CMR INSTITUTE OF TECHNOLOGY

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Internal Assesment Test – III

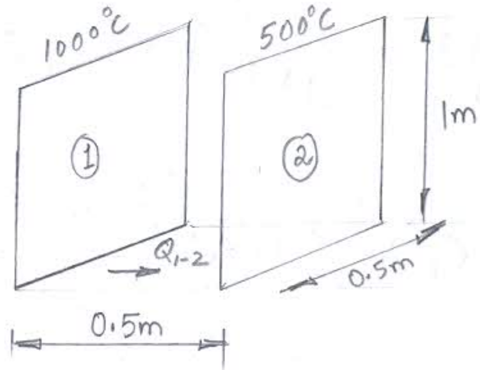
Sub:	**Heat and Mass Transfer**							Code:	10ME63
Date:	30/05/2017	Duration:	90 mins	Max Marks:	50	**Sem:**	VI	Branch:	Mechanical

Answer any five full questions

Use of HMT Data book is allowed

		Marks	OBE CO	OBE RBT
1. (a)	Two parallel black plates 0.5m × 1m are 0.5m spaced apart. One plate is maintained at 1000°C and other at 500°C. What is the net radiant heat exchange between the two plates.	[05]	C04	L3
(b)	State and prove the Wien's Displacement Law.	[05]	C04	L3
2.	Prove that the emissive power of a black body in a hemispherical closure is π times the intensity of radiation.	[10]	C04	L3
3.	Explain the following laws with mathematical relation [2.5 × 4] a) Kirchhoff's Law b) Planck's Law c) Lambert's Cosine Law d) Stefan Boltzmann Law	[10]	C04	L4
4.	Two large parallel plates with ε = 0.5 each, are maintained at different temperatures and are exchanging heat only by radiation. Two equally large radiation shields with surface emissivity 0.05 are introduced in parallel to the plates. Find the percentage reduction in net heat transfer.	[10]	C04	L3
5.	Explain the following terms [2.5 × 4] a) Radiosity and Irradiation b) Black Surface and Grey Surface	[10]	C04	L4
6. (a)	State and Explain the Fick's Law of diffusion [5]	[10]	C03	L4
(b)	Define Mass concentration and Molar concentration [5]			L1
7.	The temperature of a black surface 0.2 m^2 in area is 540°C. Calculate: i) The total rate of energy emission. ii) The intensity of normal radiation. iii) The wavelength of maximum monochromatic emissive power.	[10]	C04	L3
8.	Two parallel plates, each of $4m^2$ area, are large compared to a gap of 5mm separating them. One plate has a temperature of 1000K and surface emissivity of 0.5, while the other has a temperature of 400K and surface emissivity of 0.8. Find the net energy exchange by radiation between them. If a polished metal sheet of surface emissivity 0.4 on both sides is now located centrally between the two plates, what will be its steady temperature? How the heat transfer would be altered? Neglect the convection and edge effect if any. Comment upon the significance of this exercise.	[10]	C04	L3

Que: 1 (a)



Refer page 92 (Data book)

$$X = L/D = \frac{\text{Length side}}{\text{Distance}}$$

$$X = \frac{1}{0.5} = 2$$

$$Y = \frac{B}{D} = \frac{\text{Smaller side}}{\text{Distance}} = \frac{0.5}{0.5} = 1$$

from graph / Table $F_{1-2} = 0.285$

$$Q_{1-2} = A_1 F_{1-2} \sigma (T_1^4 - T_2^4)$$ where $T_1 = 1000 + 273 = 1273\,K$

