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14ELD11

First Semester M.Tech. Degree Examination, Dec.2017/Jan.2018
Advanced Mathematics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Construct QR decomposition for the matrix :
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$
(08 Marks)

b. Find the singular value decomposition of
$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}.$$
(12 Marks)

2 a. Find the least square solution of AX = B given :
$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$
(12 Marks)

b. Find the pseudo inverse of matrix :
$$A = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 0 & 2 \end{bmatrix}.$$
(08 Marks)

3 a. A necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y')dx$ where $y(x_1) = y_1$ and $y(x_2) = y_2$ to be an extremum is that $\frac{\partial t}{\partial y} - \frac{d}{dx}\left(\frac{\partial t}{\partial y'}\right) = 0$. **(12 Marks)**

b. Find the extremal of the functional $I = \int_0^{\pi/2} (y^2 - y'^2 - 2y \sin x)\, dx$ under the condition $y(0) = y(\pi/2) = 0$. **(08 Marks)**

4 a. Find the extremal of the functional $I = \int_0^{\pi} (y'^2 - y^2)\, dx$ under the condition $y(0) = 0$, $y(*) = 1$ and subject to the constraints $\int_0^{\pi} y\, dx = 1$. **(10 Marks)**

b. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about the x – axis gives a minimum surface area. **(10 Marks)**

5 a. A string is stretched and fixed between two points (0, 0) and (ℓ, 0) motion is initiated by displacing the string in the form $U = \lambda \sin\left(\frac{\pi x}{\ell}\right)$ and released from rest at time t = 0. Find the displacement and any point or the string at any time 't' solve by Laplace transform method. **(10 Marks)**

b. Solve the heat conduction equation given by PDE $k\frac{\partial^2 u}{\partial t^2} = \frac{\partial u}{\partial t}$ $-\infty < x < \infty$, $t > 0$ with condition u(x, t) and $u_x(x, t)$ both $\to 0$ as $|x| \to \infty$ $u(x, 0) = f(x)$, $-\infty < x < \infty$. **(10 Marks)**

6 a. Derive the Poisson equation. **(10 Marks)**

b. Show that the two dimensional Laplace equation $\nabla^2 u = 0$ is transformed by introducing plane coordinates (r, θ) denoted by the relations $x = r \sin\theta$, $y = r \sin\theta$ it takes the form $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. **(10 Marks)**

7 a. Use Simplex method to solve the following equation :

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subject to $x_1 + 2x_2 + x_3 \le 430$

$3x_1 + 2x_3 \le 460$

$x_1 + 4x_2 \le 420$

$x_1, x_2, x_3 \ge 0$. **(08 Marks)**

b. Apply dual simplex method to :

Minimize $z = 2x_1 + 2x_2 + 4x_3$

subject to the constraints $2x_1 + 3x_2 + 5x_3 \ge 2$

$3x_1 + 3x_2 + 5x_3 \ge 3$

$3x_1 + x_2 + 7x_3 \le 3$

$x_1 + 4x_2 + 6x_3 \le 5$

$x_1, x_2, x_3 \ge 0$. **(12 Marks)**

8 a. Use Two – Phase method to :

Maximize $z = 5x_1 - 4x_2 + 3x_3$

Subject to $2x_1 + x_2 - 6x_3 = 20$

$6x_1 + 5x_2 + 10x_3 \le 76$

$8x_1 - 3x_2 + 6x_3 \le 50$

$x_1, x_2, x_3 \ge 0$. **(12 Marks)**

b. Using Kuhn – Tucker conditions :

maximize $Z = x_1^2 + x_1x_2 - 2x_2^2$

subject to $4x_1 + 2x_2 \le 24$

$5x_1 + 10x_2 \le 30$

$x_1, x_2 \ge 0$. **(08 Marks)**

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