CMR
INSTITUTE OF
TECHNOLOGY

Improvement Test – Jun 2017

Sub:	Antennas and Propagation							**Code:**	10EC64
Date:	01/ 06/ 2017	Duration:	90 mins	Max Marks:	50	**Sem:**	VI	**Branch:**	ECE

Answer ANY FIVE questions

Answer legibly and draw the diagrams neatly. Give proper units wherever necessary.

			OBE	
			CO	RBT
1.	Derive the expression for resultant field strength at a point due to space wave propagation.	10	CO6	L2

Ground Reflection

If the two antennas are situated close to the ground, due to the discontinuity in the electrical properties at the air-ground interface, any wave that falls on the ground is reflected.

— Reflection amount depends on angle of incidence, polarization of the wave, electrical properties (conductivity, dielectric constant).

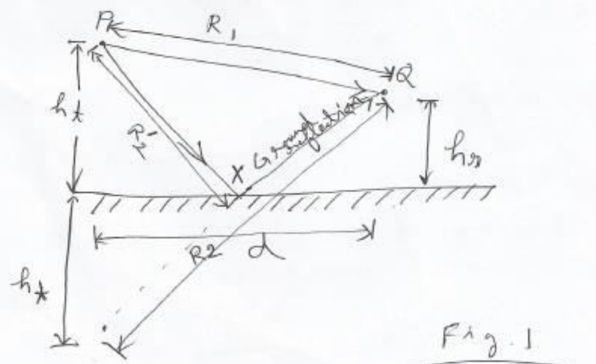


Fig. 1

Direct and ground-reflected paths of wave-propagation consider a transmit antenna located at point P at a height h_t and a receive antenna Q at a height h_R from the surface of the ground.

- Let the horizontal distance b/w the antennas be d

- (a) The direct path
  (b) Ground reflected path.

- The total electric field at any point Q is given by the vector sum of the electric fields due to the direct and ground reflected path.

- Assumed that t_x antenna and the field point are located in the y-z plane.

- The t_x antenna is located along ~~plane~~ x-axis.

- The electric field of an infinitesimal dipole oriented along the x-axis is

$$\vec{E} = -jk\eta \frac{I_0 dl}{4\pi} \frac{e^{-jkR}}{R} \left(\hat{a}_\theta \cos\theta \cos\phi - \hat{a}_\phi \sin\phi\right) \quad \text{--(1)}$$

In the yz plane, $\phi = 90^\circ$.

$\because \cos 90^\circ = 0$, the θ- component of the electric-field is zero.

- The ϕ- comp. of the electric field at Q due to the direct wave is

$$E_1 = jk\eta \frac{I_0 dl}{4\pi} \frac{e^{-jkR_1}}{R_1} \quad \text{(2)}$$

- The field at Q also has contribution from the wave that travels via the reflected path PXQ.

- The location of X depends on h_t, h_r and d.

- At X, the incident and reflected rays satisfy Snell's law of reflection.
(i.e. the angle of incidence is equal to angle of reflection)

- YZ-plane is the plane of incidence.

- The incident field at X is,

$$E_i = jk\eta \frac{I_0 dl}{4\pi} \frac{e^{-jkR_2'}}{R_2'} \quad \cdots (3)$$

R_2' → distance from the transmitter to X

Assume incident $\vec{E}$ field vector is $\perp^r$ to the plane of incidence.

∴ At X, the reflection coefficient,

$$\Gamma_\perp = \frac{E_r}{E_i} = \frac{\sin\psi - \sqrt{(\epsilon_r - j\chi) - \cos^2\psi}}{\sin\psi + \sqrt{(\epsilon_r - j\chi) - \cos^2\psi}} \quad \cdots (4)$$

where, $\chi = \frac{\sigma}{\omega\epsilon_0}$

σ → conductivity of the ground.

ω → angular freq. in rad/s.

ψ → grazing angle of incidence.

Grazing angle(ψ) :— The angle of the incident wave w.r.t the horizontal.

The electric field at the receiver due to the reflected wave is

$$E_2 = jk\eta\, \Gamma_\perp \frac{I_0 dl}{4\pi} \frac{e^{-jkR_2}}{R_2} \quad \cdots (5)$$

This is the field at Q_2 due to an equivalent (image) dipole having a strength $I_0 dl \Gamma_\perp$ located at $(0, 0, -h_t)$

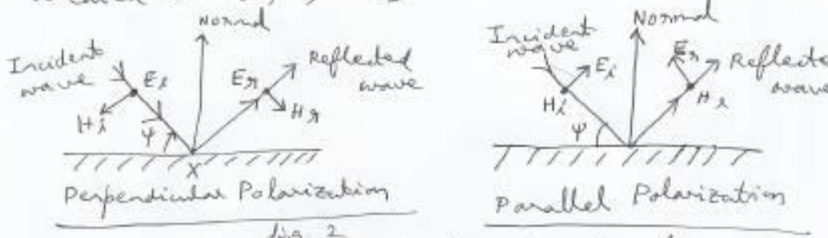


fig 2

The total electric field at Q is given by,

$$E = E_1 + E_2 = j k \eta \frac{I_0 dl}{4\pi} \left(\frac{e^{-jkR_1}}{R_1} + \Gamma_\perp \frac{e^{-jkR_2}}{R_2} \right) \quad \text{--- (6)}$$

If the field point Q is far away from the transmitter, (when R_1 and R_2 are in the denominator)

$$R_1 \simeq R_2$$

$$\therefore E = E_1 + E_2 = jk\eta \frac{I_0 dl}{4\pi} \cdot \frac{e^{-jkR_1}}{R_1} \left(1 + \Gamma_\perp e^{-jk(R_2 - R_1)} \right)$$

$\therefore$ The total field is the product of the free-space field and and an environmental factor, $F_\perp$, given by, (7)

$$F_\perp = \left(1 + \Gamma_\perp e^{-jk(R_2 - R_1)} \right) \quad \text{--- (8)}$$

Next, consider an infinitesimal dipole at $(0, 0, h_t)$ oriented along z-direction.