$T_2 = 500 + 273 = 773\,K$

$$Q_{1-2} = (0.5 \times 1)(0.285)(5.67 \times 10^{-8})(1273^4 - 773^4)$$

$$Q_{1-2} = 18333.54\ W.$$

Que: 1-b Ween's displacement law states that, the maximum ~~power~~ value of the emissive power E_b can be obtained by differentiating Planck's equation with respect to λ and equating it to zero, therefore:-

$$\frac{dE_{b\lambda}}{d\lambda} = 0$$

$$\frac{dE_{b\lambda}}{d\lambda} = \frac{d}{d\lambda}\left\{C_1\lambda^{-5}\left(e^{C_2/\lambda T}-1\right)^{-1}\right] = 0$$

$$= C_1\lambda^{-5}(-1)\left(e^{\frac{C_2}{\lambda T}}-1\right)^{-2}\left(\frac{-C_2}{\lambda T}\right)\left(e^{\frac{C_2}{\lambda T}}-0\right) + \left(e^{\frac{C_2}{\lambda T}}-1\right)^{-1}.C_1(-5)\lambda^{-6} = 0$$

$$= \left(C_1\frac{1}{\cancel{\lambda^{8}}}\left(e^{\frac{C_2}{\lambda T}}-1\right)^{-2}\cdot\frac{C_2}{\cancel{\lambda}T}\left(e^{C_2/\lambda T}\right)\right) = 5C_1\cancel{\left(e^{\frac{C_2}{\lambda T}}-1\right)^{-1}}\frac{1}{\cancel{\lambda^{6}}} = 0$$

$$\frac{e^{C_2/\lambda T}}{\left(e^{C_2/\lambda T}-1\right)} = \frac{5\lambda T}{C_2}$$

put $C_2/\lambda T = x$

$$\frac{e^x}{e^x-1} = \frac{5}{x} \Rightarrow xe^x = 5e^x-5$$

$$xe^x-5e^x+5=0$$

$$5e^x\left[\frac{x}{5}-1+e^{-x}\right] = 0$$

$$e^{-x}+\frac{x}{5}-1 = 0$$

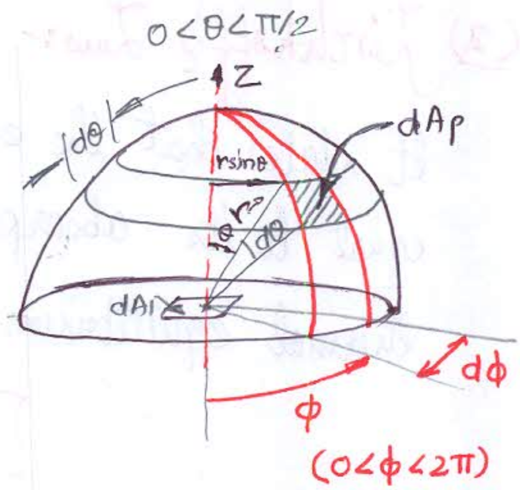
solving the above equation gives $x = 4.965 = \frac{C_2}{\lambda_{max}.T}$

$$\lambda_{max}.T = \frac{C_2}{4.965} = \frac{1.438\times10^{-2}}{4.965} = \boxed{2.898\times10^{-3}\ m\text{-}k}$$

$$\boxed{\lambda_{max}.T = 2.898\times10^{-3}\ m\text{-}k}$$

Que: 2

The amount of radiation directed towards the area dA_p



$$dE_b = I_n \, dA_1 \cos\phi \, \frac{dA}{r^2}$$

area of the small element

$$dA_p = (R \sin\theta . \, d\theta)(R . d\phi)$$

$$= R^2 \sin\theta . d\theta . d\phi$$

solid angle in the direction of $\theta = d\omega = \frac{dA_p}{R^2}$

$$d\omega = \frac{dA_p}{R^2} = \frac{\cancel{R^2} \sin\theta . d\theta . d\phi}{\cancel{R^2}} = \sin\theta . d\theta . d\phi$$

$$dE_b = I_n . \cos\theta . d\omega . dA_1$$

$$= I_n . \cos\theta . \sin\theta . d\theta . d\phi . dA_1$$

$$\int dE_b = I_n \, dA_1 \int_0^{\pi/2} \sin\theta . \cos\theta . d\theta . \int_0^{2\pi} d\phi$$

$$E_b = I_n . dA . \frac{1}{2} \times 2\pi \Rightarrow E_b = I_n . \pi . dA$$

Total emissive power of area dA

$$E_b = \sigma T^4 dA$$

$$I_n . \pi . \cancel{dA} = \sigma T^4 \cancel{dA}$$

$$E_b = I_n . \pi$$ {emissive power of a black body is equal to π times of intensity of radiation}

