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Improvement Test

Sub: Analysis of Determinate structures

Code: 15CV42

Date: 31/5/2017	Duration: 90 mins	Max Marks: 50	Sem: 4th	Branch (sections): civil (A,B,)

Answer any Two question from Part A And one question from Part B

Max Marks: 50

CO RB

Part A

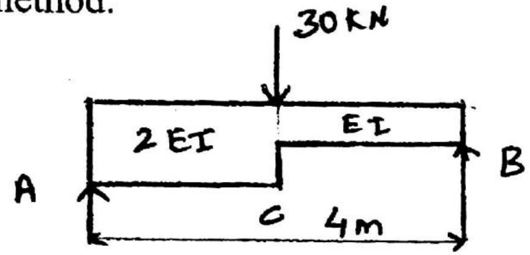
1. Simply supported beam of length 'L' carries a point load 'w'at the center of span find the slope at the supports and the maximum deflection in the span by
   b) Double integration Method. b) Conjugate beam method. [10+10] CIV412.2 L3

2 A Cantilever Beam of length 'L' carries a UD load w throughout its length. Calculate slope and deflection at free end by using
   b) Moment area method. b) Conjugate beam method. [10+10] CIV412.1 L3

3 A beam of 6m span is simply supported at the ends & is loaded with a point load of 6 kN at 2 meter from left support.UDL of 2kN/m for the second half of the beam. Find Deflection at mid span, Maximum deflection and slope at left support. Take E=200GPa, I= 20×10^6 mm^4. [20] CIV412.2 L3

Part B

1 For the fig shown below calculate slope at the support and maximum deflection using conjugate beam method. [10] CIV412.2 L3



2 A simply supported beam is subjected to a couple of 120 kNm at a point 4m from the left support. Calculate the slopes at the ends and the deflection at the point of application of load. EI is constant. [20] CIV412.2 L3

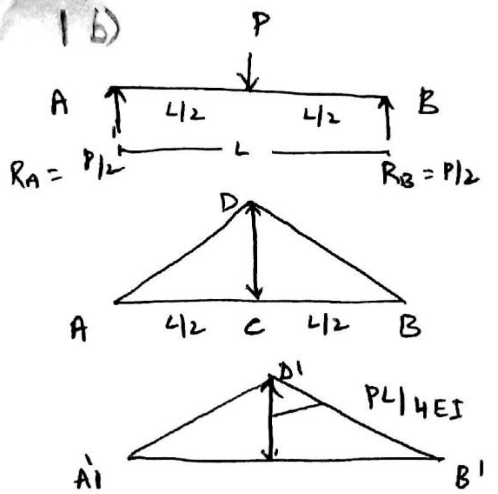
CI

CCI

HOD

...carrying a point load at the centre. 5

1 b)



① Bending Moment calculation

The B.M @ A & B is zero.

$$\therefore M_c = \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$$

$$\boxed{M_c = \frac{PL}{4}}$$

Let R_A' & R_B' be the reaction @ A & B for conjugate beam

Total load on conjugate beam = Area of the load diagram

$$= \frac{1}{2} \times (L) \times \frac{PL}{2EI}$$

$$\text{Total load} = \frac{PL^2}{8EI}$$

∴ Reaction at each support for conjugate beam will be half of the total load.

$$\therefore R_A' = R_B' = \frac{1}{2} \times \frac{PL^2}{8EI} = \frac{PL^2}{16EI}$$

→ According to conjugate beam method

Slope @ A & B :

θ_A in real beam = S.F @ A' for the conjugate beam

$$\theta_A = R_A' = \frac{PL^2}{16EI}$$

Due to symmetry, $\theta_B = -\theta_A = \dfrac{-PL^2}{16EI}$

→ Deflection @ C & maximum deflection

y_c in real beam = B.M @ C' for the conjugate beam

$$= (R_A' \times L/2) - (\text{Load corresponding to } A'C' \ldots$$

$$= \left(\frac{PL^2}{16EI} \times \frac{L}{2}\right) - \left(\frac{1}{2} \times \frac{L}{2} \times \frac{PL}{4EI}\right) \times \left(\frac{L}{6}\right)$$

