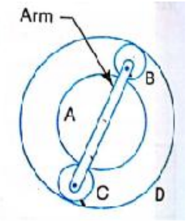
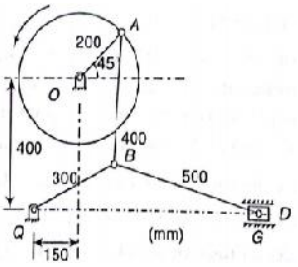
CMR INSTITUTE OF TECHNOLOGY	USN										

IMPROVEMENT TEST

Sub:	KINEMATICS OF MACHINES							Code:	15ME42
Date:	29/ 05 / 2017	Duration:	90 mins	Max Marks:	50	Sem:	IV – A, B	Branch:	MECH

		Marks	OBE CO	OBE RBT
Part – A: Compulsory				**[5]**
1	State and prove Kennedy's theorem.	05	CO2	L2
Part – B: Answer any three questions				**[15]**
2	Using complex algebra derive expressions for velocity and acceleration of piston, angular acceleration of connecting rod of a reciprocating engine mechanism. With these expressions determine the above quantities, if the crank length is 50mm, connecting rod 200mm, crank speed is constant at 3000rpm and crank angle is 30º.	15	CO2	L3
3 a.	Explain Klien's construction for slider crank mechanism.	10	CO2	L2
3 b.	The crank and connecting rod of a reciprocating engine are 200mm and 700mm respectively. The crank is rotating in clockwise direction at 120 rad/sec. Find with the help of Klien's construction: i. Velocity and acceleration of piston ii. Angular velocity and angular acceleration of the connecting rod, at the instant when the crank is at 30º to the inner dead centre.	05	CO2	L3
4	An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star shaped carrier C. The sizes of different toothed wheels are such that the planet carrier C rotates at 1/5th of the speed of the sun wheel S. The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel is 100Nm. Determine: i. Number of teeth on different wheels of the train, and ii. Torque necessary to keep the internal gear stationary.	15	CO4	L3
5 a.	What are the types of gear trains? Explain with the help of neat sketches.	05	CO4	L1
5 b.	An epicyclic train of gears is arranged as shown in the figure. How many revolutions does the arm, to which the pinions B and C are attached, make: i. When A makes one revolution clockwise and makes half a revolution counter clockwise, and ii. When A makes one revolution clockwise and D is stationary. The number of teeth on the gears A and D are 40 and 90 respectively.  	10	CO4	L3
6	In the toggle mechanism shown in the figure below, the crank OA rotates at 210rpm counter clockwise increasing at the rate of 60rad/sec^2. For the given configuration determine: i. Velocity of slider D and angular velocity of link BD ii. Acceleration of slider D and angular acceleration of link BD  	15	CO2	L3

SOLUTION TO IAT-3
KINEMATICS OF MACHINES
(15ME42)

Part-A.

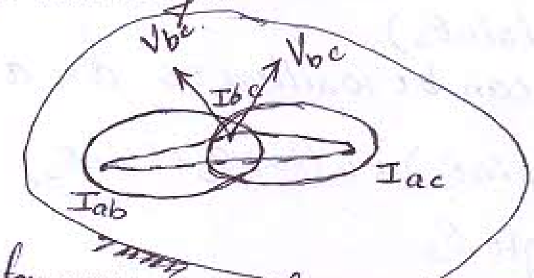
1. State and Prove kennedy's theorem.

Sol "It states that any three bodies having plane motion relative to one another have three instantaneous centres and lie on a straight line."
Consider three bodies A, B and C having relative plane motion to each other as shown in fig.

Then, $N = \frac{n(n-1)}{2}$ where, N = Number of instantaneous centers and
n = Number of bodies as links.

$\therefore N = \frac{3(3-1)}{2} = 3.$

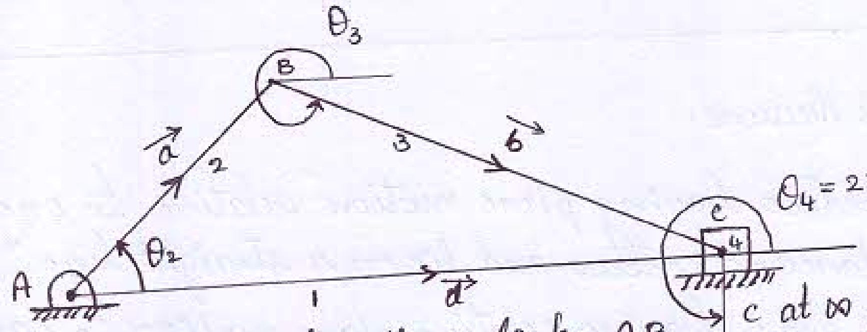
Hence it is proved, that there are three instantaneous centres. They are I_{ab}, I_{ac} and I_{bc}. Now we have to prove that these three centres lie in a straight line.
I_{ab} = Instantaneous centre of A relative to B.
I_{ac} = Instantaneous centre of A relative to C.
I_{bc} = Instantaneous centre of B relative to C.