Que: 2

(a) Kirchhoff's Law:-

It states that the emissivity of the surface of a body is equal to its absorptivity when the body is in the thermal equilibrium with its surroundings

$$\boxed{\alpha = \varepsilon}$$

(b) Planck's Law:-

The relation for the spectral blackbody emissive power $E_{b\lambda}$ was developed by max planck in 1901. This relation is known as Plank's Law and is expressed as

$$E_{b\lambda}(\lambda, t) = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}$$

where: $C_1 = 3.74177 \times 10^{-6}$ w-m^2

$C_2 = 1.438 \times 10^{-2}$ m-K.

λ = wavelength of the radiation (m)

T = Absolute temperature (K)

3 (c) :- Lembert's cosine Law:-

It states that a diffuse surface radiates energy in such a manner that the rate of energy radiated in any prepandicular direction is proportional to the cosine of the angle between the direction under consideration and the normal to the Surface.

$$E_{b\theta} = E_{bn} \cos\theta$$

3(d) Stefan Boltzmann Law:-

It states that the energy (radient) emitted by a body is directly proportional to the fourth power of the temperature in absolute scale. Mathematically it is written as

$$Q \propto T^4$$

$$Q = \sigma T^4$$

σ = Stefan Boltzmann constant $= 5.67 \times 10^{-8}\ W/m^2k^4$

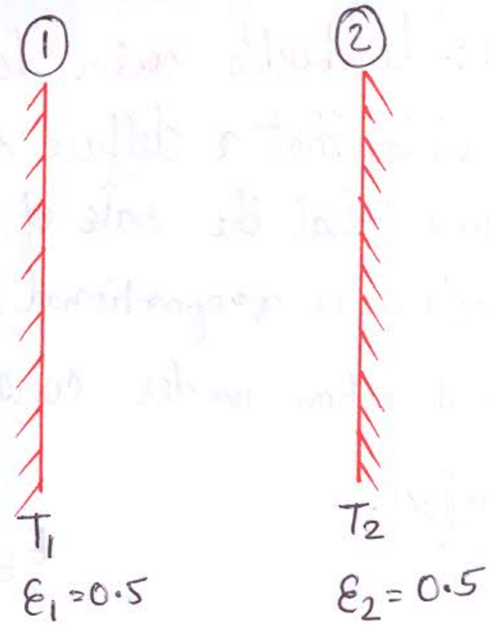
T = Temp. of body (K)

Q = Heat flux (W/m^2)

Que: 4 Given that

$$\varepsilon_1 = 0.5 = \varepsilon_2$$

Let the surface temperature be T_1 and T_2 for plate 1 and 2 respectively.

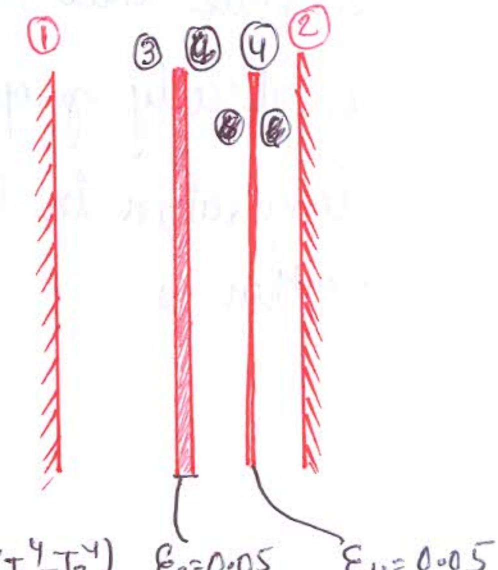


case 1 : without Shield

* **case : 1** without Shield.