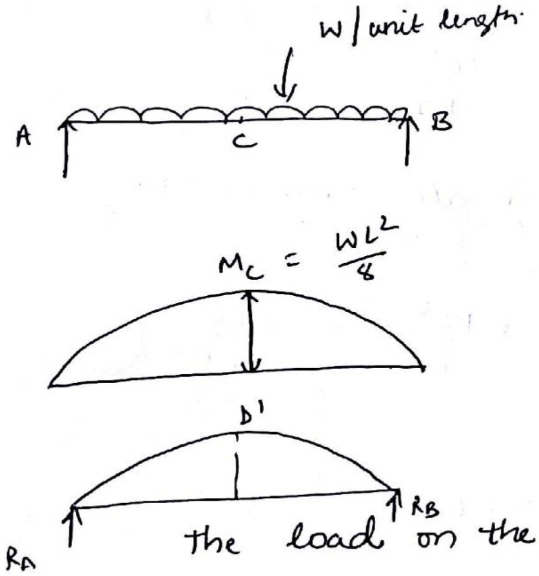
$$y_c = \left(\frac{PL^3}{32EI} - \frac{PL^3}{96EI}\right)$$

$$= \left(\frac{3PL^3 - PL^3}{96EI}\right)$$

$$\boxed{= \frac{PL^3}{48EI}}$$

Simply supported beam subjected to a UDL throught its span.

$$R_A = R_B = \frac{WL}{2}$$



① B.M. Calculation

$$M_c = \frac{WL}{2} \times \frac{1}{2} \times \frac{L}{2}$$

$$= \frac{WL^2}{8}$$

Let R_A' & R_B' be the reaction @ A & B for conjugate beam

the load on the conjugate beam $=$ Area of the load diagram

$$= \frac{2}{3} \times (A'B' \times C'D')$$

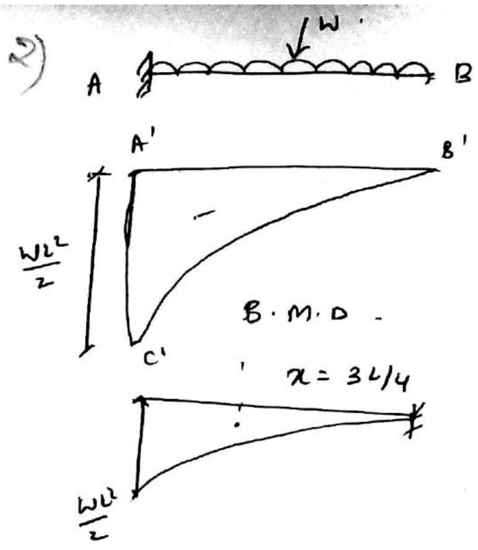
$$= \frac{2}{3} \times L \times \frac{WL^2}{8EI}$$

$$= \frac{WL^3}{12EI}$$

∴ Reaction @ each support for the conjugate beam will be half of total load.

$$R_A' = R_B' = \frac{WL^3}{12EI} \times \frac{1}{2} = \frac{WL^3}{24EI}$$

2)



B.M. Calculation,

$$M = -W \times L \times \frac{L}{2} = -\frac{WL^2}{2}$$

$\therefore$ S.F. Cal

Slope @ B in real beam = S.F @ B for conjugate beam.

$$\theta_B = \frac{1}{3} \times AB' \times AC'$$

$$= \frac{1}{3} \times L \times -\frac{WL^2}{2}$$

$$\boxed{\theta_B = -\frac{WL^3}{6EI}}$$

Deflection at B,

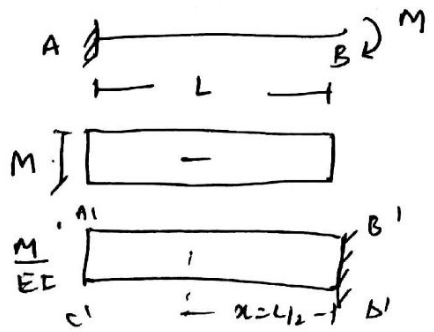
Deflection @ B in real beam = B.M @ B' for conjugate beam

= Load A'B'C' × distance of CG of B'A'C

$$= \frac{1}{3} \times L \times \left(-\frac{WL^2}{2EI}\right) \times \frac{3}{4}L$$

$$\boxed{y_B = -\frac{WL^4}{8EI}}$$

Cantilever beam subjected to a couple M @ free end.



