CMR INSTITUTE OF TECHNOLOGY

USN | | | | | | | | | | |

Internal Assesment Test – III

| Sub: | **Operations Research** | | | | | | | Code: | 10EE661 |
|---|---|---|---|---|---|---|---|---|---|
| Date: | 30/05/2017 | Duration: | 90 mins | Max Marks: | 50 | Sem: | 6 | Branch: | **EEE** |

**Answer any 5 questions**

| | | Marks | OBE CO | OBE RBT |
|---|---|---|---|---|
| 1 | **a)** A factory is engaged in manufacturing 3 products A, B and C. Each product undergoes three processes, namely, turning, grinding and assembly. The turning, grinding and assembly time for 1 unit of product A is 2 hours, 3 hours and 1 hour respectively; for product B; the processing time for product B is 3 hours, 1 hour and 3 hours for turning, grinding and assembly respectively; the processing time for product C is 1 hour, 3 hours and 1 hour for turning, grinding and assembly respectively. The profit on product A, B and C is ₹20, ₹40 and ₹20 respectively. Assuming 300 hours of lathe time, 300 hours of grinding time and 240 hours of assembly time is available, formulate a mathematical model | [05] | CO1 | L3 |
| | **b)** ABC Company owns a paint factory that produces both exterior and interior paint for wholesale distribution. The basic raw materials A and B are used to manufacture the paints. The minimum availability of A is 6 tons/day and that of B is 8 tons/day. The requirements of raw materials/ton of interior and exterior are given in the table. Market survey has established that the daily demand for interior paint cannot exceed that of the exterior paint by more than 1 ton. The survey shows that maximum demand for exterior paint is limited to 2 tons/day. The wholesale price is Rs. 3000 for exterior paint and Rs. 2000 for interior paint. How much interior and exterior paint should the company produce to maximize profit? Formulate a mathematical model. | [05] | | |

| **RAW MATERIAL** | **EXTERIOR** | **INTERIOR** | **AVAILABILITY** |
|---|---|---|---|
| A | 1 | 2 | 6 |
| B | 2 | 1 | 8 |

| | | | | |
|---|---|---|---|---|
| 2 | Solve the following problem by **Simplex Method**:<br>**Max.** $Z= X_1+2X_2+3X_3$<br>Subject to: $X_1+ 2X_2+3X_3 \leq 10$<br>$X_1+ X_2 \leq 5$<br>$X_1 \leq 1$<br>$X_1, X_2, X_3 \geq 0$ | [10] | CO3 | L3 |

<table>
<tr><td>3</td><td>Solve the following problem by Big M method:<br>Min. $Z= 10X_1+15X_2+20X_3$<br>Subject to: $2X_1+4X_2+6X_3 \geq 24$<br>$3X_1+9X_2+6X_3 \geq 30$<br>$X_1, X_2, X_3 \geq 0$</td><td>[10]</td><td>CO3</td><td>L3</td></tr>
<tr><td>4</td><td>Find the dual of the following Linear Programming Problems<br>i. Max. $Z= X_1-2X_2+3X_3$<br>Subject to: $-2X_1+X_2+3X_3 \leq 10$<br>$2X_1+3X_2+4X_3 \leq 5$<br>$X_1, X_2, X_3 \geq 0$<br>ii. Max. $Z= 3X_1+5X_2$<br>Subject to: $2X_1+6X_2 \leq 50$<br>$3X_1+2X_2 \leq 35$<br>$5X_1-3X_2 \leq 10$<br>$X_2 \leq 20$<br>$X_1, X_2 \geq 0$<br>iii. Min. $Z= 5X_1+4X_2-3X_3+6X_4$<br>Subject to: $6X_1+5X_2-3X_3+7X_4 \geq 150$<br>$X_1+3X_2+2X_3 \leq 25$<br>$X_1, X_2, X_3 X_4 \geq 0$</td><td>[03]<br>[04]<br>[03]</td><td>CO3</td><td>L3</td></tr>
<tr><td>5</td><td>Draw a neat network for the following details:<br>a)<br>i. A, B and C are starting activities.<br>ii. D, E and F can start if A is complete<br>iii. G can start after B and D are complete<br>iv. H can start after C and E are complete<br>b)<br><table><tr><th>Activity</th><th>Precedence</th></tr><tr><td>A</td><td>-</td></tr><tr><td>B</td><td>A</td></tr><tr><td>C</td><td>A</td></tr><tr><td>D</td><td>B</td></tr><tr><td>E</td><td>B</td></tr><tr><td>F</td><td>C, D</td></tr><tr><td>G</td><td>E, F</td></tr></table></td><td>[03]<br>[02]</td><td>CO4</td><td>L4</td></tr>
</table>