The location of instantaneous centres Iab and Iac are known and to Prove that Ibc also lies on the line joining Iab & Iac.
According to the definition of instantaneous centre, instant centre is a point common to two bodies and have the same linear velocity in both magnitude and direction. Since the direction of two velocities do not coincide, the point I_{bc} cannot be the instantaneous centre of body B & C. Their direction will coincide only if the instantaneous centre lies on the same line joining Iab & Iac. Therefore all the three Instantaneous centres are on the same line. Hence Proved.

Part-B.

2. Using complex algebra derive expressions for velocity and acceleration of piston, angular acceleration of connecting rod of a reciprocating engine mechanism. With these expressions determine the above quantities, if the crank length is 50mm, connecting rod 200mm crank speed is contant @ 3000rpm and crank angle is 30°.



Let a and b be the length of links AB and BC respectively and 'd' be the displacement of piston θ_2, θ_3 and θ_4 be the angular position of links 2, 3 and 4 respectively. For counter clockwise measurement, the angles are considered as positive.

Let $\vec{a}$, $\vec{b}$ and $\vec{d}$ be the relative position vectors of the links.

∴ The loop closure equations is $-\vec{d}+\vec{a}+\vec{b}=0$

i.e., $\vec{d}=\vec{a}+\vec{b}$ — (1)

Expressing the vectors in exponential (or) complex number in Polar form.

$\vec{d} = de^{i\theta_1} = d(\cos\theta_1 + i\sin\theta_1) = d \quad (\because \theta_1 = 0)$

$\vec{a} = ae^{i\theta_2} = a(\cos\theta_2 + i\sin\theta_2)$

$\vec{b} = be^{i\theta_3} = b(\cos\theta_3 + i\sin\theta_3)$

∴ The equation 1 can be written as $d = ae^{i\theta_2} + be^{i\theta_3}$ — (2)

i.e., $d = a(\cos\theta_2 + i\sin\theta_2) + b(\cos\theta_3 + i\sin\theta_3)$ — (3)

(i) Connecting rod angle θ_3.

The realpart of equation 3.

$d = a\cos\theta_2 + b\cos\theta_3$

$\therefore b\cos\theta_3 = d - a\cos\theta_2$ — (4)

The imaginary part of equation 3.

$0 = a\sin\theta_2 + b\sin\theta_3$

$b\sin\theta_3 = -a\sin\theta_2$ — (5)

$\theta_3 = \sin^{-1}\left(\frac{-a\sin\theta_2}{b}\right)$ — (6)

(ii) Angular velocity of connecting rod and velocity of slider.

Differentiating equations 4 & 5 w.r.t. time.

$$-b\omega_3 \cdot \sin\theta_3 = V_c + a\sin\theta_2 \cdot \omega_2$$

i.e $-a\omega_2\sin\theta_2 - b\omega_3 \cdot \sin\theta_3 - V_c = 0 \quad — (7)$

$$b\cos\theta_3\,\omega_3 = -a\cos\theta_2\,\omega_2$$

i.e $a\cos\theta_2\,\omega_2 + b\cos\theta_3\,\omega_3 = 0 \quad — (8)$

Multiply equation (7) by $\cos\theta_3$ and equation (8) by $\sin\theta_3$ and add.

i.e, $-a\omega_2\sin\theta_2\cos\theta_3 - b\omega_3\sin\theta_3\cos\theta_3 - V_c\cos\theta_3 + a\cos\theta_2\,\omega_2\sin\theta_3$
$+ b\cos\theta_3\,\omega_3\sin\theta_3 = 0$

i.e., $-a\omega_2\sin\theta_2\cos\theta_3 + a\omega_2\cos\theta_2\sin\theta_3 - V_c\cos\theta_3 = 0$

$$a\omega_2(\sin\theta_3\cos\theta_2 - \cos\theta_3\sin\theta_2) - V_c\cos\theta_3 = 0$$

$$a\omega_2\sin(\theta_3-\theta_2) - V_c\cos\theta_3 = 0$$

$\therefore$ Velocity of slider $V_c = \dfrac{a\omega_2\sin(\theta_3-\theta_2)}{\cos\theta_3} \quad — (9)$

From equation (8), Angular Velocity of connecting rod

$$\omega_3 = -\frac{a\omega_2\cos\theta_2}{b\cos\theta_3} \quad — (10)$$

Angular acceleration of connecting rod and acceleration of piston.

Differentiating equation (7) and (8) w.r.t time.

$-a\omega_2^2\cos\theta_2 - a\sin\theta_2\alpha_2 - b\omega_3^2\cos\theta_3 - b\sin\theta_3\alpha_3 - A_c = 0 \quad — (11)$

$-a\omega_2^2\sin\theta_2 + a\cos\theta_2\alpha_2 - b\omega_3^2\sin\theta_3 + b\cos\theta_3\alpha_3 = 0 \quad — (12)$

Multiply equation 11 by $\cos\theta_3$ and equation 12 by $\sin\theta_3$ and add.

