USN | | | | | | | | | | |

Improvement Test

Sub:	Operations Research			Code:	10CS661
Date:	31/05/2017	Duration: 90 mins	Max Marks: 50	Sem: VI	Branch: CSE

Answer any five full questions

		Marks	OBE CO	OBE RBT
1.	Use Big M method to solve the LPP, Max $Z = 6x + 4y$ subject to the constraints $2x + 3y \le 30$; $3x + 2y \le 24$, $x + y \ge 3$; $x, y \ge 0$	[10]	CO2	L3
2.	Use two phase method to solve the LPP Max $P = 2x + 3y + 4z$ subject to the constraints $3x + y + 6z \le 600$; $2x + 4y + 2z \ge 480$; $2x + 3y + 3z = 540$, $x, y, z \ge 0$	[10]	CO2	L3

3. Solve the transportation problem in which the cell entries represent the unit costs(in lakhs of rupees) of transportation.

	D1	D2	D3	demand
O1	2	7	4	5
O2	3	3	1	8
O3	5	4	7	7
O4	1	6	2	14
supply	7	9	18	

4. (a)Four professors are each capable of teaching any one of the four different subjects. Class preparation time in hours for different topics varies from professor to professor and is given in the table. Each professor should be assigned only one subject. Find the schedule so as to minimize the toal subject preparation time for all subjects/professors. [5] CO4 L3

	Subjects			
	S_1	S_2	S_3	S_4
Professor P_1	2	10	9	7
Professor P_2	15	4	14	8
Professor P_3	13	14	16	11
Professor P_4	3	15	13	8

(b)Solve the travelling salesman problem given by the following data $C_{12}= 20$, $C_{13}= 4$, $C_{14}= 10$, $C_{23}= 5$, $C_{34}= 6$, $C_{25}= 10$, $C_{35}= 6$, $C_{45}= 20$, where $C_{ij} = C_{ji}$ and there is no route between the cities i and j if the value of C_{ij} is not shown. [5] CO3 L3

5. (a)What is a saddle point? Solve the following game using dominance concept. [5] CO3 L1

Strategies	Player B		
	B1	B2	B3
Player A –A_1	4	5	8
Player A –A_2	6	4	6
Player A –A_3	4	2	4

(b)What is Mini max or Maxi min Principle. Solve the given 3 x 2 game by graphical method. [5] CO3 L2

	I	II
I	3	-2
II	-1	4
III	2	2

		Marks	CO	RBT
6.	Use the dual simplex method to solve the LPP Minimize $Z = 2x + y + 3z$ Subject to the constraints. $x - 2y + z \ge 4$, $2x + y + z \le 8$, $x - z \ge 0$ with $x, y, z \ge 0$	[10]	CO2	L1
7.	Use Revised Simplex method to solve the following LPP Maximize $Z = x + 2y$ subject to the constraints $x + y \le 3$, $x + 2y \le 5$, $3x + y \le 6$, $x, y \ge 0$	[10]	CO2	L2

Improvement test – Jun 2017

Solution Manual (CSE)

1). Max $Z = 6x + 4y$

S.t $2x + 3y + S_1 = 30$

$3x + 2y + S_2 = 24$

$x + y - S_3 + a_1 = 3$

$Z = 6x + 4y + 0S_1 + 0S_2 + 0S_3 - Ma_1$

$n = 6,\ m = 3,\ n - m = 3$

$\therefore$ put $x, y, S_3 = 0$

$S_1 = 30,\ S_2 = 24,\ a_1 = 3$ is the basic

sol. (2m)

