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Imrovement Test –May 2017

Sub:	Information Theory and Coding							Code:	10TE65
Date:	__ /05/2017	Duration:	90 mins	Max Marks:	50	Sem:	6A&B	Branch:	TCE

Answer Any Five Full Questions

		Marks	OBE CO	OBE RBT
1	Write a short note on ***RS codes*** and ***GOLAY codes***.	10	CO5	L1

*. Reed solomon codes

. RS codes are a class of maximum distance se

codes.

. Proposed by I.S. REED & G. solomon in the year

. Non binary codes

. A class of non binary BCH codes.

. The developments in VLSI processing technology m

codes pratically prominent in recent years, becau

-feasible decoding computations.

*. To show that RS codes as special case of

let us consider the primitive q-ary BCH codes

length $n = q^m - 1$ and the extension of field of

invoked for locating roots of generator polynom

let $m=1$, we get $n = q-1$, which governs the s

codes.

. For the primitive BCH codes, the generator po

$g(x) = \text{lcm}[\phi_{\alpha^j}(x), \phi_{\alpha^{j+1}}(x), \ldots \phi_{\alpha^{j+\delta-1}}(x)]$

Where $\phi_{\alpha^j}(x)$ = minimal polynomial of element

α^j = powers of a primitive element o

δ = desired value of d_{min}.

Now with $m=1$, however, the minimal polyno

of elements in $GF(q)$ are first degree of the for

$g(x)$ reduces to

$$g(x) = (x-\alpha^j)(x-\alpha^{j+1}) \ldots (x-\alpha^{j+\delta-2})$$

This is a polynomial over $GF(q)$ of

$(\delta-1)$. Thus $n-k = \delta-1$ for R-S codes.

$$\therefore \; n-k = \delta-1 \longrightarrow (3)$$

Since these are BCH codes, the true minim

must be atleast.

$$d_{min} \geq \delta = n-k+1 \Rightarrow d_{min} = n-k+1.$$

$\therefore$ The RS codes are "Maximum distance

i.e, for any (n,k) LBC, the minimum distance d_{m}

But, for a RS code, $d_{min} = n-k+1$.

The values of k may be any integer e

In eq^n (1) & (2) the starting exponent in the s

roots, is completly arbitrary and in commonly

1. GOLAY CODES

Golay codes are (23, 12) binary codes capable upto 3 errors in a block of 23 bit code word. It is a perfect code because it satisfies the H with equality sign for t = 3.

2 Write a note on ***Shortened cyclic codes*** and ***Burst error correcting codes***. 10 CO5 L3

Shortened Cyclic Codes

The cyclic codes have the generator polynomial as factors of x^n+1.

But the polynomial x^n+1 has relatively few di[visors] result there are usually very few cyclic codes fo[r]

To overcome this difficulty & to increase the [n] (n,k) for which useful codes can be constructed, are often used in shortened form.

In the shortened form, the last 'i' informat[ion] are always '0'. i.e., the last 'i' bits of the code are ~~d with~~ Hence the name zero padding codes.

These bits are not transmitted, the decoder [fo]r shortened cyclic codes can decode the ~~a~~ shortened code simply by padding the received (n-i) tuples of

Hence, for a given (n-k) cyclic code, it is alwa[ys possible to] construct a (n-i, k-i) shortened cyclic codes.

The shortened cyclic codes are a subject of th[e code] from which it is derived & hence its d_{min} & e[rror correcting] capability is atleast as great as the orginal co[de].

Advantages

* The encoding circuit, syndrome circuit & error [correction] procedures for shortened cyclic codes are identi[cal to] cyclic codes.

- All shortend cyclic codes inherit nearly all of
tion advantages of cyclic codes.
- The mathematical structure of cyclic codes serve as
shortend cyclic codes.

*. Burst error Correcting Codes

- The presence of impulse noise affects more than
or a bit to cause burst errors.
- Codes used for correcting random errors are
efficient for correcting burst errors.
- Special codes have been developed to correct

*. Burst of length q : Is defined as a vector whose components are confined to digit positions, the first & l non-zero.