$-a\omega_2^2\cos\theta_2\cos\theta_3 - a\sin\theta_2\alpha_2\cos\theta_3 - b\omega_3^2\cos^2\theta_3 - b\sin\theta_3\alpha_3\cos\theta_3 - A_c\cos\theta_3$
$- a\omega_2^2\sin\theta_2\sin\theta_3 + a\cos\theta_2\alpha_2\sin\theta_3 - b\omega_3^2\sin^2\theta_3 + b\cos\theta_3\alpha_3\sin\theta_3 = 0$

i.e, $-a\omega_2^2(\sin\theta_3\sin\theta_2 + \cos\theta_3\cos\theta_2) + a\alpha_2(\sin\theta_3\cos\theta_2 - \cos\theta_3\sin\theta_2)$
$-b\omega_3^2(\sin^2\theta_3 + \cos^2\theta_3) - A_c\cos\theta_3 = 0$

i.e., $-a\omega_2^2\cos(\theta_3-\theta_2) + a\alpha_2\sin(\theta_3-\theta_2) - b\omega_3^2 - A_c\cos\theta_3 = 0$

∴ Acceleration of piston

$$A_c = \frac{a\alpha_2 \sin(\theta_3-\theta_2) - a\omega_2^2 \cos(\theta_3-\theta_2) - b\omega_3^2}{\cos\theta_3} \quad \text{— (13)}$$

From equation (12).

Angular acceleration of connecting rod $\alpha_3 = \dfrac{a\omega_2^2 \sin\theta_2 - a\cos\theta_2\alpha_2 + b\omega_3^2 \sin\theta_2}{b\cos\theta_3}$

b) Data:-

$a = 50mm = 0.05m$

$b = 200mm = 0.2m$

$\theta_2 = 30°$

$N_2 = 3000 rpm$

Angular velocity of crank. $\omega_2 = \dfrac{2\pi N_2}{60}$

$\omega_2 = \dfrac{2\pi \times 3000}{60}$

$\omega_2 = 314.16$ rad/sec

Angular acceleration of crank, $\alpha_2 = 0$ (∵ const speed)

(i) Connecting rod angle. $\theta_3 = \sin^{-1}\left(\dfrac{-a\sin\theta_2}{b}\right) = \sin^{-1}\left(-\dfrac{0.05 \times \sin 30°}{0.2}\right)$

$\theta_3 = 352.82°$.

(ii) Angular velocity of connecting rod, $\omega_3 = -\dfrac{a\omega_2\cos\theta_2}{b\cos\theta_3} = \dfrac{-0.05 \times 314.16 \times \cos 30}{0.2 \times \cos 352.82}$

$\omega_3 = -68.56$ rad/sec.

(iii) Angular acceleration of connecting rod,

$$\alpha_3 = \frac{a\omega_2^2 \sin\theta_2 - a\cos\theta_2\alpha_2 + b\omega_3^2 \sin\theta_3}{b\cos\theta_3}$$

$$\alpha_3 = \frac{0.05 \times 314.16^2 \times \sin 30 + 0.2 \times (-68.56)^2 \sin 352.82}{0.2 \times \cos 352.82}$$

$\alpha_3 = 11842.43$ rad/sec² (ccw).

(iv) Velocity of piston, $V_c = \dfrac{a\omega_2 \sin(\theta_3-\theta_2)}{\cos\theta_3} = \dfrac{0.05 \times 314.16 \sin(352.82-30)}{\cos 352.82}$

$V_c = -9.57$ m/sec.

(v) Acceleration of piston, $A_c = \dfrac{a\alpha_2 \sin(\theta_3-\theta_2) - a\omega_2^2 \cos(\theta_3-\theta_2) - b\omega_3^2}{\cos\theta_3}$

$$A_c = \frac{0 - 0.05 \times 314.16^2 \times \cos(352.82-30) - 0.2 \times (-68.56)^2}{\cos 352.82}$$

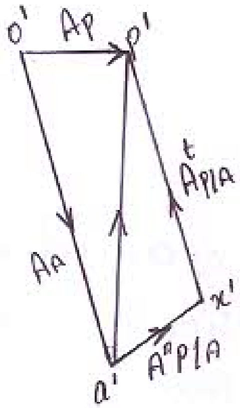
$A_c = -4910.4$ m/sec².

3a. Explain Klien's construction for slider crank mechanism.

Sol. A graphical method to find the velocity and acceleration of piston of a reciprocating engine mechanism was given by Prof. Klein.

(i) Velocity diagram.

1. Draw the slider crank mechanism OAP of the given position of crank OA as shown in fig.(a).
2. Draw a perpendicular from O to meet the extension of PA at M.
3. The triangle OAM is known as Klien's velocity diagram.



Fig(c).

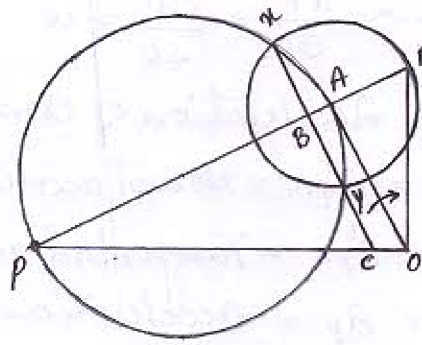
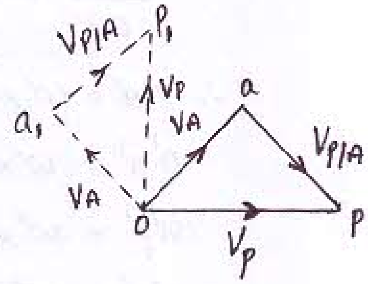


Fig (a)



Fig(b).