Basis	C_B	x_B	x (EV)	y	S_1	S_2	S_3	a_1	MR
S_1	0	30	2	3	1	0	0	0	15
S_2	0	24	3	2	0	1	0	0	8
LV ← a_1	$-M$	3	$\boxed{1}$ P.E	1	0	0	-1	1	3
	$Z = \Sigma C_B x_B = -3M$		$-M-6$	$-M-4$	0	0	$+M$	0	
S_1	0	24	0	1	1	0	$+2$	-2	$R_1 \to R_1 - 2R_3$
S_2	0	15	0	-1	0	1	$+3$	-3	$R_2 \to R_2 - 3R_3$
x	6	3	1	1	0	0	1	1	$R_4 \to R_4 + (M+6)R_3$
	$Z = \Sigma C_B x_B = 18$		0	2	0	0	-6	$M+6$	(3M)

Basis	C_B	x_B	x	y	S_1	S_2	S_3 (E.V)	a_1	M.R
S_1	0	24	0	1	1	0	2	-2	12
L.V ← S_2	0	15	0	-1	0	1	P.E $\boxed{3}$	-3	5
x	6	3	1	1	0	0	-1	1	×
	$Z = \Sigma C_B x_B = 18$		0	2	0	0	-6 ↑	M+6	
S_1	0	24	0	1	1	0	2	-2	$R_1 \rightarrow$
S_3	0	5	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	P.E $\boxed{1}$	-1	$R_1 - 2R_2$ $R_3 \to R_3 + R_2$
x	6	3	1	1	0	0	-1	1	$R_4 \to R_4 + 6R_2$
	$Z = \Sigma C_B x_B$		0	2	0	0	-6	M+6	
S_1	0	14	0	$\frac{5}{3}$	1	$-\frac{2}{3}$	0	0	
S_3	0	5	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	1	-1	
x	6	8	1	$\frac{2}{3}$	0	$\frac{1}{3}$	0	0	
	$Z = \Sigma C_B x_B = 48$		0	0	0	2	0	[illegible]	(3M)

as all $\Delta \geq 0$. The solution is optimal

$Z_{max} = 48, \ x = 8, \ y = 0$. (2M)

(2) Max $P = 2x + 3y + 4z$.

S.t $3x + y + 6z + s_1 = 600$

$2x + 4y + 2z - S_2 + a_1 = 480$

$2x + 3y + 3z + a_2 = 540$.

Sol. Phase 1 :-

$P_{max} = 0x + 0y + 0z + 0s_1 + 0s_2 - a_1 - a_2$

$n = 7, \ m = 3, \ n - m = 4$.

put $x, y, z, S_2 = 0$

$S_1 = 600, a_1 = 480, a_2 = 540$ is the basic sol. (2M)

Basis	C_B	x_B	x	y (E.V)	z	S_1	S_2	a_1	a_2	M.R
S_1	0	600	3	1	6	1	0	0	0	600
L.V ← a_1	-1	480	2	$\boxed{4}$ P.E	2	0	-1	1	0	120
a_2	-1	540	2	3	3	0	0	0	1	180
	$Z = \sum C_B x_B$		-4	-7 ↑	-5	0	1	0	0	$R_2 \to \frac{R_2}{4}$
S_1	0	600	3	1	6	1	0	0	0	
y	0	120	1/2	$\boxed{1}$ P.E	1/2	0	-1/4	1/4	0	$R_1 \to R_1 - R_2$
a_2	-1	540	2	3	3	0	0	0	1	$R_3 \to R_3 - 3R_2$, $R_4 \to R_4 + 7R_2$
	$Z = \sum C_B x_B$		-4	-7	-5	0	1	0	0	
					E.V					M.R
L.V ← S_1	0	480	5/2	0	$\boxed{11/2}$	1	1/4	-1/4	0	87.27
y	0	120	1/2	1	1/2	0	-1/4	1/4	0	240
a_2	-1	180	1/2	0	3/2	0	3/4	-3/4	1	120
	$Z = \sum C_B x_B = -180$		-1/2	0	-3/2 ↑ P.E	0	-3/4	7/4	0	$R_1 \to R_1 / \frac{11}{2}$
z	0	$\frac{960}{11}$	5/11	0	$\boxed{1}$	2/11	1/22	-1/22	0	
y	0	120	1/2	1	1/2	0	-1/4	1/4	0	
a_2	-1	180	1/2	0	3/2	0	3/4	-3/4	1	
	$Z = \sum C_B x_B$		-1/2	0	-3/2	0	-3/4	7/4	0	