The electric field of an z-directed infinitesimal dipole is

$$\vec{E} = \hat{a}_\theta \, j\eta \frac{k I_0 dl \sin\theta}{4\pi} \frac{e^{-jkR_1}}{R_1} \quad \text{--- (9)}$$

$R_1 \rightarrow$ distance from the antenna to the field point.

The electric field is $\|^{l}$ to the plane of incidence and the reflection co-efficient $\Gamma_{\|}$, at X is given by,

$$\Gamma_{\|} = \frac{(\epsilon_r - jx)\sin\psi - \sqrt{(\epsilon_r - jx) - \cos^2\psi}}{(\epsilon_r - jx)\sin\psi + \sqrt{(\epsilon_r - jx) - \cos^2\psi}} \quad \text{--}(10)$$

where, $x = \frac{\sigma}{\omega\epsilon_0}$.

$\therefore$ The total field at point Q is,

$$E = jk\eta \frac{I_0 dl}{4\pi}\left(\frac{e^{-jkR_1}}{R_1} + \Gamma_{\|}\frac{e^{-jkR_2}}{R_2}\right)$$

$$\Rightarrow E = jk\eta \frac{I_0 dl}{4\pi} \frac{e^{-jkR_1}}{R_1} F_{\|} \quad \text{--}(11)$$

where, $F_{\|} = \left(1 + \Gamma_{\|} e^{-jk(R_2 - R_1)}\right)$ --(12)

From fig. 1

$$R_1^2 = d^2 + (h_r - h_t)^2$$

$$\Rightarrow R_1 = \sqrt{d^2 + (h_r - h_t)^2} = d\sqrt{1 + \left(\frac{h_r - h_t}{d}\right)^2}$$

For $d \gg h_r$ and $d \gg h_t$, using the first two terms in binomial expansion of $\sqrt{1+x}$, $\sqrt{1+x} \simeq 1 + \frac{x}{2}$ for $x \ll 1$.

$$\therefore R_1 \simeq d\left[1 + \frac{1}{2}\left(\frac{h_r - h_t}{d}\right)^2\right] \quad \text{-}(13)$$

Similarly, R_2 can be expressed as,

$$R_2 \simeq d\left[1+\frac{1}{2}\left(\frac{h_r+h_t}{d}\right)^2\right] \quad \cdots (14) \qquad R_2^2 = d^2+(h_r+h_t)^2$$

The path diff,

$$R_2 - R_1 = \frac{1}{2}\left(\frac{h_r+h_t}{d}\right)^2 - \frac{1}{2}\left(\frac{h_r-h_t}{d}\right)^2$$

$$= \frac{d}{2d^2}\left[h_r^2+h_t^2+2h_rh_t - h_r^2-h_t^2+2h_rh_t\right]$$

$$= d\frac{2h_rh_t}{d^2} = \frac{2h_rh_t}{d} \quad \cdots (15)$$

For $\frac{h_rh_t}{d} << \lambda$,

$$\Delta\theta = k(R_2-R_1) = \frac{2\pi}{\lambda}\cdot\frac{2h_rh_t}{d} = \frac{4\pi h_rh_t}{d\lambda} \quad \cdots (16)$$

is small, so that,

$\sin x \simeq x$ and $\cos x \simeq 1$.

$$\therefore e^{-jk(R_2-R_1)} = e^{-j\Delta\theta} = \cos(\Delta\theta) - j\sin(\Delta\theta)$$

$$\simeq 1 - j\frac{2kh_rh_t}{d} \quad \cdots (17)$$

For low angles of incidence,

$$\Gamma_\perp \simeq \Gamma_\parallel \simeq -1$$

$$\therefore F = F_\perp = F_\parallel$$

$$= \left(1 + \Gamma_\parallel e^{-jk(R_2-R_1)}\right)$$

$$= \left[1 - \left(1 - j\frac{2kh_rh_t}{d}\right)\right]$$

$$= j\frac{2kh_rh_t}{d} \quad \cdots (18)$$

	considering the ground reflection, the power received by the receive antenna can be written as, $P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R_1}\right)^2 \|F\|^2$ For h_r and h_t small compared to d, $R_1 \approx d$. ∴ The received power is approximately given by, $P_r \simeq P_t G_t G_r \frac{\lambda^2}{(4\pi)^2 d^2} \frac{(4\pi)^2 h_r^2 h_t^2}{d^2 \lambda^2}$ $= \frac{P_t G_t G_r (h_r h_t)^2}{d^4}$. ∴ For large d, the received power decreases as d^4, This rate of change of power with distance is much faster than that observed in free-space propagation condition.			
2.(a)	Explain the principle of surface wave propagation. Obtain an equation for the tilt angle α for the wave. Surface-Wave tilting :- - Surface wave glide over the surface. - Waves are vertically polarized and induce charge in earth. polarity of these charges keep on - Number and location of wave-fiel changing with intensity and location ∴ Their variation constitute a current.	6	CO6	L2

- In carrying this current, earth behaves like a leaky capacitor.
- As wave propagates power loss in earth's resistance and wave becomes ~~weak~~ weaker.
- So, power keeps on reducing as the wave propagates.
- Also wave front tilt towards forward direction.
- It happens as earth is not a perfect conductor, (finite σ)

$\therefore$ There is a horizontal component of $\vec{E}$.