4. The velocity diagram obtained by relative method is shown in fig(b) as triangle Oap which is rotated by 90° and indicated by dotted lines as Oa_1P_1.

5. Since the triangle OAM obtained by Klein's construction, and the triangle Oa_1P_1 obtained by the relative velocity method are similar.

$$\frac{a_1P_1}{AM} = \frac{OP_1}{OM} = \frac{Oa_1}{OA} = \omega$$

$\therefore$ $a_1P_1 = V_{PA} = \omega . AM$ = Velocity of connecting rod.

$OP_1 = V_p = \omega . OM$ = Velocity of piston.

$Oa_1 = V_A = \omega . OA$ = Velocity of crank.

6. Also, $V_{PA} = \omega_{AP} \times AP$ $\therefore$ $\omega_{PA} \times Ap = \omega \times Am$.

$\therefore$ Angular Velocity of connecting rod $\omega_{CR} = \omega_{AP} = \omega . \frac{AM}{Ap}$.

(ii) Acceleration diagram (Klein's construction).

1. With A as centre, and radius equal to AM draw a circle.
2. With AP as diameter the second circle to intersect the first circle at points x and y.
3. Join x and y cutting AP at B and OP at C.
4. join OABC which forms a quadrilateral and known as klein's acceleration diagram.
5. Since the quadrilateral OABC obtained by klein's construction and the quadrilateral shown in fig (c) obtained by the relative acceleration method are similar.

$$\frac{o'a'}{OA} = \frac{a'x'}{AB} = \frac{x'p'}{BC} = \frac{p'o'}{CO} = \omega$$

$\therefore o'a' = \omega^2 \times OA$ = Acceleration of crank OA.

$a'x' = \omega^2 \times AB = A^n_{PA}$ = Normal acceleration of connecting rod

$x'p' = \omega^2 \times BC = A^t_{PA}$ = Tangential acceleration of —"—

$p'o' = \omega^2 \times OC = A_P$ = Acceleration of piston.

$A^t_{PA} = \alpha_{PA} \times AP$

$\therefore \omega^2 \times BC = \alpha_{PA} \times AP$

$\alpha_{PA} = \omega^2 \times \dfrac{BC}{AP}$ = Angular acceleration of connecting rod.

Proof:-

In order to prove the quadrilateral OABC represents the acceleration diagram, it is necessary to prove that the quadrilateral OABC is similar to the quadrilateral $o'a'x'p'$. For that, if it satisfies the following two conditions, then the two quadrilaterals are similar.

(i) The sides of quadrilateral OABC must be parallel to the side of quadrilateral $o'a'x'p'$.

(ii) Ratio of adjacent sides of quadrilateral must be equal to the ratio of corresponding adjacent sides to another quadrilateral.

(a) OC parallel to $o'p'$ by construction

OA parallel to $o'a'$ —"—

AB Parallel to $a'x'$ —"—

BC Parallel to $x'p'$ —"— Since XY is $\perp^r$ to AP.

Hence it satisfies the first condition.

(b) Join Px and Ax and from the triangle APx and ABx.

Angle AxP = Angle ABx = right angles

Angle PAx = Angle BAx = common angle.

$\therefore \angle APx = \angle AxB$

Therefore the two triangles are similar.

$$\therefore \frac{Ax}{AP} = \frac{AB}{Ax}$$

$$\text{i.e., } AB = \frac{Ax^2}{AP} = \frac{AM^2}{AP} \quad (\because Ax = AM)$$

Dividing b.s by OA.

$$\frac{AB}{OA} = \frac{AM^2}{AP.OA} \quad \text{— (1)}$$

$$\text{Now, } \frac{a'x'}{o'a'} = \frac{\text{Normal acceleration of connecting rod } AP}{\text{Absolute acceleration of crank } OA}$$

$$= \frac{V_{PA}^2 / AP}{\omega^2 \times OA} = \frac{\omega^2 \times AM^2}{AP.\omega^2 \times OA} \quad (\because V_{PA} = \omega.AM)$$

$$\text{i.e. } \frac{a'x'}{o'a'} = \frac{AM^2}{AP.OA} \quad \text{— (2)}$$

Equating (1) and (2), we get.

$$\frac{AB}{OA} = \frac{a'x'}{o'a'}$$

Hence it satisfies the second condition also.