Basis	C_B	x_B	x	y	z	S_1	S_2	a_2	M.R
z	0	960/11	5/11	0	$\boxed{1}$	2/11	1/22	0	
y	0	120	1/2	1	1/2	0	-1/4	0	$R_2 \to R_2 - \frac{1}{2}R_1$
a_2	-1	180	1/2	0	3/2	0	3/4	1	$R_3 \to R_3 - \frac{3}{2}R_1$
	$Z = \sum C_B x_B$		-1/2	0	-3/2	0	-3/4	0	$R_4 \to R_4 + \frac{3}{2}R_1$
z	0	960/11	5/11	0	1	2/11	1/22 E.V	0	1922.24
y	0	840/11	3/11	1	0	-1/11	-3/11	0	116
L.V ← a_2	-1	540/11	-2/11	0	0	-3/11	15/22	1	72.00
	$Z = -540/11$		2/11	0	0	3/11	-15/22 ↑	0	
z	0	960/11	7/11	0	1	2/11	1/22	0	
y	0	840/11	3/11	1	0	-1/11	-3/11	0	$R_3 \to \frac{R_3}{(15/22)}$
S_2	0	1080/15	-44/165	0	0	-66/165	P.E $\boxed{1}$	22/15	$R_1 \to R_1 - \frac{1}{22}R_3$, $R_2 \to R_2 + \frac{3}{11}R_3$
	$Z = -540/11$		2/11	0	0	3/11	-15/22	0	$R_4 \to R_4 + \frac{15}{22}R_3$
z	0	924/11	847/1815	0	1	363/1815	0		
y	0	1056/11	115/583	1	0	-363/1815	0		
S_2	0	1080/15	-44/165	0	0	-66/165	1	22/15	
	$Z = 0$		0	0	0	0	0		

(SM)

as all $\Delta \geq 0$. optimal condition is reached.

Phase II :-

$Z = 2x+3y+4z+0s_1+0s_2+0s_3$

Basis	C_B	x_B	x	y	z	S_1	S_2	M.R
z	4	$\frac{924}{11}$	$\frac{847}{1815}$	0	1	$\frac{363}{1815}$	0	
y	3	$+\frac{1056}{11}$	$\frac{115}{583}$	1	0	$\frac{-363}{1815}$	0	—
S_2	0	$+\frac{1080}{15}$	$\frac{-44}{165}$	0	0	$\frac{-66}{165}$	1	—
	$Z=\Sigma C_B x_B$		36.93	$\frac{1056}{11}$	$\frac{924}{11}$	-31.2	$+\frac{1080}{15}$	
z	4	84	0.4666	0	1	0.2	0	
y	3	96	0.1772	1	0	-0.2	0	—
S_2	0	72	-0.2666	0	0	-0.4	1	—
	$Z=\Sigma C_B x_B = 624$		36.93	$\frac{1056}{11}$	$\frac{924}{11}$	-31.2 ↑	$\frac{1080}{15}$	

(2M)

S_1 will be the entering variable & Z will be the leaving variable as it is not possible as it as already entered the basis.

∴ Optimal sol ⇒ $Z_{max} = 624$

at $x=0$, $y=96$, $z=84$. (1M)

(3) The given problem is balanced transportation problem.