| | | | | |
|---|---|---|---|---|
| | **c)** A project consists of the following activities:<br>i. A & B are starting jobs<br>ii. A controls C, D and E<br>iii. B controls F & K<br>iv. G depends on C<br>v. H depends on D<br>vi. E & F control J & M<br>vii. L depends on K<br>viii. M is controlled by L | [05] | | |
| 6 | A project schedule has the following characteristics<br><table><tr><th>Activity</th><th>Time (in days)</th></tr><tr><td>1-2</td><td>4</td></tr><tr><td>1-3</td><td>1</td></tr><tr><td>2-4</td><td>1</td></tr><tr><td>3-4</td><td>1</td></tr><tr><td>3-5</td><td>6</td></tr><tr><td>4-9</td><td>5</td></tr><tr><td>5-6</td><td>4</td></tr><tr><td>5-7</td><td>8</td></tr><tr><td>6-8</td><td>1</td></tr><tr><td>7-8</td><td>2</td></tr><tr><td>8-10</td><td>5</td></tr><tr><td>9-10</td><td>7</td></tr></table>Construct the network and determine the Critical Path and the duration. Also calculate Early Start Time, Early Finish Time, Latest Start Time, Latest Finish Time and Floats. | [10] | CO4 | L4 |
| 7 | The details of a project consisting of activities A to K are summarized in the table below.<br><table><tr><th rowspan="2">Activity</th><th rowspan="2">Immediate Predecessor</th><th colspan="3">DURATION (IN WEEKS)</th></tr><tr><th>OPTIMISTIC TIME</th><th>MOST LIKELY TIME</th><th>PESSIMISTIC TIME</th></tr><tr><td>A</td><td>-</td><td>6</td><td>7</td><td>8</td></tr><tr><td>B</td><td>-</td><td>1</td><td>2</td><td>9</td></tr><tr><td>C</td><td>-</td><td>1</td><td>4</td><td>7</td></tr><tr><td>D</td><td>A</td><td>1</td><td>2</td><td>3</td></tr><tr><td>E</td><td>A, B</td><td>1</td><td>2</td><td>9</td></tr><tr><td>F</td><td>C</td><td>1</td><td>5</td><td>9</td></tr><tr><td>G</td><td>C</td><td>2</td><td>2</td><td>8</td></tr><tr><td>H</td><td>E, F</td><td>4</td><td>4</td><td>4</td></tr><tr><td>I</td><td>E, F</td><td>4</td><td>4</td><td>10</td></tr><tr><td>J</td><td>D, H</td><td>2</td><td>5</td><td>14</td></tr><tr><td>K</td><td>I, G</td><td>2</td><td>2</td><td>8</td></tr></table> | [10] | CO4 | L4 |

| | | | | |
|---|---|---|---|---|
| | i. Construct the project network<br>ii. Find the expected duration and variance of each activity<br>iii. Find the critical path and the expected project completion time.<br>iv. What is the probability of completing the project on or before 25 weeks. | | | |
| 8 | Consider the data of a project shown in the table below.<br><br>**Activity** \| **Normal Time (in weeks)** \| **Normal Cost (in Rupees.)** \| **Crash Time (in weeks)** \| **Crash Cost (in Rupees)**<br>1-2 \| 13 \| 700 \| 9 \| 900<br>1-3 \| 5 \| 400 \| 4 \| 460<br>1-4 \| 7 \| 600 \| 4 \| 810<br>2-5 \| 12 \| 800 \| 11 \| 865<br>3-2 \| 6 \| 900 \| 4 \| 1130<br>3-4 \| 5 \| 1000 \| 3 \| 1180<br>4-5 \| 9 \| 1500 \| 6 \| 1800<br><br>If the indirect cost per week is Rs. 160, find the optimal crashed project completion time. | [10] | CO4 | L4 |

# SOLUTION BANK FOR IAT-3

(1.)