$\therefore$ Slope @ B θ_B = load A'B'C'D'

θ_B = = AC' × A'B'

$$\boxed{\theta_B = -\frac{M}{EI} \times L}$$

Deflection @ B, Load AB'C'D' × $\bar{x}$

$$y_B = -\frac{M}{EI} \times L \times \bar{x}$$

$$= -\frac{ML}{EI} \times \frac{L}{2} = -\frac{ML^2}{2EI}$$

3) A simply supported beam of length 6m ~~it~~ carries two point loads of 48 KN & 40 KN at a distance of 1m & 3m resp from the left support.

Find – Deflection under each load
– Maximum deflection.

$1\,N/mm^2 = 10^6\,N/m^2$
$1\,Pa = 1\,N/m^2 = 10^{-6}\,N/mm^2$
$1\,mm^4 = 10^{-12}\,m^4$
$1\,m = 10^3\,mm$

Given – $E = 2\times10^5\ N/mm^2 = 2\times10^{11}\ N/m^2$ $\quad I = 8.5\times10^7\,mm^4$.

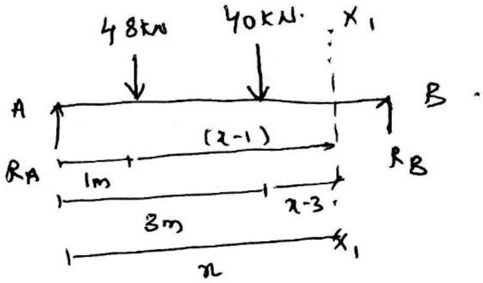
Sol?

$L = 6m$ $\quad E = 2\times10^5\,N/mm^2$

$P_1 = 48\,KN$ $\quad I = 8.5\times10^7\,mm^4 \cdot \times10^{-12} = 8.5\times10^{-5}\,m^4$

$P_2 = 40\,KN$.

$EI = \underline{17\times10^3\ KN\text{-}m^2}$



⇒ <u>To find reaction</u>..

$\Sigma Y = 0$, $R_A + R_B = 48 + 40 = 88\ KN$

$\Sigma M_A = 0$, $R_B \times 6 = (40\times3) + (48\times1)$

$= \underline{28\ KN}$

∴ Eqn $R_A = 88 - R_B = \underline{60\ KN}$

⇒ <u>Bending moment eqn</u>

$$M_x = R_A \times x - 48(x-1) - 40(x-3)$$

$$= 60x - 48(x-1) - 40(x-3) \quad — ①$$

⇒ <u>Apply Moment curvature eqn</u>.

$$EI\frac{d^2y}{dx^2} = M_x = 60x - 48(x-1) - 40(x-3)$$

$$EI\cdot\frac{dy}{dx} = M_x = \frac{60x^2}{2} + c_1 - \frac{48(x-1)^2}{2} - \frac{40(x-3)^2}{2}$$

$$EI\,dy = 30x^2 + c_1 - 24(x-1)^2 - 20(x-3)^2$$

Integrating once again eq^n (2) w.r.t.x.

$$EI\,y = \frac{30x^3}{3} + c_1 x + c_2 - \frac{24(x-1)^3}{3} - \frac{20(x-3)^3}{3}$$

$$= 10x^3 + c_1 x + c_2 - 8(x-1)^3 - 6.67(x-3)^3 \quad — (3)$$

⇒ Boundary conditions.

@ $x = 0$, $y = 0$.

∴ eq^n (3)

$$EI(0) = 10(0) + c_1(0) + c_2.$$

$$\therefore \boxed{c_2 = 0}$$

— @ $x = L = 6m$, $y = 0$

$$0 = 10(6)^3 + c_1 \times 6 - 8(6-1)^3 - 6.67(6-3)^3$$

$$\therefore 6c_1 = -980$$

$$\therefore \boxed{c_1 = -163.3}$$

⇒ Sub the c$_1$ & c$_2$ in eq^n (3) we get.

$$y = \frac{1}{EI}\left[10x^3 - 163.3x \;\overset{①}{}\; - 8(x-1)^3 \;\overset{②}{}\; - 6.66(x-3)^3 \;\overset{③}{}\right] \quad — (4)$$

⇒ To find the deflection under each load,

→ Deflection under first load, P = 48 kN.

@ $x = 1m$

$$y_c = \frac{1}{EI}\left[10 \times 1^3 - 163.3 \times 1\right] = \frac{10 - 163.3}{17 \times 10^3} =$$

$$y_a = -9.01\,mm \quad \text{or} \quad \underline{y_c = 9.01\,mm} \downarrow$$

—

Deflection under second load, $P = 40$ kN.

at $x = 3$

$$y_D = \frac{10(3)^3 - (163.33)(3) - 8(3-1)^3}{17 \times 10^3}$$

$y_D = -16.70$ mm or 16.7 mm $\downarrow$.