[5] 2	7	4	5
3	3	[8] 1	8
5	[7] 4	7	7
[2] 1	[2] 6	[10] 2	14
7	9	18	$\Sigma S = \Sigma D = 34$

I	II	III	IV
2	$\boxed{2}$		
$\boxed{2}$			
1	1	1	1
1	1	1	$\boxed{5}$

I	1	1	1
II	1	2	2
III	4	2	$\boxed{5}$
IV	$\boxed{4}$	2	

$IBFS = 5\times2 + 8\times1 + 7\times4 + 2\times1 + 2\times6 + 10\times2$

$= 80$ lakhs. (3M)

<u>optimality check.</u>

$m+n-1 =$ no of allocations $= 6$.

	v_1	v_2	v_3	
u_1	[5] 2	7	4	5
u_2	3	3	[8] 1	8
u_3	5	[7] 4	7	7
u_4	[2] 1	[2] 6	[10] 2	14
	7	9	18	

$u_4 = 0$

For occupied cells: $C_{ij} = (u_i + v_j)$

$u_1 + v_1 = 2$, $u_2 + v_3 = 1$, $u_3 + v_2 = 4$, $u_4 + v_1 = 1$

$u_4+v_2=6$, $u_4+v_3=2$.

$\Rightarrow u_1=1$, $u_2=-1$, $u_3=-2$, $v_1=1$, $v_2=6$, $v_3=2$

For unoccupied cells:

$c_{ij}-(u_i+v_j) \Rightarrow 7-(1+6)=0$

$4-(1+2)=1$, $3-(-1+1)=3$, $3-(-1+6)=-2$

$5-(-2+1)=6$; $7-(-2+2)=0$.

as all $c_{ij}-(u_i+v_j) \ngeq 0$ The Sol is not optimum. Form a closed loop starting & ending at -ve cell evaluation. ie from c_{22}.

(5) 2	7	4	5
3	(θ) 3	($8-\theta$) 1	8
5	(7) 4	7	7
(2) 1	($2-\theta$) 6	($10+\theta$) 2	14
7	9	18	

(4M)

$8-\theta=0$, $2-\theta=0 \Rightarrow \theta=2$ is the min

rewrite the table.

(5) 2	7	4	5
3	(2) 3	(6) 1	8
5	(7) 4	7	7
(2) 1	6	(12) 2	14
7	9	18	

(1M)

Again check for optimality

$u_2 = 0$

$v_2 = 3$, $v_3 = 1$, $u_4 = 1$, $v_1 = 0$, $u_3 = 1$, $v_2 = 0$

$u_1 = 2$

<u>For unoccupied cells:</u>

$7 - (2+3) = 2$, $4 - (2+1) = 1$, $3 - (0+0) = 3$

$5 - (1+0) = 5$, $7 - (1+1) = 5$, $6 - (1+5) = 4$.

$c_{ij} \geq 0$ ∴ The opt cost is

$5 \times 2 + 2 \times 3 + 6 \times 1 + 7 \times 4 + 2 \times 1 + 12 \times 2 = 76$ lakhs

(2M)

4.

a.

	S_1	S_2	S_3	S_4
P_1	2	10	9	7
P_2	15	4	14	8
P_3	13	14	16	11
P_4	3	15	13	8

Sol: It is a balanced assignment Problem

1) Row reduction

0	8	7	5
11	0	10	4
2	3	5	0
0	12	10	5

2) Column reduction

$\boxed{0}$	8	2	5
11	0	5	4
2	3	0	0
0	12	5	5

Step 3:

$\boxed{0}$	8	2	5
11	$\boxed{0}$	5	4
2	3	$\boxed{0}$	0
0	12	5	5

No of allocation $= 3 = n$

Order of Matrix $= 4 = N$

$n \neq N$

(3M)

Modification

0	8	$\boxed{0}$	2
11	$\boxed{0}$	3	2
4	5	0	$\boxed{0}$
$\boxed{0}$	12	3	3

$n = 4$

$N = 4$

$n = N$

$P_1 \to S_3$, $P_2 \to S_2$

$P_3 \to S_4$, $P_4 \to S_1$

Total duration $= 9 + 4 + 11 + 3 = 27$ hrs

(2M)

5.
a) If the Maximin value is equal to Minimax value, then the game is said to have

a saddle pt. (1M)

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ A_1 & 4 & 5 & 8 \\ A_2 & 6 & 4 & 6 \\ A_3 & 4 & 2 & 4 \end{array}$$