Tilt angle (α),

$$\tan(\alpha) = \frac{E_H}{E_V}$$

$$E_H = Z_S J_S = Z_s H$$

$$E_V = Z_0 H$$

$$\therefore \tan\alpha = \frac{Z_S J_S}{Z_0 H} = \frac{Z_s H}{Z_0 H} = \frac{Z_s}{Z_0}$$

$$\Rightarrow \boxed{\tan\alpha = \frac{Z_s}{Z_0}}$$

Z_s = Earth's surface impedance

Z_0 = atmosphere (free space) impedance $= \sqrt{\frac{\mu_0}{\epsilon_0}}$.

$$Z_s = \frac{\sqrt{j\omega\mu}}{\sqrt{\sigma + j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{1}{\sqrt{1 + \frac{\sigma}{j\omega\epsilon}}}$$

As the earth is non-magnetic, $\mu = \mu_0$, $\epsilon \doteq \epsilon_0 \epsilon_r$

$$\therefore Z_s = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} \cdot \frac{1}{\sqrt{1 + \frac{\sigma}{j\omega\epsilon}}}$$

$$= \cancel{\frac{Z_0}{\sqrt{\epsilon_r}\left(1 - \frac{\sigma}{j\omega\epsilon}\right)}} = \frac{Z_0}{\sqrt{\epsilon_r} \cdot \sqrt{1 + \frac{\sigma}{j\omega\epsilon}}} = \frac{Z_0}{\sqrt{\epsilon_r}\sqrt{\left(1 - \frac{j\sigma}{\omega\epsilon}\right)}}$$

$$\therefore |Z_s| = \frac{Z_0}{\sqrt{\epsilon_r}} \frac{1}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$$

Let, $\chi = \frac{\sigma}{\omega\epsilon}$

$$\therefore |Z_s| = \frac{Z_0}{\sqrt{\epsilon_r}\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$$

$$\therefore \tan\alpha = \frac{Z_s}{Z_0} = \frac{1}{\sqrt{\epsilon_r}\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$$

$$\Rightarrow \alpha = \tan^{-1}\left[\frac{1}{\sqrt{\epsilon_r}(1 + \chi^2)^{1/4}}\right]$$

$\rightarrow \alpha$ reduced if χ ~~reduced or~~ increased.
i.e. operating freq. reduced.

(b)	Calculate the wave tilt in degrees of the surface wave over an earth of 6 milimho conductivity and relative permittivity of 12 at 2 MHz.	4	CO6	L3

$$x = \frac{\sigma}{\omega \epsilon} = \frac{\sigma}{\omega \epsilon_0 \epsilon_r} = \frac{6 \times 10^{-3}}{2\pi \times 2 \times 10^{6}\ 12 \times 8.85 \times 10^{-12}}$$

$$= \frac{4.4938 \times 10^{-3} \times 10^{-3}}{10^{-6}}$$

$$= 4.49.$$

$$\therefore \alpha = \tan^{-1}\left(\frac{1}{\sqrt{\epsilon_r}\,(1 + x^2)^{1/4}}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{12}\ (21.25)^{1/4}}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{12} \times \sqrt{21.25}}\right)$$

$$= \tan^{-1}(0.13445) = 7.657^{o}$$

3.(a)	What are the factors affecting ground propagation. Explain the Somerfield equation for ground wave propagation.	7	CO6	L2

- Till now we assumed that antennas are situated in infinite free space. But practically it is not the case.
- The factors which affect the propagation of radio waves in actual environment are following:

(1) Spherical Shape of Earth
- waves travel in a straight line path in free space
- communication between two points is limited to horizon
- So, to establish a comm. link beyond the horizon, the radio waves need to undergo a change in the direction of propagation.

(2) Atmosphere :-

The earth's atmosphere extends upto 600 km.
- It is divided in several layers:
  - Troposphere (< 15 km)
  - Stratosphere (15 > km to 80 km)
  - ionosphere (> 80 km)

- Near the earth surface, wave propagation is affected by troposphere.
- for some frequencies, ionosphere acts as a reflecting surface, so it is used for propagation.

(3) Interaction with the objects on the ground :-
- Near the surface of earth, many obstacles are there: buildings, trees, hills, water bodies etc.
- This results in phenomena of reflection, refraction, diffraction and scattering.

'Interaction of wave/fields with medium and discontinuities' results in

(I) Reflection (II) Refraction (III) Diffraction (IV) scattering.

Reflection, Refraction:-

Snell's laws of reflection and refraction.

" Optics."

$\theta_i = \theta_r$ (Reflection)

$\epsilon_{r_1} \sin\theta_i = \epsilon_{r_2} \sin\theta_r$ (Refraction)

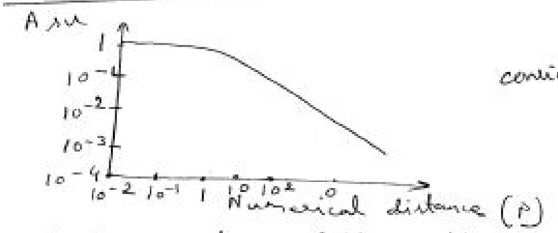
Bending of path of wave in inhomogeneous medium is known as "refraction"

These laws are applicable if radius of curvature of the interface as well as that of wavefront is large compared to the wavelength of operation.

