CBCS Scheme

USN

16/17ELD/ECS/EVE/EIE/ESP11

First Semester M.Tech. Degree Examination, Dec.2017/Jan.2018

Advanced Engineering Mathematics

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define Basis of a vector space V show that the set s = {(1, 2)(3, 4)} forms a basis of R^2. **(08 Marks)**

b. If ω_1, and ω_2 are two finite dimensional subspaces of a vector space V(F), then $\omega_1 + \omega_2$ is finite dimensional and dim ω_2 + dim ω_2 = dim($\omega_1 \cap \omega_2$) + dim($\omega_1 + \omega_2$). **(08 Marks)**

OR

2 a. Let u and v be two vector spaces over the field F. let T_1 and T_2 be two linear transformations from U into V then the function ($T_1 + T_2$) defined by $(T_1 + T_2)(\alpha) = T_1(\alpha) + T_2(\alpha)$, $\forall \alpha \in U$ is a linear transformation from U into V. If C is any element of F, then the function (CT) defined by (CT)(α) = CT(α) is a linear transformation from U into V. Then set of all transformation L(U, V) from U into V, together with the addition and scalar multiplication defined above, is a vector space over the field F. **(08 Marks)**

b. Find the matrix representation of a linear map $T : R^3 \to R^3$ defined by :
T(x, y, z) = (z, y + z, x+ y + z) relative to the basis {(1, 0, 1) (–1, 2, 1)(2, 1, 1)}. **(08 Marks)**

Module-2

3 a. Find the eigne value and eigen vectors of the matrix :

$$A = \begin{bmatrix} 2 & 0 & 1 & -3 \\ 0 & 2 & 10 & 4 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

(08 Marks)

b. Use the modified Gram – Schmidt process to construct an orthogonal set of vectors from linearly independent set {x_1, x_2, x_3} when $X_1 = \begin{bmatrix} -4 \\ 3 \\ 6 \end{bmatrix}$ $X_2 = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$ $X_3 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$. **(08 Marks)**

OR

4 a. Construct a QR decomposition for the matrix :

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

(08 Marks)

b. Solve the following system of equations in the least square sense: $x_3 + 2x_4 = 1$; $x_1 + 2x_2 + 2x_3 + 3x_4 = 2$. **(08 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-3

5 a. Find the path on which a particle in the absence of friction will slide from one point to another in the shortest time under the action of gravity. **(08 Marks)**

b. Show that the curve which extremizes the functional $I = \int_0^{\pi/4} (y''^2 - y^2 + x^2)\,dx$ under the conditions $y(0) = 0,\ y'(0) = 1,\ Y(\pi/4) = y'(\pi/4) = \frac{1}{\sqrt{2}}$ is $y = \sin x$. **(08 Marks)**

OR

6 a. Prove that the sphere is the solid figure of revolution which, for a given surface area, has maximum volume. **(08 Marks)**

b. Show that the functional $\int_0^{\pi/4} \left[2xy + \left(\frac{du}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] dt$ such that $x(0) = 0,\ x(\pi/2) = -1,\ y(0) = 0,\ y(\pi/2) = 1$ is stationary for $x = -\sin t,\ y = \sin t$. **(08 Marks)**

Module-4

7 a. A committee consist of a students two of which are from I year, three from II year and four from III year. three students are to be removed at random what is the chance that : i) the three students belong to different classes ii) two belong to the same class and third to the different class iii) the three belong to the same class. **(08 Marks)**

b. The frequency distribution of a measurable characteristic varying between 0 and 2 is as under : $f(x) = \begin{cases} x^3 & 0 \le x \le 1 \\ (2-x)^3 & 1 \le x \le 2 \end{cases}$

Calculate the standard deviation and mean deviation about the mean. **(08 Marks)**

OR

8 a. Define moment generation function and probability generating function. Find the moment generating function for $f(x) = \frac{1}{C} e^{-x/c}$, $0 \le x \le \infty$, $c > 0$. If x is a random variable with probability generating function $P_x(t)$, find the probability generating function for $x + 2$. **(08 Marks)**

b. In a certain factory turning out razor blade, there is a small chance of 0.002 for any blade to be defective. the blades are supplied in packets of 10, use Poisson distribution to calculate the approximate number of packets containing i) no defective, ii) one defective iii) two defective blade in a consignment of 10,000 packets. **(08 Marks)**

Module-5

9 a. Prove that the conditional PDF of a random variable x given that y = y is $f_{x/y}(x/y) = \frac{f_{x,y}(x,y)}{f_y(0)}$. Also find marginal PDF's and conditional PDF's for a joint PDF given by $f_{x,y}(x,y) = \frac{2abc}{(ax+by+c)^3} u(x)u(y)$. **(08 Marks)**

b. Define independent random variables. P.T is x and y are independent random variables then $E[xy] = \mu_x\mu_y$, $c_{ov}(x, y) = 0$ and $f_{x,y} = 0$. Also prove that independent random variables are necessarily uncomelated but the converse is not always time with example. **(08 Marks)**

OR

10 a. What is the probability that $X_k = x(t_k)$ is equal to 1(or 0) if T_1, T_2- - - - is a sequence of IID random variables each with an exponential distribution $f_T(s) = \lambda e^{-\lambda s}u(s)$ suppose there are exactly n switches in the time interval $[0, t_k]$. **(08 Marks)**

b. Define Gaussian random process consider the random process $x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$ where A and B are independent, zero – mean Gaussian random variables with equal variables of σ^2. Find the mean and auto correlation function of this process. **(08 Marks)**

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