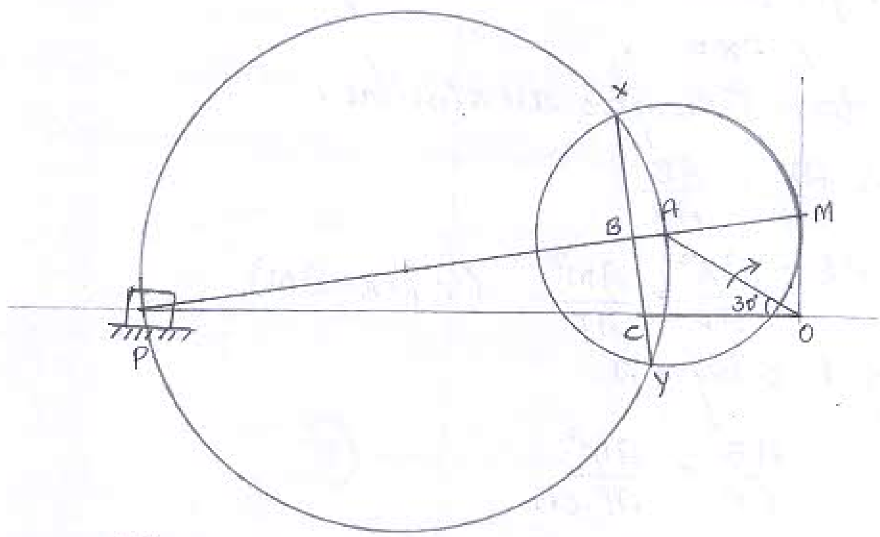
$\therefore$ Quadrilateral $OABC$ is similar to quadrilateral $o'a'x'p'$.

3b. The crank and connecting rod of a reciprocating engine are 200mm and 700mm respectively. The crank is rotating in clockwise direction at 120 rad/sec. Find with the help of Klien's construction:

(i) Velocity and acceleration of Piston.

(ii) Angular Velocity and angular acceleration of the connecting rod, at the instant when the crank is at 30° to the inner dead centre.

Scale
100mm = 1cm.



Given

$\omega_{OA} = 120 \text{rad/sec}$.

$OA = 200\text{mm} = 0.2\text{m}$

$AP = 700\text{mm} = 0.7\text{m}$.

(i) Velocity of piston, $V_{PO} = \omega \times OM$

$= 120 \times 130 \times 10^{-3}$

$= 15.6 \text{m/s}$.

acceleration of piston, $A_P = \omega^2 \times OC$

$= 120^2 \times 200 \times 10^{-3}$

$= 2880 \text{m/sec}^2$.

(ii) Angular velocity of connecting rod.

$$\omega_{AP} = \omega \times \frac{AM}{AP}$$

$$= \frac{120 \times 180}{700} = 30.85 \text{rad/sec}.$$

Angular acceleration of connecting rod.

$$\alpha_{AP} = \omega^2 \times \frac{BC}{AP}$$

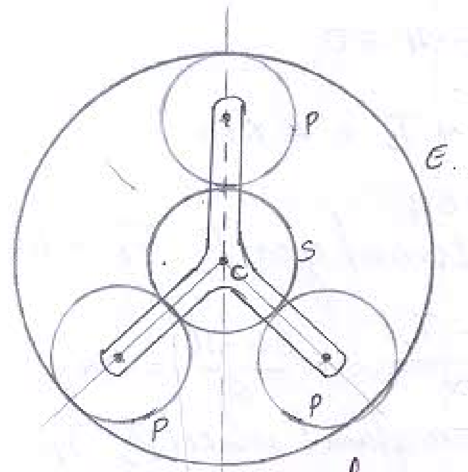
$$\alpha_{AP} = \frac{120^2 \times 100}{700} = 2057.14 \text{rad/sec}^2$$

4. An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star shaped carrier C. The sizes of different toothed wheels are such that the planet carrier C rotates at $1/5^{th}$ of the speed of the sun wheels. The minimum number of teeth on any wheel is 16. The driving torque on the sun wheels: The minimum number of teeth on any wheel is 100 Nm

(i) Number of teeth on different wheels of the train, and

(ii) Torque necessary to keep the internal gear stationary.

Sol



(i) Number of teeth on different wheels.

As the minimum number of teeth on any wheel is 16, take the number of teeth on sun wheel $T_S = 16$.

Since the pitch circle radius is proportional to number of teeth and the gears have same pitch.

$$r_E = r_S + 2r_p$$

$$\text{i.e } T_E = T_S + 2T_p$$

$$\therefore T_p = \frac{T_E - T_S}{2} \quad -(1)$$

Tabular column :-

Condition of motion.	Planet carrier C	Sun wheel S	Planet wheel P	Internal gear E.
Fix the planet C and give +1 rev to S	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_E}$
Multiply by x	0	x	$\frac{-T_S}{T_P} \cdot x$	$\frac{-T_S}{T_E} \cdot x$
Add y	y	$x+y$	$y - \frac{T_S}{T_P} \cdot x$	$y - \frac{T_S}{T_E} x$.

Planet carrier C rotates @ 1/5 of the speed of the Sunwheel S
i.e for every 5 revolutions of the sunwheels, planet carrier C will make 1 revolution.

$$\therefore y = 1 \text{ and } x + y = 5$$

$$\therefore x = 4.$$

Internal gear E is stationary

$$\text{i.e.,} \quad y - \frac{T_S}{T_E} x = 0$$

$$\text{i.e,} \quad 1 - \frac{T_S}{T_E} \cdot 4 = 0$$

$$\therefore T_E = 4 T_S = 4 \times 16$$

$$T_E = 64$$

i.e. Number of teeth on internal gear E, $T_E = 64$.