$$\left(\frac{Q_{1-2}}{A}\right)_{case:1} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left\{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right\}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{3}$$

* **case : 2**



$$\frac{Q_{1-2}}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + 2\sum_{i=1}^{N}\frac{1}{\varepsilon_{3i}} - (N+1)}$$

$N = 2$ {No of shield}

$$\left(\frac{Q_{12}}{A}\right)_{case2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + 2\left\{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_4}\right\} - 3} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{41}$$

$$\text{Q \% reduction} = \frac{\left(Q/A\right)_{case1} - \left(Q/A\right)_{case2}}{\left(Q/A\right)_{case1}} = \cancel{92.68\%}$$

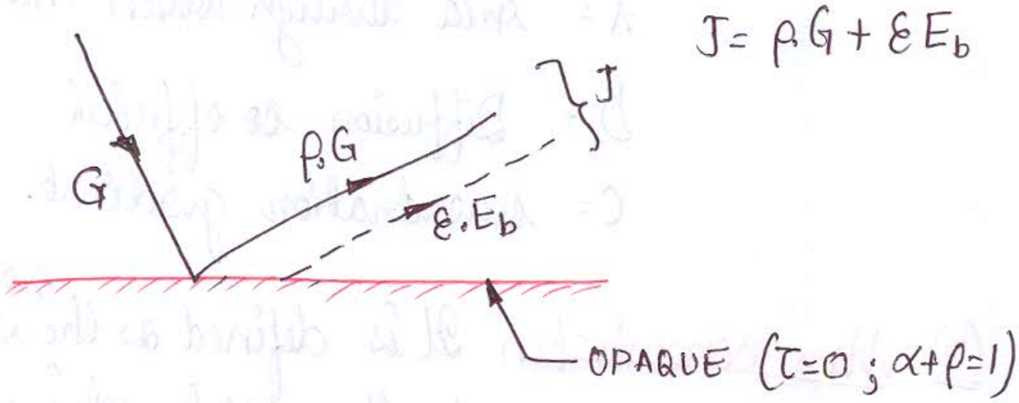
96.25%

Que:5

(a) Radiosity:- It is defined as the total amount of radiation leaving a surface per unit time per unit area.
It is ~~given~~ written as J and expressed in W/m^2

(b) Irradiation:- It is defined as the total radiation incident upon a surface per unit time per unit area.
It is expressed in W/m^2

$$J = \rho G + \varepsilon E_b$$



Black Surface:- A body is said to be black, if it absorbs all incident radiation. For a black body $\alpha = 1$ and $\rho = \tau = 0$.
There is no such perfectly black body in nature.

Grey Surface:- A grey body has a constant monochromatic emissivity ε_λ with respect to wavelength. Thus gray body has a characteristics emissivity value <1. which does not vary with temp.

Que:6

(a) Fick's law of diffusion:-

It states that the mass flux of a constituent per unit area is proportional to the concentration gradient.

$$\frac{M_a}{A} \propto \frac{dC_A}{dx}$$

$$\frac{M_a}{A} = -D\frac{dC_A}{dx}$$

where M_A = Mass flow rate

A = Area through which mass flows

D = Diffusion co-efficient

C = concentration gradient.

(b) Mass concentration It is defined as the ratio species A of a body to the total volume of mixture.

$$\text{Mass concentration} = \frac{\text{Mass concentration } A}{\text{Volume of mixture.}} \quad (Kg/m^3)$$

Molar concentration:- It is defined as the no. of mole of species A ~~ie~~ per unit volume of mixture.

$$\text{Molar Concentration} = \frac{\text{No. of mole of species } A}{\text{Volume of mixture}} \quad (mol/m^3)$$