(a.) Let $X_1$, $X_2$ and $X_3$ be the production quantities of products A, B & C respectively.

$$Z_{max} = 20X_1 + 40X_2 + 20X_3$$

Subject to constraints:

$$2X_1 + 3X_2 + X_3 \leq 300$$
$$2X_1 + X_2 + 3X_3 \leq 300$$
$$X_1 + 3X_2 + X_3 \leq 240$$

(b.) Let $X_1$ & $X_2$ be the requirement of exterior & interior paint.

$$Z_{max} = 3000X_1 + 2000X_2$$

Constraints: $X_1 + 2X_2 \leq 6$

$$2X_1 + X_2 \leq 8$$
$$X_2 - X_1 \leq 1.$$
$$X_1 \leq 2.$$

(2.)

$$\text{Max } Z = X_1 + 2X_2 + 3X_3 + 0S_1 + 0S_2$$

$$X_1 + 2X_2 + 3X_3 + S_1 = 10$$

$$X_1 + X_2 + S_2 = 5$$

| $C_{Bi}$ | $C_j$ | 1 | 2 | 3 | 0 | 0 | Soln | Ratio |
|---|---|---|---|---|---|---|---|---|
| | Basic Var | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | | |
| 0 | $S_1$ | 1 | 2 | 3 | 1 | 0 | 10 | 10/3 |
| 0 | $S_2$ | 1 | 1 | 0 | 0 | 1 | 5 | |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | | |
| | $C_j - Z_j$ | 1 | 2 | 3 | 1 | 0 | | |

| $C_{Bi}$ | $C_j$ | 1 | 2 | 3 | 0 | 0 | Soln | Ratio |
|---|---|---|---|---|---|---|---|---|
| | Basic V | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | | |
| 3 | $X_3$ | 1/3 | 2/3 | 1 | 1/3 | 0 | 10/3 | |
| 0 | $S_2$ | 1 | 1 | 0 | 0 | 1 | 5 | |
| | $Z_j$ | 1 | 2 | 3 | 1 | 0 | | |
| | $C_j - Z_j$ | 0 | 0 | 0 | -1 | 0 | | |

Optimality is attained

$\therefore X_1 = 0,\ X_2 = 0,\ X_3 = 10/3$

$$\text{Max } Z = 0 + 2(0) + 3\left(\frac{10}{3}\right) = 10$$

(3.)

$$\text{Min } Z = 10X_1 + 15X_2 + 20X_3 + 0S_1 + 0S_2 + MA_1 + MA_2$$

$$2X_1 + 4X_2 + 6X_3 - S_1 + A_1 = 24$$

$$3X_1 + 9X_2 + 6X_3 - S_2 + A_2 = 30$$

| CB'L | $C_j$ | 10 | 15 | 20 | 0 | 0 | M | M | Sol. | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| | Basic V | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | | |
| M | $A_1$ | 2 | 4 | 6 | -1 | 0 | 1 | 0 | 24 | 6 |
| M | $A_2$ | 3 | 9 | 6 | 0 | -1 | 0 | 1 | 30 | 30/9 |
| | $Z_j$ | 5M | 13M | 12M | -M | -M | M | M | | |
| | $C_j - Z_j$ | 10-5M | 15-13M | 20-12M | M | M | 0 | 0 | | |

| $C_j$ | | 10 | 15 | 20 | 0 | 0 | M | | |
|---|---|---|---|---|---|---|---|---|---|
| $CB_i$ | Basic V | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | $A_1$ | Sol | Ratio |
| M | $A_1$ | $2/3$ | 0 | $10/3$ | $-1$ | $4/9$ | 1 | $32/3$ | |
| $X_2$ | 15 | $1/3$ | 1 | $2/3$ | 0 | $-1/9$ | 0 | $10/3$ | |
| | $Z_j$ | $\frac{-2M}{3}-5$ | $-15$ | $\frac{-10M}{3}-10$ | M | $\frac{-4M}{9}+\frac{5}{3}$ | $-M$ | | |
| | $c_j - z_j$ | $\frac{-2M}{3}+5$ | 0 | $\frac{-10M}{3}+10$ | M | $\frac{-4M}{9}-\frac{5}{3}$ | | | |

| $C_j$ | | 10 | 15 | 20 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|
| $CB_i$ | B.V | $X_1$ | $X_2$ | $X_3$ | $S_1$ | $S_2$ | Sol. | Ratio |
| 20 | $X_3$ | $1/5$ | 0 | 1 | $-3/10$ | $2/15$ | $16/5$ | |
| 15 | $X_2$ | $1/5$ | 1 | 0 | $1/5$ | $-1/5$ | $6/5$ | |
| | $Z_j$ | $-7$ | $-15$ | $-20$ | $-9$ | $1/3$ | | |
| | $C_j - Z_j$ | 17 | 0 | 0 | $+9$ | $1/3$ | | |

