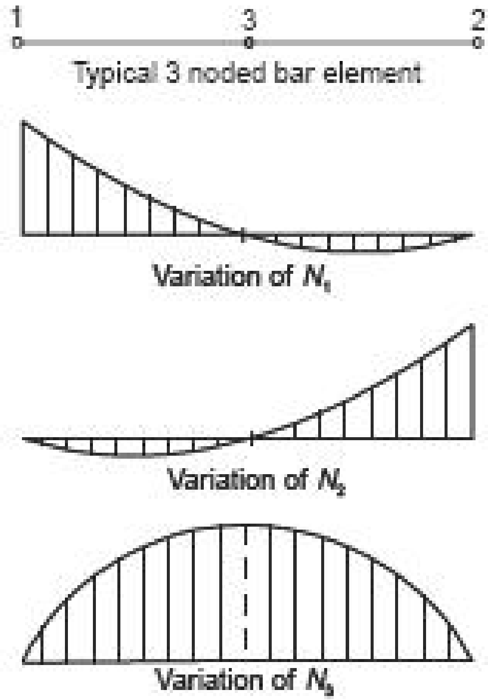


Fig. 5.17 ***(contd)*** *(b) Three noded bar element*

The typical element and the variation of its shape functions are shown in Fig. 5.17 (b).