CMR
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110

USN

Improvement Test

Sub:	Operations Research							Code:	10CS661
Date:	1/ 06 / 2017	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	ISE

Answer any five full questions

	Question	Marks	OBE CO	OBE RBT
1.	Use Big -M method to solve the LPP, Maximize $Z = -2x - y$ subject to the constraints $3x+y = 3$, $4x + 3y \geq 6$, $x+2y \leq 4$, $x, y \geq 0$.	[10]	CO2	L3
2.	Use Two -phase method to solve the LPP, Maximize $Z = 3x - y$ subject to the constraints $2x + y \geq 2$, $x + 3y \leq 2$, $y \leq 4$, $x, y \geq 0$.	[10]	CO2	L3
3.	a) Define balanced and unbalanced transportation problem with an example.	[2]	C04	L1
	b) Find the optimal solution for the given transportation problem by first finding the initial basic feasible solution by Vogel's approximation method.	[8]	C04	L3

Destination

Origin

	D1	D2	D3	Supply
O1	6	8	4	14
O2	4	9	3	12
O3	1	2	6	5
Demand	6	10	15	

	Question	Marks	OBE CO	OBE RBT
4.	a) Define assignment problem. Mention the algorithm used to solve.	[2]	C04	L1
	b) A company has 5 tasks and 5 persons to perform the same. The matrix shows the returns (profit) in hundreds of rupees for assigning jobs to the persons. Assign the 5 tasks to 5 persons to maximize the total return.	[8]	C04	L3

Person

Task

	P1	P2	P3	P4	P5
J1	5	11	10	12	4
J2	2	4	6	3	5
J3	3	12	5	14	6
J4	6	14	4	11	7
J5	7	9	8	12	5

	Question	Marks	OBE CO	OBE RBT
5.	a) Solve the following game by dominance principle whose payoff matrix for player A is given by	[6]	C06	L3

Player B

Player A

	B1	B2	B3	B4
A1	3	2	4	0
A2	3	4	2	4
A3	4	2	4	0
A4	0	4	0	8

b) Solve the following 6x2 game by graphical method. [illegible] CO6 L3

PLAYER B

PLAYER A

Strategies	B1	B2
A1	1	-3
A2	3	5
A3	-1	6
A4	4	1
A5	2	2
A6	-5	0

6. Use dual simplex method to solve the LPP, [10] CO3 L3

Minimize $Z = 2x+y$

subject to the constraints $3x+y \geq 3$, $4x+3y \geq 6$, $x+2y \geq 3$, $x,y \geq 0$.

7. Use revised simplex method to solve the LPP, [10] CO2 L3

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

subject to the constraints $x_1 + 2x_2 + x_3 \leq 430$, $3x_1 + 2x_3 \leq 460$, $x_1 + 4x_2 \leq 420$, $x_1, x_2, x_3 \geq 0$.

S.[illegible] 29/5/2017

CCI

Improvement test - Jun 2017

Solution Manual (ISE)

1. $Z_{max} = -2x - y - Ma_1 + 0S_1 - Ma_2 + 0S_2$

S.t $3x + y + a_1 = 3$, $4x + 3y - S_1 + a_2 = 6$

$x + 2y + S_2 = 4$.

$n = 6, m = 3, n - m = 3$.

Put $x, y, S_1 = 0 \Rightarrow a_1 = 3, a_2 = 6, S_2 = 4$ is the basic sol. (2M)

Basis	C_B	x_B	x (E.V)	y	S_1	S_2	a_1	a_2	M.R
L.V ← a_1	$-M$	3	$\boxed{3}$ P.E	1	0	0	1	0	1
a_2	$-M$	6	4	3	-1	0	0	1	1.5
S_2	0	4	1	2	0	1	0	0	4
	$Z = \sum C_B x_B$	$= -9M$	$-7M+2$ ↑	$-4M+1$	M	0	0	0	
x	-2	1	1	$1/3$ ↓	0	0	$1/3$	0	$R_1 \to \frac{R_1}{3}$
a_2	$-M$	2	0	$5/3$	-1	0	$-4/3$	1	$R_2 \to R_2 - 4R_1$
S_2	0	3	0	$5/3$	0	1	$-1/3$	1	$R_3 \to R_3 - R_1$
	$Z = -2 - 2M$		0	$(-5M+1)/3$	M	0	$\frac{7M-2}{3}$	0	$R_4 \to R_4 + (M-2)R_1$
x	-2	1	1	$1/3$ P.E	0	0	$1/3$	0	3
← a_2	$-M$	2	0	$\boxed{5/3}$	-1	0	$-4/3$	1	1.2
S_2	0	3	0	$5/3$	0	1	$-1/3$	0	1.8
	$Z = -2 - 2M$		0	$(1-5M)/3$	M	0	$\frac{7M-2}{3}$	0	(3M)