Diffraction:- Whenever an edge is encountered by the electromagnetic fields, the induced currents can flow around the edge to the opposite side of surface and produce fields

Scattering:- The interaction of fields with discontinuities or inhomogeneities which are small compared to wavelength is generally known as 'scattering'. Ex:- due to presence of rain drop.

Ground-wave Attenuation factor



considered, $b = 0°$

At the surface of the earth, the attenuation factor is also known as the ground wave attenuation factor, A_{su}.

$$p = \frac{\pi R}{\lambda x} \cos b$$

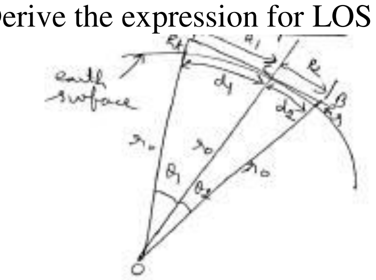
where, $R \to$ distance b/w T_X and R_X antenna

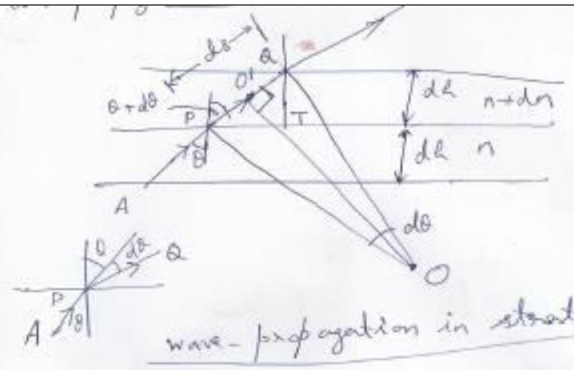
$\lambda \to$ wavelength

$x = \frac{\sigma}{\omega \epsilon_0}$ = loss factor = $\frac{J_c}{J_d}$

b = power factor angle

$$b = \tan^{-1}\left(\frac{\epsilon_r + 1}{x}\right)$$

	According to Sommerfeld eqn. $E = E_0 \frac{A_{su}}{R}$ As $A_{su} \propto \frac{1}{p}$ and $p \propto R$ i.e. $A_{su} \propto \frac{1}{R}$ $\therefore E \propto \frac{1}{R^2}$ as $R\uparrow$ $E\downarrow$ $f\uparrow$ $A_{su}\downarrow$ and $E\downarrow$ $f\downarrow$ cond$\uparrow \Rightarrow p\uparrow \Rightarrow A_{su}\downarrow$ $f\uparrow$ cond$\downarrow \Rightarrow p\downarrow \Rightarrow A_{su}\uparrow$ ∴ surface wave propagation is limited to VLF and LF. – Because of low attenuation in case of vertically polarized wave it is preferred.			
(b)	Derive the expression for LOS distance between transmitting and receiving antenna.  Propagation path of radio waves over the surface of the earth with $\frac{dN}{dh} = 0$ In ΔAOC $R_1^2 = (r_0 + h_t)^2 - r_0^2 = 2r_0 h_t + h_t^2$ ∵ $r_0 >> h_t$, $R_1^2 \simeq 2r_0 h_t$ similarly from ΔBOC, $R_2^2 \simeq 2r_0 h_r$ ∴ The LOS dist. b/w the tx and Rx antenna is, $\boxed{R = R_1 + R_2 = \sqrt{2r_0}(\sqrt{h_t} + \sqrt{h_r})}$	3	CO6	L2
4.	In case of tropospheric propagation, show that the radius of curvature of the path of an electromagnetic wave is a function of the rate of change of dielectric constant with height. Also explain duct propagation of wave.	10	CO6	L2



wave-propagation in stratified medium

- Earth is flat
- Tropo sphere is made up of stratified layers parallel to the surface of the earth.

i.e. r.i is a fn. of height only.

- A ray incident from the lower layers at point P is refracted through the layer dh and touches the upper layer at point Q.

$\theta \rightarrow$ angle of incidence with the normal drawn to the plane at P

$(\theta+d\theta) \rightarrow$ angle of refraction at P.

Aim: To calculate radius of curvature r of the ray.

- Draw angle-bisectors at P and Q.

let them meet at O.

$\angle APQ = \pi - d\theta$ -- (1)

OP bisects angle $\angle APQ$.

$\therefore \angle OPQ = \frac{1}{2}\angle APQ = \frac{\pi}{2} - \frac{d\theta}{2}$.

Draw $OO' \perp^r$ to PQ.

In $\Delta POO'$, $\angle POO' = \frac{\pi}{2} - \angle OPQ$

∴ The angle subtended by the segment ds at O is,

$\angle POQ = 2\angle POO' = d\theta$ ... (3)

∴ $ds = r\,d\theta$. ... (4)

From ΔPQT,

$\frac{dh}{ds} = \sin(90 - (\theta + d\theta)) = \cos(\theta + d\theta)$

$\Rightarrow ds = \frac{dh}{\cos(\theta + d\theta)} \simeq \frac{dh}{\cos\theta}$ for small $d\theta$ ... (5)

Using eqn. (4),

$\Rightarrow r = \frac{ds}{d\theta} = \frac{dh}{\cos\theta\, d\theta}$ ... (6)

According to law of refraction,

$n \sin\theta \simeq (n + dn)\sin(\theta + d\theta)$

$\Rightarrow n\sin\theta = (n + dn)(\sin\theta\cos d\theta + \cos\theta \sin d\theta)$

For small $d\theta$, $\cos d\theta \simeq 1$, $\sin d\theta = d\theta$

$\Rightarrow n\sin\theta \simeq (n + dn)(\sin\theta + d\theta\cos\theta)$

~~$\Rightarrow n\sin\theta \simeq n\,d\theta + n\cos\theta + dn\cos\theta + dn\,d\theta$~~

∴ $(n\cancel{\sin\theta}) \simeq n\cancel{\sin\theta} + n\cos\theta\, d\theta + dn\sin\theta + dn\,d\theta\cos\theta$

ignored.