Eg: The vector V = (00101001000) has a burst of

A code which is capable of correcting all lengths q or less is called q-burst-error cor and is said to be burst error correcting

The condition for a burst-q-correcting that the no of parity bits must atleast 2q.

ie., $n-k \geq 2q$

The upper bound on the burst error correcti of an (n,k) code is $\boxed{q \leq \frac{n-k}{2}}$ bits.

3	For the $(7,4)$ cyclic codes, $g(x) = 1 + x + x^3$ and $D(x) = d_0 + d_1x + d_2x^2 + d_3x^3$. Find all the 16 code vectors in ***Systematic*** and ***Non systematic*** form.	10	CO 5	L3

Non-systematic Cyclic Code

6

It is defined by $V(x) = D(x)\,g(x)$.

Eg 1: For a (7,4) sec, the generator polynomial is $g(x) = 1+x+x^3$, find all the valid possible c

i> Non-systematic form

ii> systematic form.

Sol^n: Given $(n,k) = (7,4) \rightarrow (1)$

$$\Rightarrow x^n+1 = x^7+1 \rightarrow (2)$$

$$g(x) = 1+x+x^3 \rightarrow (3)$$

$$D(x) = d_0 + d_1x + d_2x^2 + d_3x^3 \rightarrow (4)$$

Valid possible code comb^ns $= 2^k = 2^4 = 16.$

$\therefore$ The non-systematic cyclic codes are given by

$$V(x) = D(x)\,g(x) \rightarrow (5)$$

substituting eq^n (3) & (4) in (5) yields.

$$V(x) = (d_0 + d_1x + d_2x^2 + d_3x^3)(1+x+x$$

$$= d_0 + d_1x + d_2x^2 + d_3x^3$$

$$+ d_0x + d_1x^2 + d_2x^3 + d_3x^4$$

$$+ d_0x^3 + d_1x^4$$

$$V(x) = d_0x^0 + (d_0+d_1)x + (d_1+d_2)x^2 + (d_0+d$$

$$(d_1+d_3)x^4 + d_2x^5 + d_3x^6$$

$$V(x) = v_0(x^0) + v_1x + v_2x^2 + v_3x^3 + v_4x^4 + v_5$$

Possible valid $2^k = 2^4 = 16$ combination inputs (D) & its code vectors are

msg (D)				Code Vector						
d_0	d_1	d_2	d_3	v_0 d_0	v_1 (d_0+d_1)	v_2 (d_1+d_2)	v_3 $(d_0+d_2+d_3)$	v_4 (d_1+d_3)	v_5 d_2	v_6 d_3
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	1	0	1
0	0	1	0	0	0	1	1	0	1	0
0	0	1	1	0	0	1	0	1	1	1
0	1	0	0	0	1	1	0	1	0	0
0	1	0	1	0	1	1	1	0	0	1
0	1	1	0	0	1	0	1	1	1	0
0	1	1	1	0	1	0	0	0	1	1
1	0	0	0	1	1	0	1	0	0	0
1	0	0	1	1	1	0	0	1	0	1
1	0	1	0	1	1	1	0	0	1	0
1	0	1	1	1	1	1	1	1	1	1
1	1	0	0	1	0	1	1	1	0	0
1	1	0	1	1	0	1	0	0	0	1
1	1	1	0	1	0	0	0	1	1	0
1	1	1	1	1	0	0	1	0	1	1

Systematic Cyclic Codes

*. In systematic cyclic codes the left most check bits and the rest rightmost k bits are

*. To derive this set let us derive the gene in standard form.

i.e., $G = [P \vdots I_k]_{k \times n}$

$\Rightarrow G = [P \vdots I_4]_{4 \times 7}$

The first row of G matrix, can be deri generator polynomial as.