Basis	C_B	x_B	x	y	S_1	S_2	a_1	a_2	M.R
x	-2	1	1	$1/3$	0	0	$1/3$	0	
y	-1	$6/5$	0	1	$-3/5$	0	$-4/5$	$3/5$	$R_2 \rightarrow \frac{R_2}{(5/3)}$
S_3	0	3	0	$5/3$	0	1	$-1/3$	0	
	$Z = \Sigma C_B x_B$		0	$\frac{1-5M}{3}$	M	0	$\frac{7M-2}{3}$	0	
x	-2	$3/5$	1	0	$1/5$	0	$3/5$	$-1/5$	$R_1 \rightarrow R_1 - \frac{1}{3}R_2$ $R_3 \rightarrow R_3 - \frac{5}{3}R_2$
y	-1	$6/5$	0	1	$-3/5$	0	$-4/5$	$3/5$	$R_4 \rightarrow R_4 + \left(\frac{5M-1}{3}\right)R_2$
S_3	0	1	0	0	1	1	1	-1	
	$Z = \Sigma C_B x_B = -12/5$		0	0	$1/5$	0	$\frac{M-2}{5}$	$\frac{M-1}{5}$	(3M)

$\Delta \geq 0$, opt sol is reached.

$Z_{min} = -12/5$, $x = 3/5$, $y = 6/5$ (2M)

2. $Z = 3x - y$

S.t $2x + y - S_1 + a_1 = 2$

$x + 3y + S_2 = 2$

$0x + y + S_3 = 4$

$n = 6$, $m = 3$, $n - m = 3$

Put $x, y, S_1 = 0 \Rightarrow a_1 = 2$, $S_2 = 2$, $S_3 = 4$

is the basic sol (2M)

Phase I: $Z^*_{max} = 0x + 0y + 0S_1 + 0S_2$ ... a_1

Basis	C_B	x_B	$x^{E.V}$	y	S_1	S_2	S_3	a_1	M.R
L.V ← a_1	-1	2	$\boxed{2}^{P.E}$	1	-1	0	0	1	1
S_2	0	2	1	3	0	1	0	0	2
S_3	0	4	0	1	0	0	1	0	X
	$Z^* = \sum C_B x_B$	$= -2$	-2	-1	1	0	0	0	
x	0	1	1	$1/2$	$-1/2$	0	0	$1/2$	
S_2	0	1	0	$5/2$	$1/2$	1	0	$-1/2$	
S_3	0	4	0	1	0	0	1	0	
	$Z = 0$		0	0	0	0	0	1	

$\Delta \geq 0$ optimal cond is reached. (3M)

Phase II: $Z_{max} = 3x - y + 0S_1 + 0S_2 + 0S_3$

Basis	C_B	x_B	x	y	S_1 (E.V)	S_2	S_3	M.R
x	3	1	1	$1/2$	$-1/2$	0	0	—
← S_2	0	1	0	$5/2$	$\boxed{1/2}^{P.E}$	1	0	2
S_3	0	4	0	1	0	0	1	—
	$Z = \sum C_B x_B$	$= 3$	0	$5/2$	$-3/2$	0	0	
x	3	2	1	3	0	1	0	$R_2 \to \frac{R_2}{(1/2)}$
S_1	0	2	0	5	1	2	0	$R_1 \to R_1 + \frac{1}{2} R_2$
S_3	0	4	0	1	0	0	1	$R_4 \to R_4 + \frac{3}{2} R_2$
	$Z = \sum C_B x_B$	$= 6$	0	10	0	3	0	(3M)

$\Delta \geq 0$, $Z_{max} = 6$, $x = 2$, $y = 0$. (2M)

3.

a. Balanced transportation problem:

when $\Sigma S = \Sigma D$ the TP is balanced (1M)

Ex.