$X_3 = 16/5$, $X_2 = 6/5$

Min $Z = 82$.

(4.)

(i) Min $Z_y = 10y_1 + 5y_2$

$$2y_1 + 2y_2 \geq 1$$

$$y_1 + 3y_2 \geq -2$$

$$3y_1 + 4y_2 \geq 3.$$

(ii) Min $Z = 50y_1 + 35y_2 + 10y_3$

$$2y_1 + 3y_2 + 5y_3 \geq 3$$

$$6y_2 + 2y_2 - 3y_3 + y_4 \geq 5.$$

(iii) ~~max~~ $Z_y = 150y_1 - 25y_2$

$$6y_1 - y_2 \leq 5$$

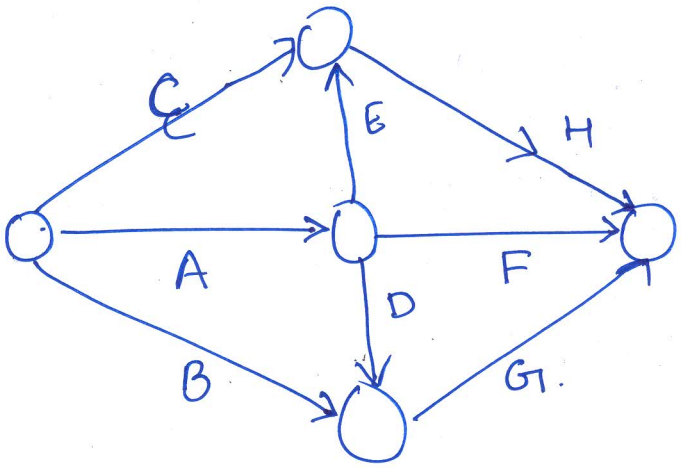
$$5y_1 - 3y_2 \leq 4$$

$$-3y_1 - 2y_2 \leq -3$$

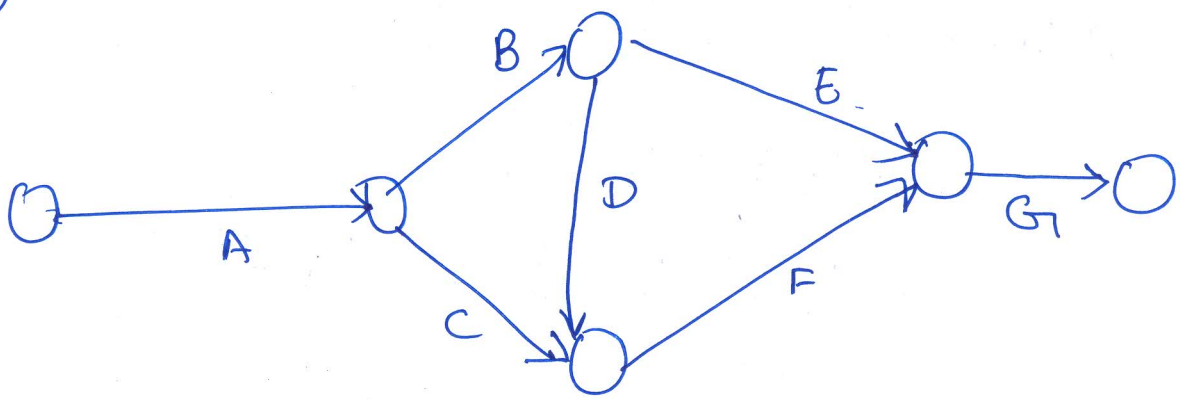
$$7y_1 \leq 6.$$

(6.)