given $g(x) = 1 + x + x^3$

$\Rightarrow g(x) = 1.x^0 + 1.x + 0.x^2 + 1.x^3 + 0.x^4 + 0.$

since $(n=7)$

$\therefore$ The first row is 1101000

$|||^{ly}$ 3 more rows are obtained as follo

2nd row is generated using

$x\,g(x) = 0.x^0 + 1.x^1 + 1.x^2 + 0.x^3 + 1.$

$\Rightarrow$ The 2nd row is $0110100 \rightarrow x$

$|||^{ly}$ 3rd row is $0011010 \rightarrow x^2$

4th row is $0001101 \rightarrow x$

$$\Rightarrow G = \Bigg[\underbrace{\begin{matrix}1&1&0\\0&1&1\\0&0&1\\0&0&0\end{matrix}}_{P}\ \Bigg|\ \underbrace{\begin{matrix}1&0&0&0\\0&1&0&0\\1&0&1&0\\1&1&0&1\end{matrix}}_{I_{n-k}}\Bigg]_{k\times n}$$

since the augmented matrix, Identity matrix is standard form.

Performing Elementary row transformation

i.e., $R_3 \to R_3 + R_1$

$R_4 \to R_4 + R_2 + R_1$

$$\Rightarrow \quad G = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\underbrace{\qquad\qquad}_{I_4}$$

$$\Rightarrow \quad P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}_{4\times 3}$$

Now since the 'G' matrix is in standard form, code can be obtained as follows.

$$V = DG$$

$$= [d_0 \ d_1 \ d_2 \ d_3] \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \overset{v_0}{(d_0 + d_2 + d_3)} \ \overset{v_1}{(d_0 + d_1 + d_2)} \ \overset{v_2}{(d_1 + d_2 + d_3)} \ \overset{v_3}{(d_1)}$$

The valid codes and their msg vectors are as follows

d_0	d_1	d_2	d_3	v_0	v_1	v_2	v_3	v_4	v_5	v_6	d_0	d_1	d_2	d_3	v_0	v_1
0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0
0	0	0	1	1	0	1	0	0	0	1	1	1	0	0	1	0
0	0	1	0	1	1	1	0	0	1	0	1	1	0	1	0	0
0	0	1	1	0	1	0	0	0	1	1	1	1	1	0	0	1
1	0	0	0	1	1	0	1	0	0	0	1	1	1	1	1	1
1	0	0	1	0	1	1	1	0	0	1						
1	0	1	0	0	0	1	1	0	1	0						

4	Design an encoder for $(7, 4)$ binary cyclic code generated by $g(x) = 1 + x + x^3$ and verify its operation using the message vector (1001) and (1011).	10	CO 5	L4

Soln: In general we have generator polynomial as

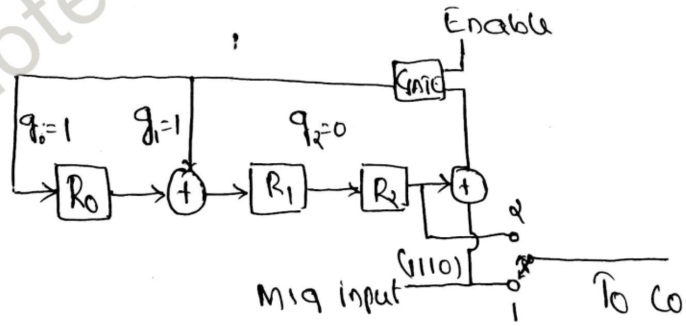
$$g(x) = g_0 + g_1 x + g_2 x^2 + \dots + g_{n-k} x^{n-k}$$

in the given data we have

$$g(x) = 1 + x + x^3$$

$\Rightarrow g_0 = 1,\ g_1 = 1,\ g_2 = 0\ \&\ g_3 = 1$

The encoding circuit for the given generator pc



Encoding circuit for (7,4) ~~given~~ BCC wire

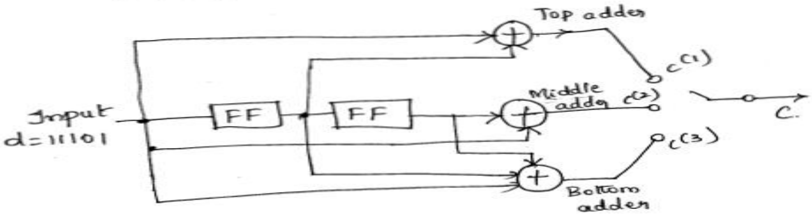
For the input msg vector D=[1110] the Operation in the table below.