Maximin = 4
Minimax = 5
Maximin ≠ Minimax

$A_2 \geq A_3$ (A_2 dominates A_3).

delete A_3

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ A_1 & 4 & 5 & 8 \\ A_2 & 6 & 4 & 6 \end{array}$$

$(B_1 \leq B_3)$

B_1 dominates B_3

delete B_3

$$\begin{array}{c|cc|c} & B_1 & B_2 & \\ A_1 & 4 & 5 & 4 \\ A_2 & 6 & 4 & 4 \\ \hline & 6 & \boxed{5} & \end{array}$$

Maximin

minimax

Maximin ≠ Minimax

No saddle pt. (2M)

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{21} + a_{12})} = \frac{4 - 6}{8 - 11} = \frac{-2}{-3} = 2/3$$

$$x_2 = 1 - x_1 = 1 - 2/3 = 1/3$$

$$y_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{21} + a_{12})} = 1/3, \; y_2 = 2/3$$

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11} + a_{22}) - (a_{21} + a_{12})} = 2/3$$

$S_A = (2/3, 1/3, 0)$, $S_B = (1/3, 2/3, 0)$ (2M)

b. <u>Minimax Principle</u>: Here the player tries to minimize his maximum loss

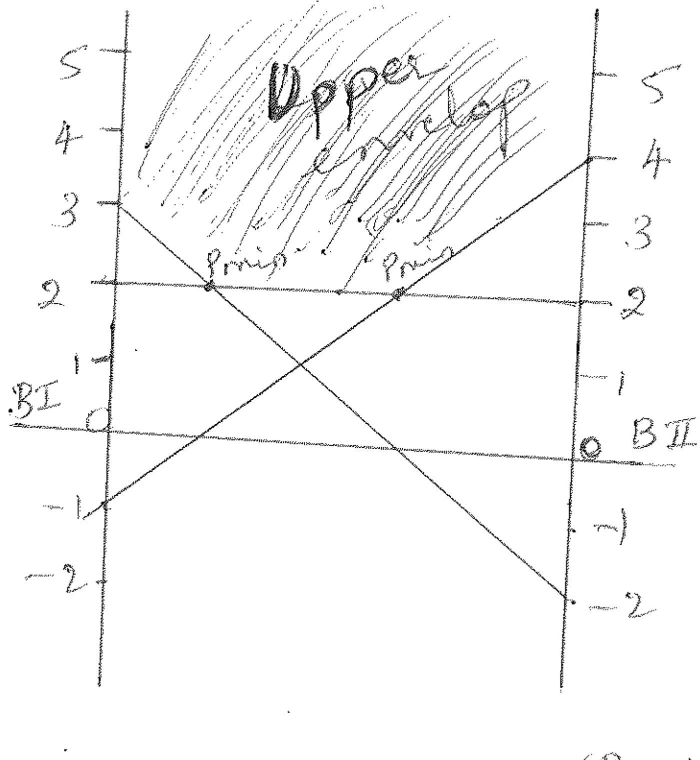
(or)

<u>Maximin Principle</u>: Here the player tries to maximize his min gains.

(1M)

$$\begin{array}{c|c} & \begin{array}{cc} BI & BII \end{array} \\ \hline \begin{array}{c} AI \\ AII \\ AIII \end{array} & \begin{bmatrix} 3 & -2 \\ -1 & 4 \\ 2 & 2 \end{bmatrix}_{3\times 2} \end{array}$$

It is a 3×2 game. Here we are interested in column player. Column player tries to minimize his max loss. Pmin is the lowest point in the upper envelop



<u>case 1:</u>

Pmin is obtained from

$$\begin{array}{c|c|c} & \begin{array}{cc} BI & BII \end{array} & \\ \hline \begin{array}{c} AI \\ AIII \end{array} & \begin{bmatrix} 3 & -2 \\ 2 & 2 \end{bmatrix} & \begin{array}{c} -2 \\ \boxed{2} \end{array} \\ \hline & \begin{array}{cc} 3 & \boxed{2} \end{array} & \end{array}$$