	D_1	D_2	D_3	S
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	8	14
D	8	8	18	

$\Sigma S = 34$

$\Sigma D = 34$

Unbalanced transportation Problem:

when ever Σ supply > Σ demand or Σ demand > Σ supply, then the problem is unbalanced TP. (1M)

Ex:

	D_1	D_2	D_3	S
O_1	4	7	8	10
O_2	1	2	4	20
O_3	3	1	5	10
D	10	15	25	

$\Sigma S = 40$

$\Sigma D = 50$

b. $\Sigma S = \Sigma D = 31$

The given TP is a balanced one

	D_1	D_2	D_3	S
O_1	6	8	4	14
O_2	4	9	3	12
O_3	1	2	6	5
D	6	10	15	$\Sigma S = 31$

$\Sigma D = 31$

IBFS: By vogel's method

	D_1'	D_2'	D_3'	S
O_1	6	$\boxed{5}$ 8	$\boxed{9}$ 4	14
O_2	$\boxed{6}$ 4	9	$\boxed{6}$ 3	12
O_3	1	$\boxed{5}$ 2	6	5
D	6	10	15	

Row difference

I	II	III	IV
2	2	4	4
1	1	$\boxed{6}$	
1			

Column difference

I	3	$\boxed{6}$	1
II	2	1	1
III		1	1

IBFS $= 5\times8 + 9\times4 + 6\times4 + 6\times3 + 5\times2$

$= 40 + 36 + 24 + 18 + 10$

$= 128$ (3M)

optimality check;

$m+n-1 = 3+3-1 = 5$.

It is a non degenerate Problem.

For occupied cells:

$u_1 = 0$

$u_1 + v_2 = 8 \Rightarrow v_2 = 8$, $u_2 + v_1 = 4 \Rightarrow v_1 = 5$

$u_1 + v_3 = 4 \Rightarrow v_3 = 4$ / $u_2 + v_3 = 3 \Rightarrow u_2 = -1$

$|||^{ly}$ $u_3 = -6$. (2M)

<u>For unoccupied cells:</u>

$\Delta_{ij} = C_{ij} - (u_i + v_j)$

$\Delta_{11} = 6 \Rightarrow 6 - (u_1 + v_1) = 6 - 5 = 1$.

$\Delta_{22} \Rightarrow 9 - (u_2 + v_2) = 9 - 7 = 2$

$\Delta_{31} \Rightarrow 1 - (u_3 + v_1) = 1 - (-1) = 2$

$\Delta_{33} \Rightarrow 6 - (u_3 + v_3) = 6 - (-2) = 8$ (2M)

as all $\Delta_{ij} > 0$ optimal sol is reached

opt sol = IBFS = 128. (1M)

4. a. <u>Assignment Problem</u>: Here the objective of the problem is to assign a <u>no of</u> origins equal to no of destinations at a min cost or max profit. The assignment Problem is solved by Hungarian method. (2M)

b. As the problem is of max type it is to be converted into minimum form.

as the no of rows = <u>no</u> of columns.
it is a balanced assignment Problem

Select the highest & subtract from all the other elements

9	3	4	2	10
12	10	8	11	9
11	2	9	0	8
8	0	10	3	7
7	5	6	2	9

(1M)

Row reduction:

7	1	2	0	8
4	2	0	3	1
11	2	9	0	8
8	0	10	3	7
5	3	4	0	7

Column reduction:

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	3	6
1	3	4	0	6

Allocations:

3	1	2	$\boxed{0}$	7
$\boxed{0}$	2	0	3	0
7	2	9	0	7
4	$\boxed{0}$	10	3	6
1	3	4	0	6

$n = 3$
$N = 5$ $n \neq N$

Modification:
Select the least element which is not covered under the lines & subtract it with all the other elements & add to intersection nods (3M)

$$\begin{array}{ccccc} 2 & 1 & 1 & \boxed{0} & 6 \\ 0 & 3 & \boxed{0} & 4 & 0 \\ 6 & 2 & 8 & 0 & 6 \\ 3 & \boxed{0} & 9 & 3 & 5 \\ \boxed{0} & 3 & 3 & 0 & 5 \end{array}$$

$n = 4$

$N = 5$

$n \neq N$.

Modify the matrix by repeating the procedure.