∴ $n\cos\theta\, d\theta = -dn\sin\theta$

$\Rightarrow \cos\theta\, d\theta = -\sin\theta \frac{dn}{n}$

Substituting this into eqn. (6),

$$\boxed{r = \frac{dh}{-\sin\theta\frac{dn}{n}} = \frac{n}{\sin\theta\left(-\frac{dn}{dh}\right)}} \quad \text{... (7)}$$

In terms of angle of incidence, the radius of curvature,

$$r = \frac{n}{\sin\left(\frac{\pi}{2} - \phi\right)\left(-\frac{dn}{dh}\right)} = \frac{n}{\cos\phi\left(-\frac{dn}{dh}\right)} \quad \text{--- (8)}$$

for $\phi \approx 0$

$$r = \frac{1}{-\frac{dn}{dh}} \quad \text{--- (9)}$$

[$\because$ the n is also very close to unity]

In terms of refractivity gradient,

$$N = (n-1)\times 10^{6} = n\times 10^{6} - 1\times 10^{6}$$

$$\Rightarrow \frac{d(n \times 10^{6})}{dh} = \frac{dN}{dh}$$

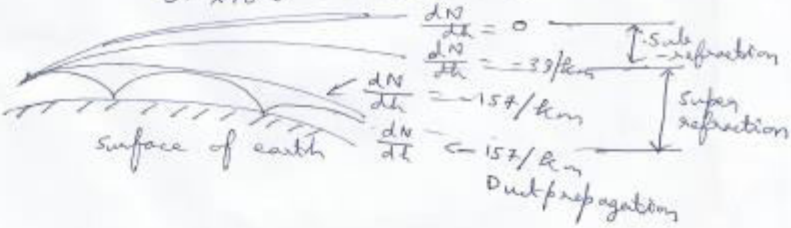
$$\Rightarrow \frac{dn}{dh} = \frac{dN}{dh}\times 10^{-6}$$

$$\therefore r = \frac{1}{-\frac{dn}{dh}} = \frac{10^{6}}{-\frac{dN}{dh}} \quad \text{--- (10)}$$

If the ray is propagating in standard atmosphere

$$\frac{dN}{dh} = -39 \text{ per km}$$

$$\therefore r = \frac{1}{39\times 10^{-6}} = 25641 \text{ km}$$



If ~~$0 \le \frac{dN}{dh} < 39/km$~~ $\frac{dN}{dh}$ is within 39/km and 0, the refraction of em. wave is lower than that in standard atmosphere → sub-refraction

If $\frac{dN}{dh}$ is less than that of standard atmosphere, the wave is refracted i.e. $\frac{dN}{dh} < -39/km$, the wave is refracted in standard atmosphere and is more than in standard atmosphere and is known as super refraction.

for $\frac{dN}{dh} = -157/km$,

$$r = \frac{1}{157 \times 10^{-6}} \simeq 6370 \text{ km}.$$

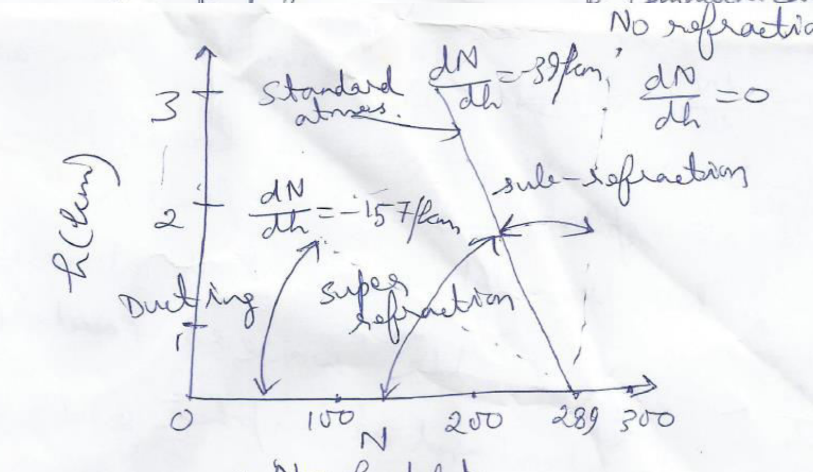
which is equal to the radius of the earth.

∴ Horizontally incident wave travells parallel to the surface of the earth.

If $\frac{dN}{dh} < -157/km$ — Ray can touch the surface of the earth and get reflected from the surface.

— This is known as ~~troposcopic~~ tropospheric duct propagation.

communication distance



A N – h plot

5.	Write short notes on : i. MUF — Maximum usable frequency: The highest freq that gets reflected by the ionosphere for a given value of angle of incidence (θ_m) is known as the maximum usable freq, f_{MUF}. Substituting $\theta_i = \theta_m$ and $f = f_{MUF}$, $\boxed{\sin\theta_m = \sqrt{1 - \frac{81N}{f_{MUF}^2}}}$ as $\theta_m > 0$, L.H.S. increases	4 +4 +2	CO6	L2

From eqn ⑤, $81N = f_{cr}^2$.