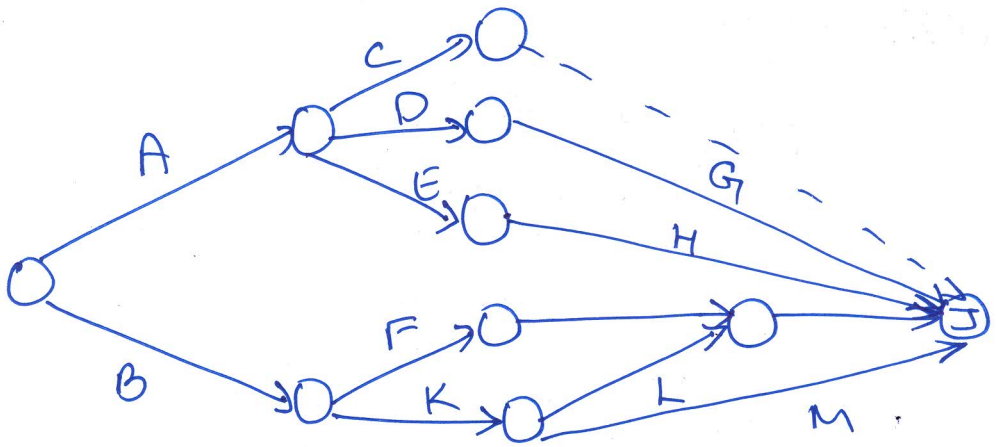
(a.)



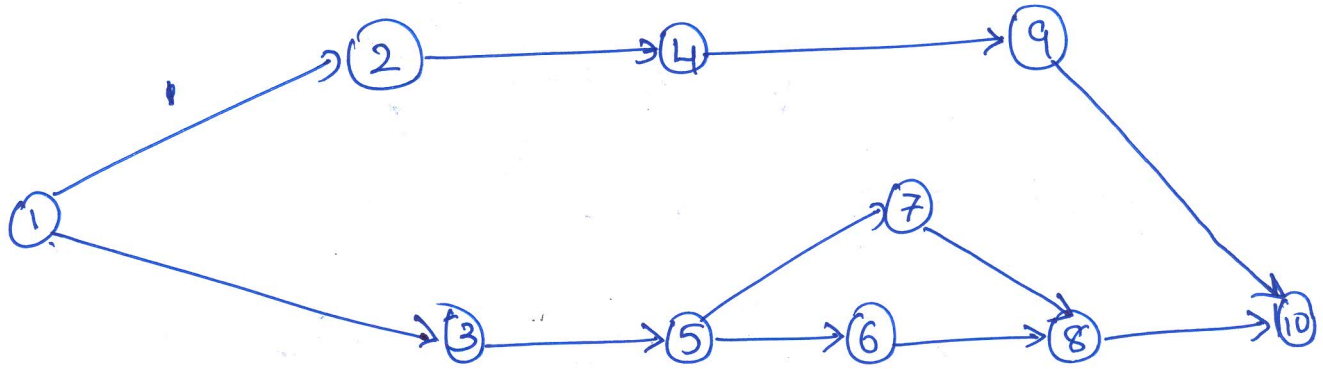

(b.)



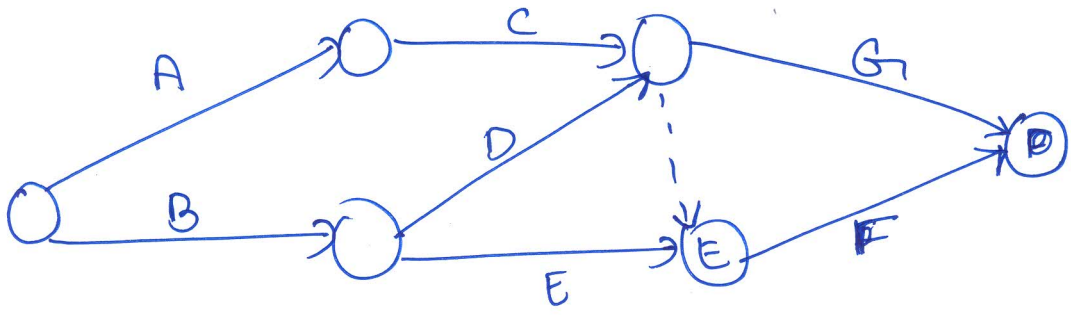

(c.)



(6.)




(7)



(8)