Modified matrix

$$\begin{array}{ccccc} 2 & 1 & \boxed{0} & 0 & 5 \\ 4 & 4 & 0 & 5 & \boxed{0} \\ 6 & 2 & 7 & \boxed{0} & 5 \\ 3 & \boxed{0} & 8 & 3 & 4 \\ \boxed{0} & 3 & 2 & 0 & 4 \end{array}$$

$n = 5$

$N = 5$

$n = N$.

opt scheduling is

$J_1 \to P_3$, $J_2 \to P_5$, $J_3 \to P_4$, $J_4 \to P_2$, $J_5 \to P_1$ (3M)

Total profit $= 10 + 5 + 14 + 14 + 7$

$= 50$ units

$= 50 \times 100 = $ Rs 5000/- (1M)

5. a.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix} \end{array}$$

(1M)

Maximin $= 2$

Minimax $= 4$.

Maximin $\neq$ Minimax

Saddle pt does not exist

Step 1: $R_3 \geq R_1$ delete R_1

$$\begin{array}{c|cccc} & BI & BII & BIII & BIV \\ \hline A\ II & 3 & 4 & 2 & 4 \\ A\ III & 4 & 2 & 4 & 0 \\ A\ IV & 0 & 4 & 0 & 8 \end{array}$$

$C_2 \leq C_1$, delete C_1

$$\begin{array}{c|ccc} & II & III & IV \\ \hline II & 4 & 2 & 4 \\ III & 2 & 4 & 0 \\ IV & 4 & 0 & 8 \end{array}$$

No pure row or column dominance

The average of $(C_3 \ \& \ C_4) \leq C_2$

delete C_2

$$\begin{array}{c|cc} & III & IV \\ \hline II & 2 & 4 \\ III & 4 & 0 \\ IV & 0 & 8 \end{array}$$

avg of $(R_3 \ \& \ R_4) \geq R_2$

delete R_2

$$\begin{array}{c|cc|c} & III & IV & \\ \hline III & 4 & 0 & 0 \\ IV & 0 & 8 & 0 \\ \hline & \boxed{4} & 8 & \end{array}$$

$\}$ Maximin

Minimax

Maximin $\neq$ Minimax (3M)

$x_1 =$ formula $= 2/3$, $x_2 = 1/3$

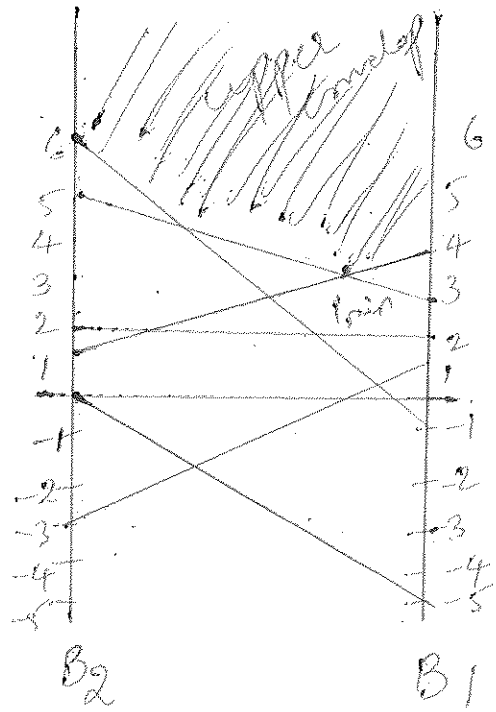
$y_1 =$ formula $= 2/3$, $y_2 = 1/3$

$V = 8/3$ (2M)

b. Maximin $\neq$ Minimax

$3 \neq 4$

It is a $N \times 2$ matrix. Here we are interest in column player. He tries to minimize his maximum loss. we find the upper envelop & select the min pt.



Pmin is from the strategies

$$\begin{array}{c|cc|c} & B_1 & B_2 & \\ A_2 & 3 & 5 & 3 \\ A_4 & 4 & 1 & 1 \\ \hline & 4 & 5 & \end{array}$$

Maximin $\neq$ Minimax
$3 \neq 4$.
No Saddle pt

(2M)

$$P_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{3}{5}$$

$$P_2 = 1 - P_1 = 2/5$$

$$q_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = 4/5$$

$$q_2 = 1 - q_1 = 1/5$$

$$V = \frac{a_{11} \cdot a_{22} - a_{12}\, a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = 17/5$$

(2M).

6.

a.