No of shifts	Input D	Shift Register Contents R_0	R_1	R_2	Rema
switch position 1 Gate Enable					
		0	0	0	← power
1	0	0	0	0	-
2	1	1	1	0	-
3	1	1	0	1	-
4	1	0	1	0	
switch position 2 Gate Disable					
5	x	0	0	1	0
6	x	0	0	0	1
7	x	0	0	0	0

Explanation for the same in simple words is ex

5 Consider a $(3, 1, 2)$ convolution code with $g^{(1)} = (110), g^{(2)} = (101)$ and $g^{(3)} = (111)$. Draw the ***Encoding Circuit*** and obtain the ***Generator Matrix*** 10 CO5 L4

(3,1,2) convolution encoder.



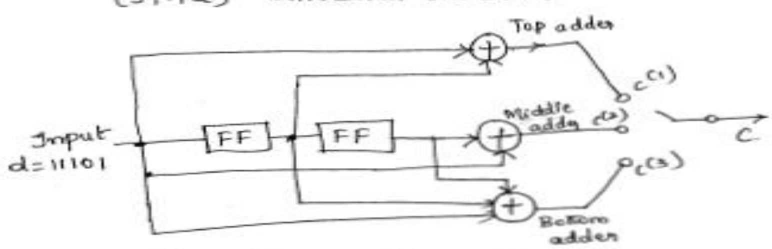
G is of order $L \times [n(L+m)]$
$5 \times [3(5+2)]$ = 5 rows × 21 columns.

given $g^{(1)} = 110$
$g^{(2)} = 101$
$g^{(3)} = 111$

$$\therefore G = \begin{bmatrix} 111 & 101 & 011 & 000 & 000 & 000 & 000 \\ 000 & 111 & 101 & 011 & 000 & 000 & 000 \\ 000 & 000 & 111 & 101 & 011 & 000 & 000 \\ 000 & 000 & 000 & 111 & 101 & 011 & 000 \\ 000 & 000 & 000 & 000 & 111 & 101 & 011 \end{bmatrix}_{5\times 21}$$

6	Consider a (3, 1, 2) convolution code with $g^{(1)} = (110), g^{(2)} = (101)$ and $g^{(3)} = (111)$. Find the code-word corresponding to the information sequence (11101) in ***Time Domain*** approach and ***Transform Domain*** approach	10	CO5	L4

(3,1,2) convolution encoder.



G is of order $L \times [n(L+m)]$
$5 \times [3(5+2)]$ = 5 rows × 21 columns.

given $g^{(1)} = 110$
$g^{(2)} = 101$
$g^{(3)} = 111$

$$\therefore G = \begin{bmatrix} 111 & 101 & 011 & 000 & 000 & 000 & 000 \\ 000 & 111 & 101 & 011 & 000 & 000 & 000 \\ 000 & 000 & 111 & 101 & 011 & 000 & 000 \\ 000 & 000 & 000 & 111 & 101 & 011 & 000 \\ 000 & 000 & 000 & 000 & 111 & 101 & 011 \end{bmatrix}_{5\times 21}$$

$\therefore [C] = [D]\,[G]$

$$= [11101] \begin{bmatrix} 111 & 101 & 011 & 000 & 000 & 000 & 000 \\ 000 & 111 & 101 & 011 & 000 & 000 & 000 \\ 000 & 000 & 111 & 101 & 011 & 000 & 000 \\ 000 & 000 & 000 & 111 & 101 & 011 & 000 \\ 000 & 000 & 000 & 000 & 111 & 101 & 011 \end{bmatrix}_{5\times 21}$$

$\therefore [c] = [111 \;\; 010 \;\; 001 \;\; 110 \;\; 100 \;\; 101 \;\; 011]$

Transfer Domain Method.