$\therefore \sin\theta_m = \sqrt{1 - \frac{f_{cR}^2}{f_{MUF}^2}}$

$\Rightarrow \sin^2\theta_m = 1 - \frac{f_{cR}^2}{f_{MUF}^2}$

$\Rightarrow 1 - \sin^2\theta_m = \cos^2\theta_m = \frac{f_{cR}^2}{f_{MUF}^2}$

$\Rightarrow \boxed{\cos^2\theta_m = \frac{f_{cR}^2}{f_{MUF}^2}}$

∴ The critical freq. and the maximum usable freq are related by the expression,

$$\boxed{f_{MUF} = f_{cR} \sec\theta_m}$$

If $f_{cr} = 9\ MHz$, for $\theta_m = 45°$,

$f_{MUF} = 9 \times \sec 45° = 12.73\ MHz$.

ii. Skip distance

Skip Distance

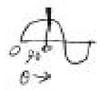
In the region of distance less than D_{skip}, it is not possible to establish a communication link by the waves reflected from the ionosphere.

— Assumed that ionosphere can be modelled as a flat reflecting surface at a height h (virtual height) from the surface of the flat earth.

— Let $\theta_m \rightarrow$ angle of incidence of a wave of freq. f_{MUF} which gets reflected from the ionosphere.

— If angle of incidence $> \theta_m$ the wave is reflected back by ionosphere.

— For $\theta_i > \theta_m$ $\quad \sin\theta_i = \sqrt{1 - \frac{81N}{f^2}}$

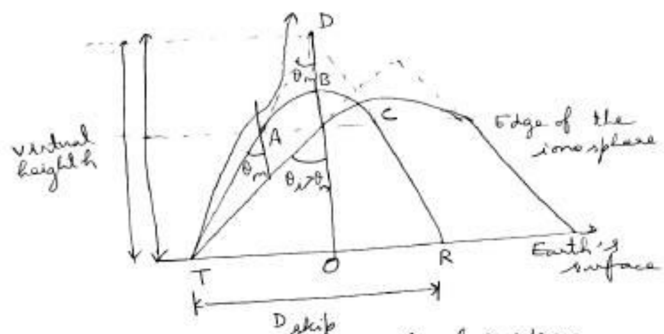


is satisfied for ~~larger~~ lower values of N, which occurs at a height less than h.

— for $\theta_i < \theta_m$, ionosphere can't reflect the wave back.

— Let when $\theta_i = \theta_m$ reach the surface of the earth at P, at a distance of D_{skip} from the transmitter.

— The distance D_{skip} is known as the skip distance



Ray-paths for different angles of incidence, illustrating skip distance.

From ΔDOT,

$$\left(\frac{DO}{DT}\right)^2 = \cos^2\theta_m = \left[\frac{h}{\sqrt{h^2+\left(\frac{D_{skip}}{2}\right)^2}}\right]^2$$

$$= \left\{\frac{1}{\left(1+\left(\frac{D_{skip}}{2h}\right)^2\right)^{1/2}}\right\}^2$$

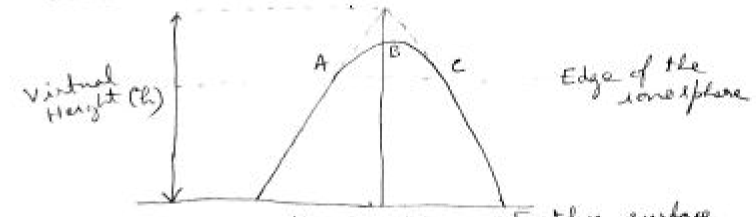
$$\Rightarrow \cos^2\theta_m = \frac{1}{1+\left(\frac{D_{skip}}{2h}\right)^2}$$

$$\Rightarrow \left(\frac{D_{skip}}{2h}\right)^2 + 1 = \frac{1}{\cos^2\theta_m} = \frac{f_{MUF}^2}{f_{cR}^2}$$

$$\Rightarrow \left(\frac{D_{skip}}{2h}\right) = \sqrt{\left(\frac{f_{MUF}}{f_{cR}}\right)^2 - 1}$$

$$\Rightarrow \boxed{D_{skip} = 2h\sqrt{\left(\frac{f_{MUF}}{f_{cR}}\right)^2 - 1}}$$

iii. Virtual Height



Propagation path in the ionosphere and the depiction of virtual height

— Consider an e.m. wave from a Tx reaching the receiver after being reflected by the ionosphere as shown in fig.

— If incident and reflected waves are extended they meet at the point D.

— The vertical height from the ground to the point D is known as the virtual height which is higher than the lower edge of the atmosphere.

6.(a)	Calculate the value of the operating frequency of the ionosphere's layer specified by refractive index of 0.85 and an electron density of 5×10^5 electrons/m^3. Calculate the critical frequency and MUF of the system with $\theta_i = 30^\circ$.	6	CO6	L3