From eqn ①

$$T_P = \frac{T_E - T_S}{2} = \frac{64 - 16}{2} = 24.$$

i.e Number of teeth on planet wheel P, $T_P = 24$.

(ii) Torque necessary to keep the internal gear stationary

From energy equation

$$\tau_S N_S + \tau_C N_C + \tau_E N_E = 0$$

$$\text{i.e} \quad \tau_S N_S + \tau_C N_C = 0 \quad (\because N_E = 0)$$

$$100 \times 5 + \tau_C \times 1 = 0$$

$$\tau_C = -500 \text{ N-m}$$

$\therefore$ Torque on planet carrier $C = -500$ N-m.

From torque equation.

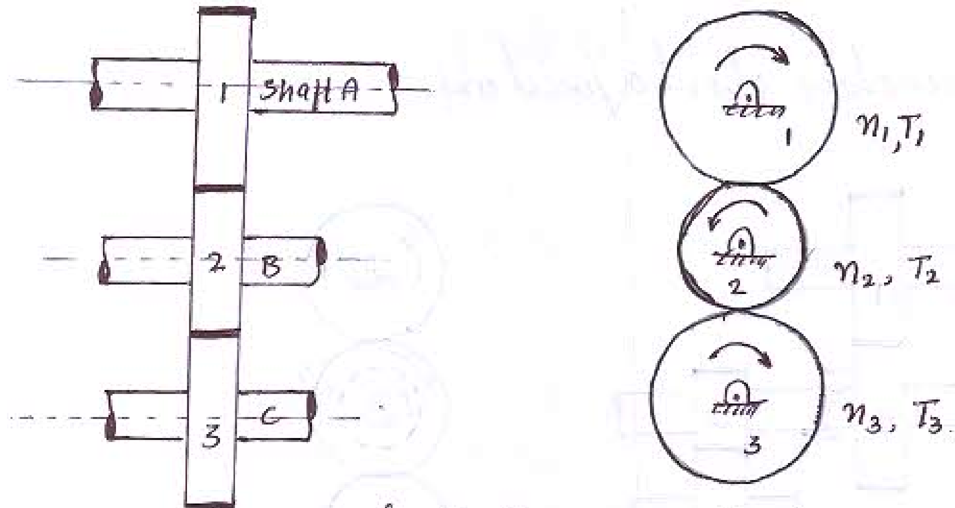
$$\tau_S + \tau_C + \tau_E = 0$$

$$100 - 500 + \tau_E = 0$$

Torque necessary to keep the internal gear E stationary $\tau_E = 400$ N-m.

5a. What are the types of gear trains? Explain with the help of neat sketches.

Sol. Simple gear train is as shown in fig.



1. In simple gear train each shaft carries only one gear.
2. All the gears revolve about fixed axis.
3. Velocity ratio (V.R):

let n_1, n_2 and n_3 are speeds of gears 1, 2 and 3 respectively. T_1, T_2 & T_3 are number of teeth on gears 1, 2 and 3 respectively.

Consider gear 1 to be the driver.

$\therefore$ V.R for gear 1 and gear 2 is, $\frac{n_1}{n_2} = \frac{T_2}{T_1}$ —(i)

V.R for gear 2 and gear 3 is, $\frac{n_2}{n_3} = \frac{T_3}{T_2}$ —(ii)

Multiplying (i) and (ii), we get

$$\frac{n_1}{n_2} \times \frac{n_2}{n_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$

i.e $V.R = \frac{n_1}{n_3} = \frac{T_3}{T_1}$

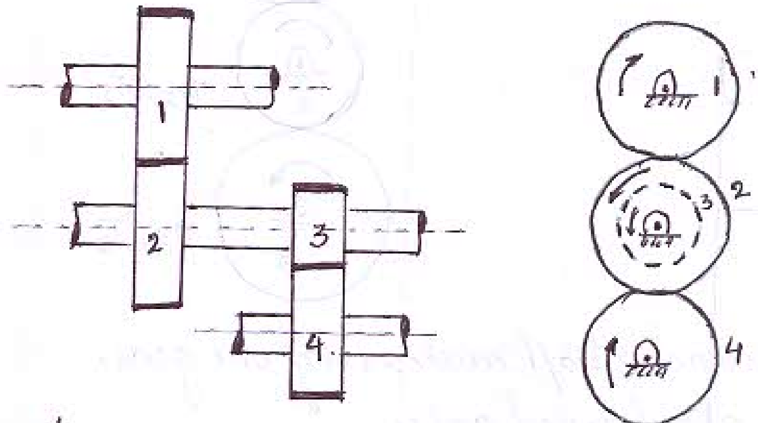
Therefore, the velocity ratio is independent of number of intermediate gears used. Hence Gear wheel 2 is called an idler.

Train value $= \frac{n_3}{n_1} = \frac{T_1}{T_3}$

Compound Gear train

(i) In compound gear train each shaft carries two or more gears except the first and last, one of which acts as a follower and the other as the driver.

(ii) All the gears revolves about a fixed axis.