Maximin = Minimax

(2M) Saddle pt exists

opt strategy for player A is A III, opt strategy for player B is B II. value of the game = 2

Case II :-

Pmin is obtained from.

$$\begin{array}{c|cc|l} & B\,I & B\,II & \\ A\,II & 1 & 4 & 1 \\ A\,III & 2 & 2 & \boxed{2} \\ \hline & \boxed{2} & 4 & \text{Maximin} \\ & \text{Minimax} & & \end{array}$$

Maximin = Minimax

Saddle pt exists

$V = 2$ (2M)

$S_A = A\,III$, $S_B = B\,I$

$$P_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{2-(2)}{1-(+6)} = \frac{0}{-5} = 0$$

$$q_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{6}{10} = 3/5$$

$q_2 = 1 - q_1 = 1 - 3/5 = 2/5$, $P_2 = 1 - P_1 = 1 - 0 = 0$

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})} = 2$$

$S_A = (1/2, 1/2, 0)$, $S_B = (3/5, 2/5)$

4 b.

–	20	4	10	–
20	–	5	–	10
4	5	–	6	6
10	–	6	–	20
–	10	6	20	–

It is a balanced ass Problem

row reduction:

-	16	0	6	-
15	-	0	-	5
0	1	-	2	2
4	-	0	-	14
-	4	0	14	-

Column reduction:

-	15	0	4	-
15	-	0	-	3
0	0	-	0	0
4	-	0	-	12
-	3	0	12	-

Allocations:

-	15	$\boxed{0}$	4	-
15	-	0	-	3
$\boxed{0}$	0	-	0	0
4	-	0	-	12
-	3	0	12	-

$n = 2$
$N = 5$
$n \neq N$

Select the least element which is not covered by lines & subtract it from other elements & add to intersection nodes

-	12	$\boxed{0}$	1	-
12	-	0	-	$\boxed{0}$
$\boxed{0}$	0	-	0	0
1	-	0	-	9
-	$\boxed{0}$	0	9	-

Again repeat the process
1 is the least uncovered element

$n = 4$
$N = 5$ $\quad n \neq N$

	1	2	3	4	5
1	–	12	0	$\boxed{0}$	–
2	11	–	0	–	$\boxed{0}$
3	$\boxed{0}$	1	–	0	1
4	0	–	$\boxed{0}$	–	9
5	–	$\boxed{0}$	0	8	–

$n = N = 5$

Schedule is

$1 \to 4$
$2 \to 5$
$3 \to 1$
$4 \to 3$
$5 \to 2$

(3M)

TSP:

$1 \to 4 \to 3 \to 1$. In this path 2 & 5 is not covered.

least element is 1 which is not covered

Case (1)

	1	2	3	4	5
1	–	12	0	$\boxed{0}$	–
2	$\boxed{11}$	–	0	–	$\boxed{\cancel{0}}$
3	$\boxed{\cancel{0}}$	1	–	0	$\boxed{1}$
4	0	–	$\boxed{0}$	–	9
5	–	$\boxed{0}$	0	8	–

schedule

$1 \to 4$, $2 \to 1$, $3 \to 5$
$4 \to 3$, $5 \to 2$.

path is

$1 \to 4 \to 3 \to 5 \to 2 \to 1$

opt cost $= 10 + 6 + 6 + 10 + 20$
$= 52$

Case (2):-

–	12	0	$\boxed{0}$	–
11	–	0	–	$\boxed{0}$
$\boxed{\cancel{0}}$	$\boxed{1}$	–	0	1
0	–	$\boxed{0}$	–	9
–	$\boxed{\cancel{0}}$	0	8	–

Here the route is not possible

(2M)