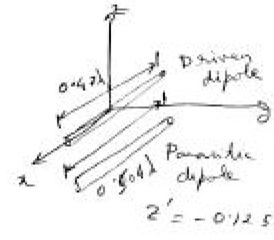
	Sdn. $\epsilon_r = 1 - \frac{81N}{f^2}$ $\Rightarrow \sqrt{\epsilon_r} = r.i = \sqrt{1 - \frac{81N}{f^2}}$ $\Rightarrow 0.85 = \sqrt{1 - \frac{81 \times 5 \times 10^5}{f^2}}$ $\Rightarrow 0.7225 = 1 - \frac{405 \times 10^5}{f^2}$ $\Rightarrow \frac{405 \times 10^5}{f^2} = 1 - 0.7225 = 0.2775$ $\Rightarrow f^2 = \frac{405 \times 10^5}{0.2775} = 1459.45 \times 10^5 \Rightarrow f = 12080.80\ Hz$ $f_{CR} = \sqrt{81N} = \sqrt{81 \times 5 \times 10^5} = 6363.96\ Hz$ $f_{MUF} = f_{CR} \sec\theta_m = \frac{f_{CR}}{\cos\theta_m}$ $\Rightarrow f_{MUF} = \frac{6363.96}{\cos(30^\circ)} = 7348.468\ Hz.$			
(b)	In an ionospheric propagation the angle of incidence made at a particular layer at a height of 200 km. is 45° with critical frequency of 6 MHz. Calculate the skip distance. Sdn $f_{CR} = 6\ MHz, \quad \theta_m = 45^\circ.$ $\therefore f_{MUF} = f_{CR} \sec\theta_m = \frac{f_{CR}}{\cos 45^\circ} = \frac{6 \times 10^6}{0.707} = 8.48\ MHz$ $\therefore D_{skip} = 2 \times 200 \times 10^3 \sqrt{\left(\frac{8.48}{6}\right)^2 - 1}$ $= 400 \times 10^3 \times \sqrt{1.9975 - 1} = 399.5\ km$	4	CO6	L3
7.	With a neat figure explain the working of Yagi-Uda antenna. Write the design formulae for different components used in Yagi-Uda antenna. Also mention the applications of Yagi-Uda antenna.	10	CO5	L2

Yagi - Uda Array

Dipole array with only one excited dipole and all other dipoles parasitically coupled to it.

- The induced currents and hence the radiation characteristics depend on the length of the dipole and the spacing b/w the dipoles.

- consider a dipole radiating in free space.
- A second dipole with its terminals short - circuited is kept ‖^{l} to it.
- A current is established on the second dipole due to e.m. induction.



$z' = -0.125\lambda$

Let current on element 1 = 1A

Induced current $= 0.7\angle 140° A$

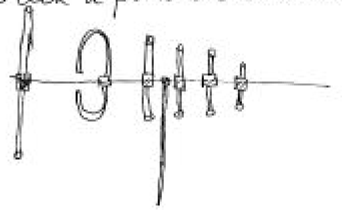
$$AF = I_1 e^{jkz_1'\cos\theta} + I_2 e^{jkz_2'\cos\theta}$$

if $z_1' = 0$, $z_2' = -0.125\lambda$,

θ measured from z-axis.

$$AF = 1 + 0.7 e^{j\left(\frac{7\pi}{9} - \frac{\pi}{4}\right)\cos\theta}$$

Antenna radiates more power along + z direction as compared to the -z direction.
- Parasitic element reflecting the field incident on it.
- such a parasitic element is called a reflector.



6 element Yagi-Uda antenna with folded dipole.

The spacing b/w the elements and the lengths of the parasitic elements determine the phases of the currents.

Directors: Dipoles shorter than $\frac{\lambda}{2}$ acts as a director and add the fields of the driven elements in the direction away from driven element.
- If more than one directors are employed, each director excites the next.
- Director elements are capacitive and induced current leads the induced voltage.

Reflector: Element of length greater than $\lambda/2$ acts as a reflector and add-up the fields of driven element in the direction from reflector.
- Usually only one reflector for Yagi-Uda array. (As behind the reflector field is very less and hence parasitic element kept behind it is not effective).

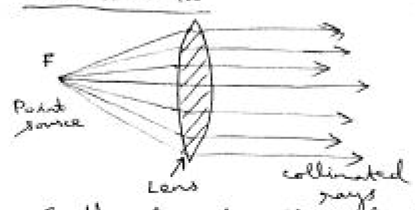
Reflector length $= \frac{500}{f\,(MHz)}$ feet

Driven element length $= \frac{475}{f\,(MHz)}$ feet

Director length $= \frac{455}{f\,(MHz)}$ feet.

	Applications 1) Yagi-Uda array is the most popular antenna for the reception of terrestrial television signal in the VHF band (30 MHz – 300 MHz) 2) Yagi-Uda arrays ~~can~~ be used in the HF, VHF, UHF and microwave frequency bands. - In the HF band, the array is constructed using wires and at VHF and UHF, hollow pipes are used for the construction of Yagi-Uda arrays General characteristics 1) It provides gain of the order of 8 db or front to back ratio of about 20db 2) It is also known as super-directive antenna due to its high gain, ~~and~~ [illegible] 3) It is frequency sensitive and a band-width of about 3% is obtainable. 4) Arrays can be stacked to increase directivity. 5) It has unidirectional beam of moderate directivity with light weight, low cost and simplicity in design.			
8.(a)	With a neat sketch explain the principle of lens antenna. Also list the merits and demerits of lens antenna.	5	CO5	L2

Lens Antenna



convex lens used in optics.

Snell's law of ~~refl~~ refraction is satisfied at the dielectric - dielectric interface as,

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}}$$

$\theta_i \rightarrow$ angle of incidence $\}$ measured w.r.t

$\theta_t \rightarrow$ - - refraction. $\}$ normal to the interfa

ϵ_{r_1} and $\epsilon_{r_2} \rightarrow$ relative dielectric constants of regions 1 and 2 respectively with radiation incident on the side of region 1.

Geometry of single-surface convex lens antenna

A spherical wavefront emanating from F emerges as a plane wave on the other side of the lens antenna, if

1) The rays must emerge ∥ to the axis after refraction at the first curved interface.
 - Normal incidence on the second surface does not produce any refraction.

2) The phase-shift along the ray path $FP + PB =$ ~~FO~~ phase shift along $FO + OA$

3) Snell's law of refraction must be satisfied at the dielectric - air interface.