(iii) Velocity ratio :-

Gear 1 is in mesh with gear 2.

$$\therefore V.R = \frac{n_1}{n_2} = \frac{T_2}{T_1} \quad \text{—(i)}$$

Similarly, VR for gear 3 & gear 4 is

$$\frac{n_3}{n_4} = \frac{T_4}{T_3} \quad \text{—(ii)}$$

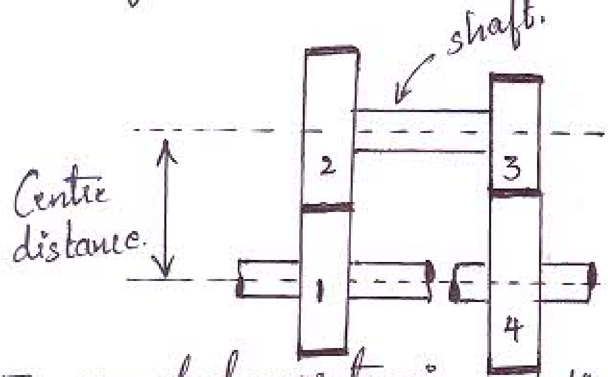
Multiplying (i) & (ii)

$$\frac{n_1}{n_2} \cdot \frac{n_3}{n_4} = \frac{T_2}{T_1} \cdot \frac{T_4}{T_3}$$

$$\text{i.e } V.R = \frac{n_1}{n_4} = \frac{T_2 \cdot T_4}{T_1 \cdot T_3} \quad (\because n_2 = n_3).$$

$$\therefore T.V = \frac{n_4}{n_1} = \frac{T_1 T_3}{T_2 \cdot T_4} = \frac{\text{Product of number of teeth on driver gears}}{\text{Product of number of teeth on driven gears.}}$$

Reverted gear train



(i) In reverted gear train the first and last gears are on the same axis.

(ii) In reverted gear train, the centre distances of the two pairs of gears must be the same.

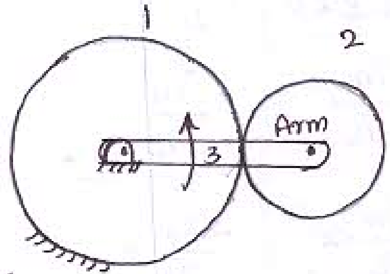
$$\therefore r_1 + r_2 = r_3 + r_4.$$

$$\therefore T_1 + T_2 = T_3 + T_4.$$

(iii) Train value = $\frac{T_1 T_3}{T_2 \cdot T_4}$, since gear 2 and gear 3 are compound gear.

(iv) All the gears revolve about a fixed axis.

Epicyclic gear train



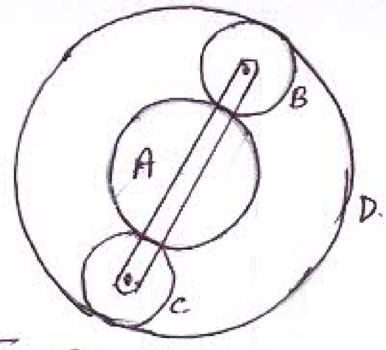
In Epicyclic gear train, the axes of rotation of all the wheels are not fixed. In epicyclic gear train, axes of some gears having relative motion with respect to other (or) relative to the frame. The gear 2 revolves about its own axis as well as about the centre of the fixed gear. Epicyclic gear train is also called planetary gear train.

5b. An epicyclic train of gears is arranged as shown in the fig. How many revolution does the arm, to which the pinions B and C are attached, make:

(i) when A makes one revolution clockwise and D makes half a revolution counter clockwise, and

(ii) when A makes one revolution clockwise and D is stationary.

The number of teeth on the gears A and D are 40 & 90 respectively.



Given:- $T_A = 40$, $T_D = 90$.

From fig, $r_A + 2r_B = r_D$

$$T_D = T_A + 2T_B$$

$$T_B = \frac{T_D - T_A}{2} = \frac{90-40}{2}$$

$$T_B = 25, \therefore T_C = 25$$

Tabular column :-

Condition of motion.	Arm	Gear A	Compound Gear B & C	Gear D
Fix arm & give +1 rev to A.	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_D}$
Multiply by x	0	x	$-\frac{T_A}{T_B}\cdot x$	$-\frac{T_A}{T_D}x$.
Add y	y	$x+y$	$y-\frac{T_A}{T_B}\cdot x$	$y-\frac{T_A}{T_D}x$

1. Speed of arm when A makes 1 revolution CW, & D makes half revolution CCW.

$\therefore x+y=1$ (1) and $N_D = y - \frac{T_A}{T_D}x \Rightarrow y - \frac{T_A}{T_D}x = -\frac{1}{2}$

$$y - \frac{40}{90}x = -0.5$$

$$y - 0.44x = -0.5 \quad (2)$$

equation (1) & (2). we get.

$x = 1.04$

$y = -0.0416$

$\therefore y = 0.0416$ CCW

2. Speed of arm when A makes 1 revolution CW & D is stationary.

$$x+y=1 \quad \text{— (1)} \qquad \text{and } N_D = y - \frac{T_A}{T_D}x = 0$$

$$y - \frac{40}{90}x = 0$$

$$y - 0.44x = 0 \quad \text{— (2)}$$

Equating (1) & (2), we get.