6. Min $Z = 2x + y + 3z$ s.t

$x - 2y + z \geq 4 \Rightarrow -x + 2y - z + s_1 = -4$

$n = 6, m = 3, n-m = 3$ $\quad 2x + y + z + s_2 = 8$

$-x + z + s_3 = 0$

$Z_{max} = -2x - y - 3z$ $\quad s_1 = -4, s_2 = 8$

$s_3 = 0$

(2M)

Basis	C_B	x_B	x ↓E.V	y	z	s_1	s_2	s_3	Min Ratio
←L.V s_1	0	-4	-1 (P.E.)	+2	-1	1	0	0	Max $\left\{\frac{2}{-1}, \frac{3}{-1}\right\} = (-2, -3)$
s_2	0	8	2	1	1	0	1	0	
s_3	0	0	-1	0	1	0	0	1	
	$Z^* = \Sigma C_B x_B$		+2	1	3	0	0	0	
x	-2	4	$\boxed{1}$	-2	1	-1	0	0	$R_1 \to \frac{R_1}{(-1)}$
s_2	0	8	2	1	1	0	1	0	$R_2 \to R_2 - 2R_1$
s_3	0	0	-1	0	1	0	0	1	$R_3 \to R_3 + R_1$
	$Z^* = \Sigma C_B x_B$		2	1	3	0	0	0	$R_4 \to R_4 + 2R_1$
x	-2	4	1	-2	1	-1	0	0	
s_2	0	0	0	5	-1	2	1	0	
s_3	0	4	0	-2	2	-1	0	1	
	$Z^* = \Sigma C_B x_B = -8$		0	5	1	2	0	0	

(6M)

$Z^* = -8, \; x = 4, \; y = 0, \; z = 0$

$\therefore Z_{min} = 8$ (2M)

7. Max $Z = x+2y$, s.t $x+y \le 3$, $x+2y \le 5$, $3x+y \le 6$

Sol. $Z_{max} = x+2y+0u_1+0u_2+0u_3$

S.t to the constraints

$x+y+u_1=3$

$x+2y+u_2=5$

$3x+y+u_3=6$

$u_1=3, u_2=5; u_3=6$

is the starting sol.

Max $Z = C^T x$ st

$Ax = B$ (2M)

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} A \\ C^T \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 1 \\ -1 & -2 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 6 \\ 0 \end{bmatrix}$$

$$B_{u}^{-1} = \begin{bmatrix} B^{-1} & 0 \\ C_B^T B^{-1} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_j - Z_j = [C_B^T B^{-1} \; 1]\hat{A}$$

$$= (0\ 0\ 0\ 1)\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \end{pmatrix}$$

$$= [1\ \ 2\ \ 0\ 0\ 0]$$

$Z_2 - C_2$ is most +ve, a_2 enters the basis (3M)

$\hat{a}_2 = B_u^{-1} \hat{a}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -2 \end{bmatrix}$

$\hat{x}_B = B_u^{-1} \hat{b} =$ Min of $(3, 2.5, 6) = 2.5$

$\therefore u_2$ leaves the basis

$\hat{y}_B$	$\hat{B}^{-1}$				$\hat{a}_2$	$\hat{x}_B$	M.R
u_1	1	0	0	0	1	3	3
u_2	0	1	0	0	2	5	2.5
u_3	0	0	1	0	1	6	6
Z	0	0	0	1	-2	—	

Converting the leading element into one & remaining elements as zero

$$\hat{B}_u^{-1} = \begin{bmatrix} 0 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 2 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$C_j - Z_j = [C_B^T \; B^{-1}\ 1](\hat{A})$$

$$= [0\ 1\ 0\ 1]\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \end{bmatrix}$$

$$= (0, 0, 0, -1, 0)$$

$\therefore$ all $C_j - Z_j \le 0$ sol is opt (3M)

$$\hat{x}_B = B_u^{-1} \hat{b} = \begin{bmatrix} 0 & -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{5}{2} \\ \frac{7}{2} \\ 5 \end{bmatrix}$$ (2M)

$x_1 = 0,\ = \frac{5}{2}$ $Z = 5$