Assuming the dimensions of the lens and the radius of curvature to be large as compared to the operating wavelength, we can assume the propagation constant within the lens medium as $k\sqrt{\epsilon_r}$

where, $\epsilon_r \rightarrow$ dielectric constant of lens material.

The equation of the curved surface is

$$kf + k\sqrt{\epsilon_r}\, x = k\sqrt{(f+x)^2 + y^2} \quad \ldots (1)$$

Rearranging, $k^2 f^2 + k^2 \epsilon_r x^2 = k^2 \left[f^2 + x^2 + 2fx + y \right.$

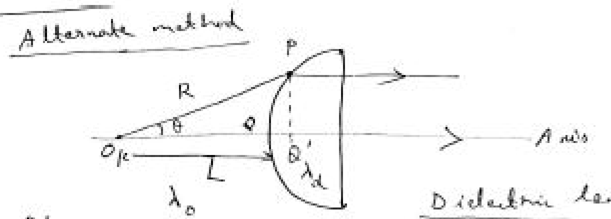
$\Rightarrow \cancel{k^2 f^2} + k^2 \epsilon_r x^2 = \cancel{k^2 f^2} + k^2 x^2 + k^2 2f x + k^2$

$\Rightarrow \quad x^2(\epsilon_r - 1) + 2fx(\sqrt{\epsilon_r} - 1) - y^2 = 0$

Above is an equation of a hyperbola.

A rotation of the hyperbola about the x-axis gives the hyperboloidal surface for the lens antenna.

Alternate method



Dielectric lens antenna

Planar wave-fronts can be obtained at the aperture when the electric path OQ' and OP remains same

$$\frac{OP}{\lambda_0} = \frac{OQ}{\lambda_0} + \frac{QQ'}{\lambda_d}$$

$$\Rightarrow \frac{R}{\lambda_0} = \frac{L}{\lambda_0} + \frac{R\cos\theta - L}{\lambda_d}$$

$$\Rightarrow R = L + \frac{\lambda_0}{\lambda_d}(R\cos\theta - L)$$

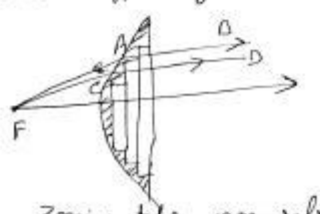
$$\Rightarrow R = L + a(R\cos\theta - L) \quad [a \rightarrow r.i]$$

$$\Rightarrow R(1 - a\cos\theta) = L(1-a)$$

$$\Rightarrow R = \frac{L(a-1)}{(a\cos\theta - 1)}$$

- Advantage is absence of blockage
- But a lens is generally heavy and bulky
- To reduce the bulk of the antenna, the method used is called zoning.

Zoning: Lens is divided into several circular zones and the dielectric material is removed from each zone, so that the electrical path length b/w adjacent zones differ by an integer multiple of wavelength.



Zoning the non-refracting surface

Zoning the refracting surface.

(b)	A parabolic reflector of 2 m diameter is used at 10 GHz. Calculate the beam width between first nulls (BWFN), HPBW and gain in dB.	5	CO5	L3

Soln

$$BWFN = 140\left(\frac{\lambda}{D}\right) = 140\left(\frac{c}{f}\right)\cdot\frac{1}{D}$$

$$= 140 \times \left(\frac{3\times10^8}{10\times10^9}\right) \times \frac{1}{2}$$

$$= 2.1^\circ$$

$$HPBW = 1.05^\circ$$

$$G = \frac{4\pi}{\lambda^2}\left(\frac{\pi D^2}{4}\right) \quad [\text{considering area-factor} = 1]$$

$$= \frac{4\pi}{\lambda^2}\cdot\left(\frac{\pi\, 4}{4}\right) = \frac{4\pi^2}{\lambda^2} = 4\pi^2 \times \left(\frac{10\times10^9}{3\times10^8}\right)^2 = 4\pi^2 \times (1111.11)$$

$$= 43864$$

Gain in dB = 46.42 dB

Improvement Test, 2017
(Scheme and solution)

1. Resultant field strength due to space-wave propagation:-
Explanation with neat sketch: 2m
Expression for E field and Γ for horizontal polarization: 4m
and for vertical polarization: 4m

2.(a) Surface-wave propagation:
explanation: 2m
expression for Z_s and derivation: 2m
final result: 2m

2.(b) Numerical: Approach: 2m
Final result: 2m.

3.(a) Factors affecting ground wave propagation: 4 m.
Sommerfeld equation expression: 2m.
explanation: 1 m

3.(b) Diagram: 1 m
derivation and final expression: 2m

4. Calculation of radius of curvature: 6m
explanation of duct propagation with neat sketches: 4m

5. i) MUF: Definition, mathematical expression: 2m+2m
ii) Skip distance: Diagram: 1m
Definition: 1 m
Final expression: 4m
iii) Virtual Height: Diagram: 1m.
~~Exact~~ explanation: 1m.

6.(a) Calculation of operating frequency: 2m
critical frequency: 2m
MUF: 2m.

6.(b) Approach: 2m
Final result: 2m.

7. $\left.\begin{array}{l}\text{Sketch: 2m} \\ \text{explanation: 4m.}\end{array}\right\}$ Working of Yagi-Uda antenna
Design Formulae: 2m
Applications: 2m

8.(a) Lens Antenna: Sketch: 1 m
Working principle: 2 m.
Merits and demerits: 2m.

8.(b) Parabolic reflector: BWFN: 2m
HPBW: 1m
Gain: 2m.