Given : $[d] = [11101] \Rightarrow d(x) = 1 + x + x^2 + x^4$

$g^{(1)} = (110) \rightarrow g^{(1)}(x) = 1 + x$

$g^{(2)} = (101) \rightarrow g^{(2)}(x) = 1 + x^2$

$g^{(3)} = (111) \rightarrow g^{(3)}(x) = 1 + x + x^2$

$\therefore$ we have the equ^n

$$\boxed{C(x) = C^{(1)}(x^n) + x\, C^{(2)}(x^n) + x^2 C^{(3)}(x^n)}$$

Before that we find $[C] = [D]\,[G]$ or

for top adder $\Rightarrow C^{(1)}(x) = d(x)\, g^{(1)}(x)$

$= 1 + x + x^2 + x^4 \,(1 + x)$

$c^{(1)}(x) = 1 + \cancel{x} + \cancel{x} + \cancel{x^2} + \cancel{x^2} + x^3 + x^4 + x^5$

$c^{(1)}(x) = 1 + x^3 + x^4 + x^5$.

For ~~bottom~~ middle adder $\Rightarrow c^{(2)}(x) = d(x)\, g^{(2)}(x)$

$= 1 + x + x^2 + x^4 \,(1 + x^2)$

$= 1 + \cancel{x^2} + x + x^3 + \cancel{x^2} + \cancel{x^4} + \cancel{x^4} + x^6$

$c^{(2)}(x) = 1 + x + x^3 + x^6$

for bottom adder $\Rightarrow c^{(3)}(x) = d(x)\, g^{(3)}(x)$

$= 1 + x + x^2 + x^4 \,(1 + x + x^2)$

$c^{(3)}(x) = 1 + \cancel{x} + \cancel{x^2} + \cancel{x} + \cancel{x^2} + \cancel{x^3} + x^2 + \cancel{x^3} + \cancel{x^4} + \cancel{x^4} + x^5 + x^6$

$c^{(3)}(x) = 1 + x^2 + x^5 + x^6$.

$\therefore$ for encoder equ^n

$C(x) = c^{(1)}(x^3) + x\, c^{(2)}(x^3) + x^2 c^{(3)}(x^3)$

$= 1 + x^9 + x^{12} + x^{15} + x(1 + x^3 + x^9 + x^{18}) + x^2(1 + x^6 + x^{15} + x^{18})$

$= 1 + x^9 + x^{12} + x^{15} + x + x^4 + x^{10} + x^{19} + x^2 + x^8 + x^{17} + x^{20}$

$= 1 + x + x^2 + x^4 + x^8 + x^9 + x^{10} + x^{12} + x^{15} + x^{17} + x^{19} + x^{20}$

$C(x) \Rightarrow$ o/p of encoder is $[111\ 010\ 001\ 110\ 100\ 101\ 011]$

$\therefore$ Transfer Domain & Time Domain Codevectors are same.

7 Consider the binary convolution encoder shown in Fig. 7. Draw the state table, state transition table, state diagram and corresponding code tree. Using the code tree find the encoded sequence for (10111). 10 CO5 L4