Min $Z = 2x + y$

S.t $3x + y \geq 3$, $4x + 3y \geq 6$, $x + 2y \geq 3$

Min $Z = -\text{Max}(-Z) = -Z^*$

$Z^* = -2x - y + 0S_1 + 0S_2 + 0S_3$ (1M)

$-3x - y + S_1 = -3$, $-4x - 3y + S_2 = -6$

$-x - 2y + S_3 = -3$

$n = 5$, $m = 3$ $n - m = 5 - 3 = 2$ (2M)

put $x, y = 0 \Rightarrow S_1 = -3$, $S_2 = -6$, $S_3 = -3$

is the basic sol.

Basis	C_B	x_B	x	y (↓ E.V)	S_1	S_2	S_3	M.R
S_1	0	-3	-3	-1	1	0	0	Max
L.V ← S_2	0	-6	-4	$\boxed{-3}$ R.E	0	1	0	$\left\{\frac{2}{-4}, \frac{1}{-3}\right\}$
S_3	0	-3	-1	-2	0	0	1	$= -1/3$
	$Z^* = \sum C_B x_B = 0$		2	1	0	0	0	
S_1	0	-3	-3	-1	1	0	0	
y	-1	2	$4/3$	$\boxed{1}$	0	$-1/3$	0	$R_2 \to \frac{R_2}{(-3)}$
S_3	0	-3	-1	-2	0	0	1	
	$Z^* = \sum C_B x_B$		2	1	0	0	0	
L.V ← S_1	0	-1	$\boxed{-5/3}$ R.E	0	1	$-1/3$	0	$R_1 \to R_1 + R_2$
y	-1	2	$4/3$	1	0	$-1/3$	0	$R_3 \to R_3 + 2R_2$
S_3	0	1	$5/3$	0	0	$-2/3$	1	$R_4 \to R_4 - R_2$
	$Z^* = -2$		$2/3$ ↑ E.V	0	0	$1/3$	0	Max $= \left(\frac{2}{3} \div -5/3, \frac{1}{3} \div -1/3\right)$

(4M) $= \left(-2/5, -1\right)$

Basis	C_B	x_B	x	y	S_1	S_2	S_3	M.R
x	-2	-1	$-5/3$	0	1	$-1/3$	0	$R_1 \to R_1 \div -\frac{5}{3}$
y	-1	2	$4/3$	1	0	$-1/3$	0	$R_2 \to R_2 - \frac{4}{3}R_1$
S_3	0	1	$5/3$	0	0	$-2/3$	1	$R_3 \to R_3 - \frac{5}{3}R_1$
	$Z^* = \Sigma C_B x_B$		$2/3$	0	0	$1/3$	0	$R_4 \to R_4 - \frac{2}{3}R_1$
x	-2	$3/5$	1	0	$-3/5$	$1/5$	0	
y	-1	$6/5$	0	1	$4/5$	$-3/5$	0	
S_3	0	0	0	0	1	-1	0	
	$Z^* = -12/5$		0	0	$2/5$	$1/5$	0	

(2M)

$\Delta \geq 0$. The opt sol is reached.

$Z^*_{\cancel{min}} = -12/5$, $x = 3/5$, $y = 6/5$

$Z_{min} = -Z^* = 12/5$. 1M

7

Revised Simplex method:

(7) $Z = 3x_1 + 2x_2 + 5x_3$ s.t $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$, $x_1 + 4x_2 \le 420$

Sol: Max $Z = 3x_1 + 2x_2 + 5x_3 + 0S_1 + 0S_2 + 0S_3$

S.t $x_1 + 2x_2 + 3x_3 + S_1 = 430$

$3x_1 + 0x_2 + 2x_3 + S_2 = 460$

$x_1 + 4x_2 + 0x_3 + S_3 = 420$.

S.t $Ax = B$, $x \ge 0$, $Z = C^T x$

where $C^T = [3 \;.\; 2 \; 5 \; 0 \; 0 \; 0]$

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{bmatrix}$$ x is a matrix of unknowns.

$$b = \begin{bmatrix} 430 \\ 460 \\ 420 \end{bmatrix}$$ Basic sol. $S_1 = 430$, $S_2 = 460$, $S_3 = 420$ (2M)

Basis Matrix $= B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ formed by basic variables

$$\hat{A} = \begin{bmatrix} A \\ -C^T \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \\ -3 & -2 & -5 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{aux}^{-1} = \begin{bmatrix} B^{-1} & 0 \\ -C_B^T B^{-1} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

compute $C_B^T B^{-1}$ ∋ $C_B^T = C$ values w.r.to basic variables

$B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$C_B^T B^{-1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}_{3\times3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

