CBCS SCHEME

USN

16/17ELD/ECS/EVE/EIE/ESP11

First Semester M.Tech. Degree Examination, Dec.2018/Jan.2019
Advanced Engineering Mathematics

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Define vector spaces, sub spaces. Show that the set S = {(1, 0, 1), (1, 1, 0), (−1, 0, −1)} is linearly dependent in $V_3(R)$. **(08 Marks)**

b. Let T:V→W be a linear transformation defined by T(x, y, z) = (x+y, x−y, 2x+z). Find the range, null space, rank and nullity. Also verify the rank-nullity theorem. **(08 Marks)**

OR

2 a. Find the matrix representation of linear transformation $T:R_2 \to R_3$ such that T(-1, 1) = (-1, 0, 2), T(2, 1) = (1, 2, 1). **(08 Marks)**

b. Find the linear transformation T: $R^3 \to R^3$. Such that T(1, 1, 1) = (1, 1, 1); T(1, 2, 3) = (-1, -2, 3); T(1, 1, 2) = (2, 2, 4) **(08 Marks)**

Module-2

3 a. Use Given's method to find eigen values of the symmetric matrix.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$$ **(08 Marks)**

b. Find the singular value decomposition of

$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$$ **(08 Marks)**

OR

4 a. Use the Gram-Schmidt orthogonalization process to construct an orthogonal set of vectors form the linearly independent set {x_1, x_2, x_3} where $x_1 = \begin{bmatrix} -4 \\ 3 \\ 6 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}, x_3 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$. **(08 Marks)**

b. Construct a QR decomposition for the matrix

$$A = \begin{pmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{pmatrix}$$ **(08 Marks)**

Module-3

5 a. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ and deduce $\frac{d}{dx}\left(f - y'\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial x} = 0$. **(08 Marks)**

b. Find the extremals of the functional $\int_0^{\pi}\left[(y')^2 - y^2 + 4y\cos x\right]dx$ satisfying y(0) = 0; y(π) = 0. **(08 Marks)**

OR

6 a. Find a function y(x) for which $\int_0^1 [(y')^2 + x^2]dx$ is a stationary function given that $\int_0^1 y\,dx = \frac{1}{6}$; $y(0) = 0$; $y(1) = 0$. **(08 Marks)**

b. Find the shortest distance between parabola $y = x^2$ and straight line $x - y = 5$. **(08 Marks)**

Module-4

7 a. A random variable x has the following probability function:

x	0	1	2	3	4	5	6	7
p(x)	0	k	2'k	2k	3k	k^2	$2k^2$	$7k^2+k$

i) Find the value of the k.
ii) Evaluate: (1) $p(x < 6)$ (2) $p(x \geq 6)$ (3) $p(0 < x < 5)$
iii) Mean and variance of the distribution. **(08 Marks)**

b. Find the moment generating function of the exponential distribution

$$f(x) = \frac{e^{-\frac{x}{c}}}{c}; \quad 0 \leq x \leq \infty, c > 0$$

Hence find mean and standard deviation. **(08 Marks)**

OR

8 a. The length of telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth: (i) Ends less than 5 minutes (ii) Between 5 and 15 minutes. **(08 Marks)**

b. The mean weight of 500 students is 151 kg and standard deviation 15 kg. Assuming the weights are normally distributed. Find how many students weight:
(i) Between 120 and 150 (ii) Less than 150 (iii) More than 151 **(08 Marks)**

Module-5

9 a. The joint pdf of a two continuous random variables x and y is given by

$$f(x,y) = \begin{cases} \frac{x+y}{3} & 0 \leq x \leq 1;\ 0 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find: i) E(X) ii) E(Y) iii) E(XY) **(08 Marks)**

b. Define:
i) Stationary random process
ii) Ergodic random process
iii) Time auto correlation
iv) Gaussian random process **(08 Marks)**

OR

10 a. Find the probability that (i) $x(10) \leq 8$ (ii) $|x(10) - x(6)| \leq 4$; where x(t) is a Gaussian process with $r(t) = 10$ and $c(t_1, t_2) = 16\ e^{-|t_1 - t_2|}$. **(08 Marks)**

b. Determine: (i) Marginal distribution of x and y (ii) Covariance (iii) Correlation coefficient for the following joint distribution. **(08 Marks)**

x \ y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

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