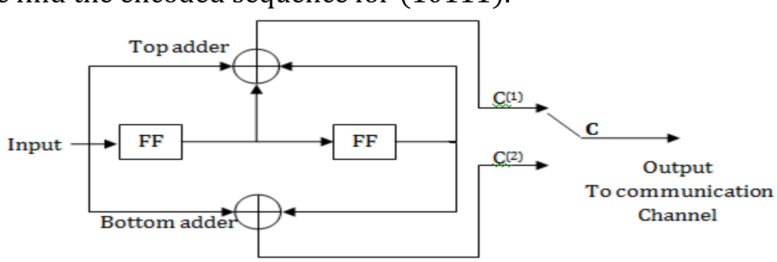
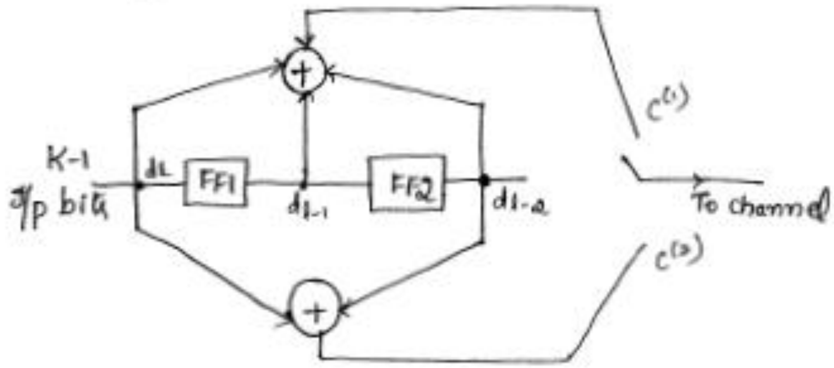


Fig. 7. (2, 1, 2) Convolution encoder.

State table, State transition table, State diagram, Code tree.

1. Draw the state transition table, state diagram & code tree for (2,1,2) Convolution encoder with impulse response $g^{(1)} = 111$, $g^{(2)} = 101$ & input $D = [10111]$

Soln



① State table

There are 2 flip flops in the shift register i.e 2
∴ there are $2^2 = 4$ States with binary data, that represented in the table below

State	S_0	S_1	S_2	S_3
Binary Description	00	10	01	11

② State Transition Table.

It is a table indicating the transition between the States and also the corresponding code bits output during each transition.