1st iteration: Last row of B_{cur}^{-1} × variable that come when S_1, S_2, S_3 goes out

$$\Delta_j = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 3 & 0 & 2 \\ 1 & 4 & 0 \\ -3 & -2 & -5 \end{pmatrix}$$

$$= \begin{bmatrix} -3 & -2 & -5 \end{bmatrix}$$

↑ entering variable

$\therefore x_3$ is the entering v

(2M)

BV	x_B	B_{cur}^{-1}				x_3	Min Ratio
S_1	430	1	0	0	0	1	430
S_2	460	0	1	0	0	$\boxed{2}$	230
S_3	420	0	0	1	0	0	
	0	0	0	0	1	-5	

$R_1 \to R_1 - R_2$
$R_2 \to R_2/2$
$R_4 \to R_4 + 5R_2$

BV	x_B	B_{cur}^{-1}				
S_1	200	1	$-1/2$	0	0	0
x_3	230	0	$1/2$	0	0	1
S_3	420	0	0	1	0	0
	1150	0	$5/2$	0	1	0

already x_3 entered

2nd iteration

$\Delta_j =$ ~~last of B_{cur}^{-1}~~ $\quad x_1 \quad x_2$

$$= \begin{bmatrix} 0 & 5/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 1 & 4 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9/2 & -2 \end{bmatrix}$$

↑ x_2 entering variable

$\therefore x_2$ – e.v.

(2M)

B.V	x_B	B_{new}^{-1}				x_2	M.R
S_1	200	1	$-1/2$	0	0	$\boxed{2}$	100
x_3	230	0	$1/2$	0	0	0	—
S_3	420	0	0	1	0	4	105
	1150	0	$5/2$	0	1	-2	

x_2	100	$1/2$	$-1/4$	0	0	1	$R_1 \to R_1/2$
x_3	230	0	$1/2$	0	0	0	$R_3 \to R_3 - 4R_1$
S_3	420	-2	1	1	0	0	$R_4 \to R_4 + 2R_1$
	1350	1	2	0	1	0	
		B_{new}^{-1}					(2M)

3rd iteration

$$\Delta = \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \\ -3 \end{bmatrix}$$

$= 4.$

$\Delta_j \geq 0$

$\therefore$ Sol is opt.

$z = 1350,\ x_1 = 0,\ x_2 = 100,\ x_3 = 230$ (2M)

② ~~Max $Z = x_1 + x_2$.~~

~~S.t $2x_1 + 5x_2 \leq 6$~~

~~$x_1 + x_2 \geq 2$~~

~~$x_1, x_2 \geq 0$.~~

Sol: $Z_{max} = x_1 + x_2 + 0S_1 - Ma_1 + 0S_2$.

S.t $2x_1 + 5x_2 + S_1 = 6$, $x_1 + x_2 + a_1 - S_2 = 2$.

B.S $S_1 = 6$, $a_1 = 2$.

$Ax = B$ $\quad z = c^T x$, $c^T = [1 \; 1 \; 0 \; 0 \; -M]$

$$A = \begin{bmatrix} 2 & 5 & 1 & 0 & 0 \\ 1 & 1 & 0 & +1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} A \\ -C_T \end{bmatrix} = \begin{bmatrix} 2 & 5 & 1 & 0 & 0 \\ 1 & 1 & 0 & -1 & +1 \\ -1 & -1 & 0 & 0 & +M \end{bmatrix}$$

$$B_{curr}^{-1} = \begin{bmatrix} B^{-1} & 0 \\ -C_B^T B^{-1} & 1 \end{bmatrix} \Rightarrow C_B^T B^{-1} = \underset{1\times3}{[0 \; 0 \; -M]} \underset{2\times2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$$= \underset{1\times3}{[0 \; 0 \; +M]} \underset{3\times3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -M \end{bmatrix}}$$

$$= [0 \; 0 \; M]$$

$$= \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & +M & 0 \\ 0 & 0 & M & 1 \end{matrix}$$

$$[0 \; 0 \; M] \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & M \end{bmatrix}$$

$$[0 \; 0 \; M]$$

24/5 $\quad -8 -10$

26/5 $\quad -8 -10$