Que:7 Given that

* Temperature of surface = $540^\circ C + 273 = 813 K$

* Black surface with $0.2 m^2$ area

(i)
we know that the total power emitted by a body is given by Stefan Boltzmann law:

$$E_b = \sigma A T^4 = 5.67 \times 10^{-8} \times 0.2 \times (813^4)$$

$$\boxed{E_b = 4954.219 \ W}$$

(ii) Intensity of normal radiation:-

The $E_b = \pi \times I_n$ where I_n = Normal Intensity of radiation

$$\boxed{I_n = \frac{E_b}{\pi} = 15.76.97 \ W}$$

(iii) Maximum monochromatic wave length:- The maximum mono chromatic emissive power is given by the wien's displacement law

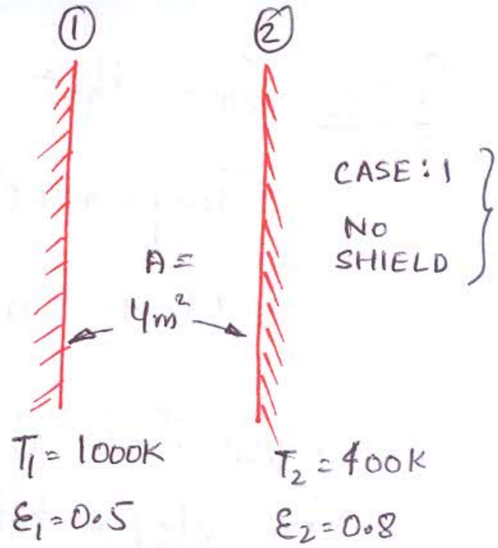
$$\lambda_{max} . T = 2.898 \times 10^{-3} \ m\text{-}K$$

$$\boxed{\lambda_{max} = \frac{2.898 \times 10^{-3}}{813} = 3.564 \times 10^{-6} \ m}$$

Que: 8 Given that

$T_1 = 1000\,K \qquad \varepsilon_1 = 0.5$

$T_2 = 400\,K \qquad \varepsilon_2 = 0.8 \qquad A_1 = A_2 = 4m^2$

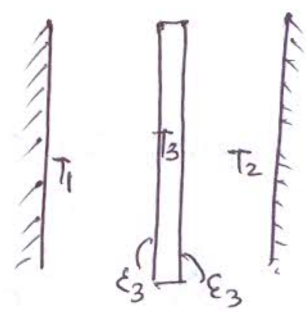


we know that

Case 1

$$\left(Q_{1-2}\right)_{\text{w/o shield}} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$= \frac{5.67 \times 10^{-8} \times 4 \times \left(1000^4 - 400^4\right)}{\frac{1}{0.8} + \frac{1}{0.5} - 1} = 98219.52\ W$$



case: 2 with Shield $\quad N = 1$

$$\left(Q_{1-2}\right)_{\text{with shield}} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + 2\sum_{i=1}^{N}\frac{1}{\varepsilon_i} - (N+1)}$$

→ FORMULA ONLY APPLICABLE WHEN BOTH SIDE OF SHIELD HAS EQUAL ε

Since N = no of shield = 1

$$Q_{1-2} = \frac{\sigma A\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + 2\frac{1}{\varepsilon_3} - 2} \Rightarrow \frac{5.67 \times 10^{-8} \times 4\left(1000^4 - 400^4\right)}{\frac{1}{0.8} + \frac{1}{0.5} + \frac{2}{0.4} - 2}$$

$$\left(Q_{1-2}\right)_{\text{with shield}} = 35359.0272\ W$$

$$\%\ \text{reduction} = \frac{98219.52 - 35359.0272}{98219.52} \times 100 = 64\%$$

(ii) Stedy temperature

from energy balance

$$Q_{1-3} = Q_{3-2}$$

$$\frac{\cancel{\sigma A}(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{\cancel{\sigma A}(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1}$$

$$\frac{1000^4 - T_3^4}{\frac{1}{0.5} - \frac{1}{0.4} - 1} = \frac{(T_3^4 - 400^4)}{\frac{1}{0.4} + \frac{1}{0.8} - 1}$$

upon soluing for T_3', we get $\boxed{T_3 = 821.002\ K}$

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