The output of the mod-2 adders can be represented as

$$c^{(1)} = d_l + d_{l-1} + d_{l-2}$$
$$c^{(2)} = d_l + d_{l-2}$$

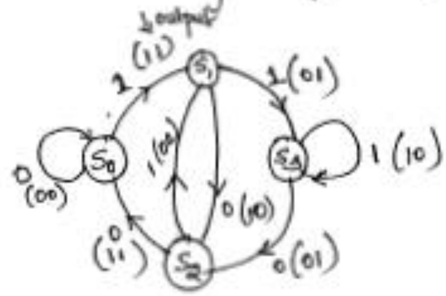
present

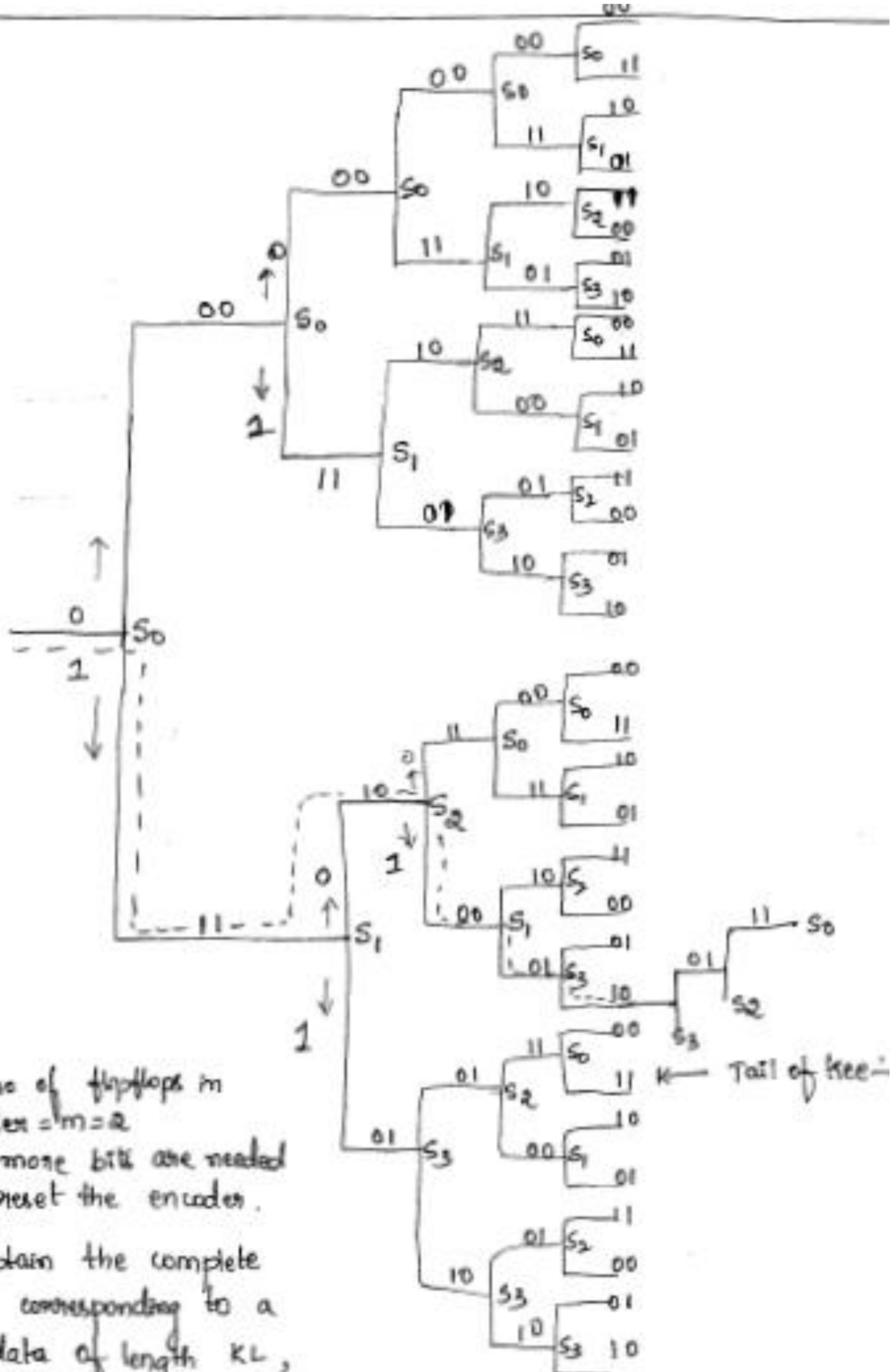
Present State	Input bit	Next State	d_l	d_{l-1}	d_{l-2}	Output $c^{(1)}$	$c^{(2)}$
$S_0 = 00$	0	$S_0 = 00$	0	0	0	0	0
	1	$S_1 = 10$	1	0	0	1	1
$S_1 = 10$	0	$S_2 = 01$	0	1	0	1	0
	1	$S_3 = 11$	1	1	0	0	1
$S_2 = 01$	0	$S_0 = 00$	0	0	1	1	1
	1	$S_1 = 10$	1	0	1	0	0
$S_3 = 11$	0	$S_2 = 01$	0	1	1	0	1
	1	$S_3 = 11$	1	1	1	1	0

Construction: Initially, let the flip flops be cleared i.e '00'.

- If i/p = 0 ⇒ shift register at '00' → S_0, ∴ o/p of add $c^{(1)} c^{(2)} = 00$
- If i/p = 1 the flip flop FF_1 changes state from 0 to 1 the contents of FF_1 (which was '0') get shifted to FF_2.
  ∴ The new shift register contents are '10' → S_1
  this transition is caused by a i/p = 1 & $d_l d_{l-1} d_{l-2} = 100$ & $c^{(1)} c^{(2)} = 11$.

(3) State diagram.





- The no of flipflops in encoder = m = 2

∴ 2 more bits are needed to reset the encoder.

To obtain the complete code corresponding to a i/p data of length KL, the tree graph has to be extended by n(m-1) time units & this extended part is called 'tail of the tree'