⇒ <u>To find the maximum deflection</u>.

The maximum deflection takes place b/w C & D.

For max deflection $\frac{dy}{dx} = 0$.

∴ equate Slope eqn i.e. eqn (2) to zero untill second term.

$$0 = 30x^2 - 163.33 - 24(x-1)^2$$

$$= 30x^2 - 163.3 - 24(x^2 + 1 - 2x)$$

$$\Rightarrow 30x^2 - 163.3 - 24x^2 + 24 - 48x$$

$$\therefore 6x^2 + 48x - 187.3 = 0$$

$$\therefore \boxed{x = 2.87 \text{ m}}$$

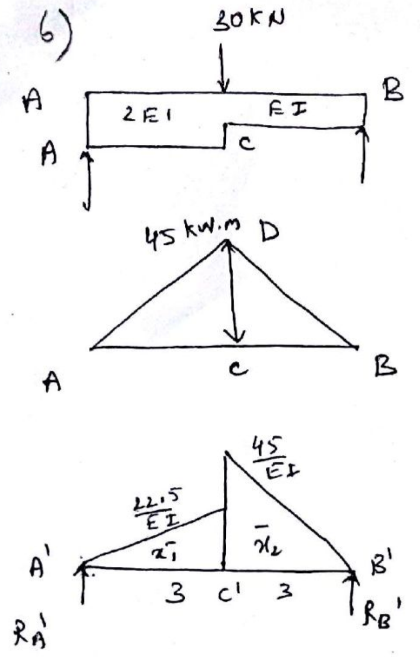
∴ eqn (4)

$$EI\,y = 10 \times (2.87)^3 - 163.3 \times 2.87 - 8(2.87-1)^3$$

$$\underline{y = 16.74 \text{ mm}}$$

Draw BMD for given of real beam & M/EI diagram

6)



$\rightarrow$ $R_A = R_B = \frac{30}{2} = 15$ kN.

$M_c = \frac{WL}{4} = \frac{30 \times 6}{4} = 45$ kN.m.

$\rightarrow$ <u>Draw BMD for given of real beam</u>

* Load b/w A' & C' = Area of the load diag

$= \frac{1}{2} \times AC' \times CD'$

$= \frac{1}{2} \times 3 \times \frac{22.5}{EI}$

It's C.G from B'

$\bar{x}_1 = \left(\frac{1}{3} \times 3 + 3\right) = \underline{4m}$

* Load b/w C' & B' = Area of load dia.

$= \frac{1}{2} \times C'B' \times C'E'$

$= \frac{1}{2} \times 3 \times \frac{45}{EI} = \underline{\frac{67.5}{EI}}$

It's C.G. from B', $\bar{x}_2 = \frac{2}{3} \times 3 = \underline{2m}$

Let R_A' & R_B' be the reaction at A' & B' for conjugate beam

$\Sigma M_B' = 0$

$R_A' \times 6 = (\text{load b/w } A' \& C') \times \bar{x}_1 + (\text{load b/w } C' \& B') \times \bar{x}_2$

$= \left(\frac{33.75}{EI} \times 4\right) + \frac{67.5}{EI} \times 2$

$R_A' = \frac{45}{EI}$

$\Sigma x = 0$,

$R_A' + R_B' = \frac{33.75}{EI} + \frac{67.5}{EI} \Rightarrow R_B' = \frac{33.75}{EI} + \frac{67.5}{EI} - \frac{45}{EI}$

$\Rightarrow$ Slope at A, B & C.

θ_A = S.F @ A' for the conjugate beam = R_A'

$$\theta_A = \frac{45}{EI}$$

θ_B = S.F at B' = $-R_B'$

$$\theta_B = \frac{-56.25}{EI}$$

θ_C = R_A' − load b/w A' & C'

$$= \frac{45}{EI} - \frac{33.75}{EI}$$

$$= \frac{11.25}{EI}$$

(b) Deflection at 'C' or max defl.

y_c = ($R_A' \times 3$) − load b/w A' & C' $\times \bar{x}$ from C

$$= \left(\frac{45}{EI} \times 3\right) - \left(\frac{33.75}{EI} \times \frac{1}{3} \times 3\right)$$

$$= \frac{101.25}{EI}$$