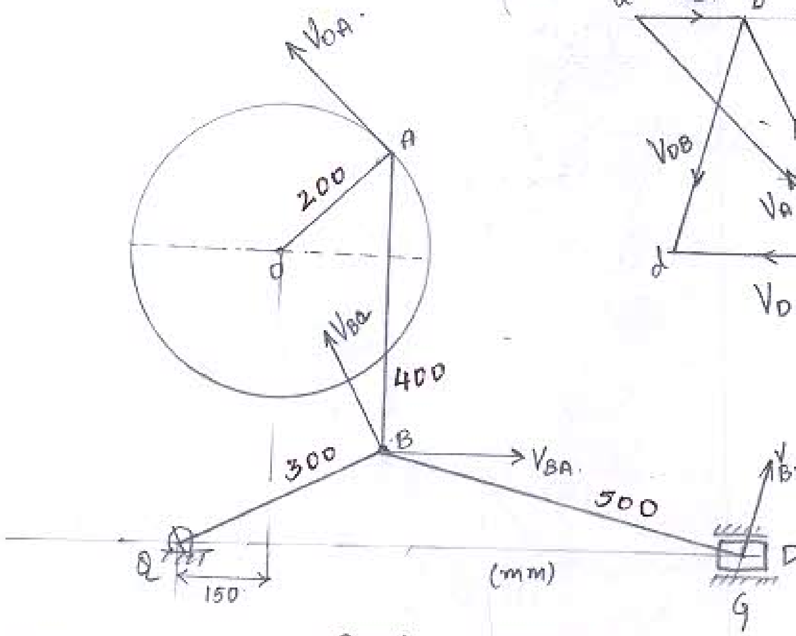
$$x = 0.694$$

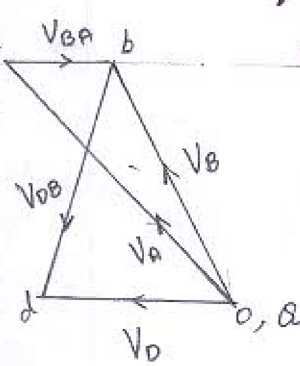
$$y = 0.305 \text{ CW}$$

6 In the toggle mechanism shown in the figure below, the crank OA rotates at 210 rpm counter clockwise increasing at the rate of $60\ rad/sec^2$. For the given configuration determine.

(i) Velocity of slider D and angular velocity of link BD.

(ii) Acceleration of slider D and angular acceleration of link BD.





$$\omega_{DB} = V_{DB}/BD = \frac{3.2}{0.5}$$

$$\omega_{DB} = 6.4\ rad/s.$$

$$a^n_{AB} = \frac{V_{AB}^2}{AB} = \frac{1.5^2}{0.4} = 5.625\ m/sec^2$$

$$a^n_{OA} = \frac{V_{OA}^2}{OA} = \frac{4.398^2}{0.2} = 96.71\ m/sec^2$$

$$a^n_{BD} = \frac{V_{BD}^2}{BD} = \frac{3.2^2}{0.5} = 20.48\ m/sec^2$$

$$a^n_{BQ} = \frac{V_{BQ}^2}{BQ} = \frac{3.6^2}{0.3} = 43.2\ m/sec^2.$$

$N_{OA} = 210\ rpm.$ $\quad \omega_{OA} = \frac{2\pi N}{60} = \frac{2\times\pi\times 210}{60}$

$\omega_{OA} = 21.99\ rad/sec$

$V_{OA} = \omega_{OA} \times OA$

$= \cancel{210 \times 0.2}$

$V_{OA} = 21.99 \times 0.2$

$V_{OA} = 4.398\ m/s.$

$a^t_{OA} = \alpha_{OA} \times OA$

$= 60 \times 0.2$

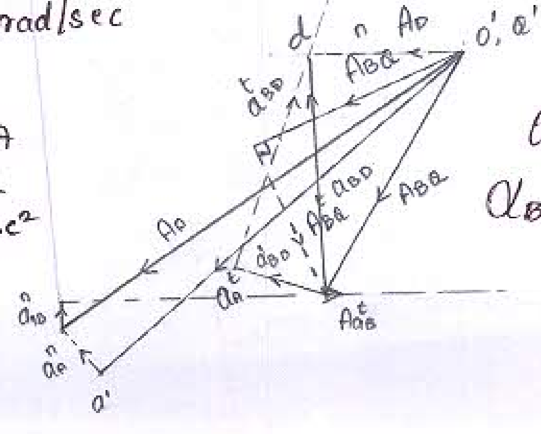
$= 12\ m/sec^2$

$V_{AB} = 1.5\ m/s$

$V_D = 2.5\ m/s$

$V_{BD} = 3.2\ m/s$

$V_B = 3.6\ m/s.$



$a_D = 2 \times 15 = 30\ m/sec^2$

$$\alpha_{BD} = \frac{a^t_{BD}}{BD} = \frac{3 \times 15}{0.5}$$

$$= 90\ rad/sec^2$$