CMR INSTITUTE OF TECHNOLOGY

Internal Assessment Test III – May 2017

Sub:	Error Control Coding			Code:	15ECS41
Date:		Duration: 90 mins	Max Marks: 50	Sem: M.Tech IV(DCE)	Branch: TCE

Note:Attempt any five full Questions. Each Question carries 10 marks.

1.Q Explain the construction procedure for standard array for decoding (n, k) linear block code. (10 Marks)

2.Q In a systematic [7 4] linear code whose parity matrix 'P' is given by

$$H = \begin{bmatrix} 110 \\ 011 \\ 111 \\ 101 \end{bmatrix}$$

i. Find all possible valid code vectors.
ii. Find the probability of undetected error considering BSC if transition probability $p = 10^{-2}$
iii. Find the minimum distance of this code.
iv. Determine the error detection and correction capability. (3+3+2+2=10 Marks)

3.Q In a cyclic code the generator polynomial is given by $g(x) = 1 + x + x^3$. Find the codeword for the message 1011 and 1111 using ENCODER Circuit. If the received data is 1011011 find the syndrome using DECODER circuit. Show the contents of registers in each step. (10 Marks)

4.Q Let $g(x) = g0\, x^0 + g1\, x^1 + g2\, x^2 \ldots\ldots +gn-k\, x^{n-k}$ be non-zero code polynomial of minimum degree cyclic code 'C'. Show that:

a. The first tem of $g(x)$ i;e $g0 = 1$
b. The generator polynomial g(x) of an (n, k) cyclic code is a factor of $x^n + 1$.
c. The polynomial generates an (n, k) cyclic code. (10 Marks)

5.Q With the help of Meggitt decoder diagram, explain the general decoder operation for (n, k) cyclic code. (10 Marks)

6.Q

a. Explain the decoding procedure for fire codes. (04Marks)
b. Write a note on inter leaved convolution code. (06Marks)

7.Q Explain the error trapping decoder structure for bust-error-correcting codes with block diagram. Also explain the decoding procedure. (10Marks)

1Q Standard array for decoding definition

Let 'C' be an (n, k) linear code.
Let $V_1, V_2 \cdots V_{2^k}$ be the codewords of C
The received vector 'r' for a codeword transmitted over a noisy channel will be one of the 2^n n-tuples over $GF(2)$.

To decode received vector 'r', the 2^n possible received vectors are partitioned into 2^k disjoint subsets $D_1, D_2, \cdots D_{2^k}$.
Where, each subsets D_i is one-to-one correspondance to a codeword V_i.

The received vector 'r' is decoded as 'V_i' if 'r' is found in the subset of 'D_i'.
One such method to partition 2^n possible received vector into 2^k disjoint subsets such that each subset contains one & only one codeword is <u>standard array decoding</u>.

Procedure:-

(i) The 2^k codeword of 'C' are placed in a row with all zero codeword $V_1 = (0, 0 \cdots\cdots 0)$ being the first element of that row.

(ii) From remaining $2^n - 2^k$ an n-tuple e_2 is selected & placed under V_1.

(iii) Sum of $e_2 + V_2$ is placed under V_2, similarly sum of $e_2 + V_i$ is placed under V_i & 2nd row is completed.

(iv) From the remaining n-tuples, an n-tuple e_3 is selected & placed under V_1 in 3rd row; & corresponding $e_3 + V_i$ is placed under V_i.

(v) This process is continued till all the n-tuples are used.

(vi) This array is called standard array & is represented as shown in fig(1)

$$
\begin{array}{cccccc}
V_1 = 0 & V_2 & V_3 & \cdots & V_i \cdots & V_{2^k} \\
e_2 & e_2+V_2 & e_2+V_3 & \cdots & e_2+V_i \cdots & e_2+V_{2^k} \\
e_3 & e_3+V_2 & e_3+V_3 & \cdots & e_3+V_i \cdots & e_3+V_{2^k} \\
e_4 & e_4+V_2 & e_4+V_3 & \cdots & e_4+V_i \cdots & e_4+V_{2^k} \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
e_l & \vdots & \vdots & & \vdots & \vdots \\
\vdots & \vdots & \vdots & & \vdots & \vdots \\
e_{2^{n-k}} & e_{2^{n-k}}+V_2 & e_{2^{n-k}}+V_3 & \cdots & e_{2^{n-k}}+V_i \cdots & e_{2^{n-k}}+V_{2^k}
\end{array}
$$

$\boxed{28}$ Given a [7,4] Systematic linear code,

$$
'P' = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}
$$

(i) All possible code words,

Message	code word	weight	
0000	0000000	0	$A_0 = 1$
1000	1101000	3	
0100	0110100	3	
1100	1011100	4	$A_3 = 7$
0010	1110010	3	$A_4 = 7$
1010	0011010	3	
0110	1000110	3	$A_1 = A_2 = A_5 = A_6 = 0$
1110	0101110	4	
0001	1010001	3	
1001	0111001	4	
0101	1100101	4	
1101	0001101	3	
0011	0100011	3	
1011	1001011	4	
0111	0010111	4	
1111	1111111	7	$A_7 = 1$

$$G = \begin{bmatrix} P_{k\times(n-k)} & I_{k\times k} \end{bmatrix} \qquad n=7,\ k=4$$

$$G = \begin{bmatrix} 1&1&0&1&0&0&0 \\ 0&1&1&0&1&0&0 \\ 1&1&1&0&0&1&0 \\ 1&0&1&0&0&0&1 \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix}$$

$$u = [u_0\ u_1\ u_2\ u_3]_{1\times 4}.$$

$$v = u \cdot G = u_0 g_0 + u_1 g_1 + u_2 g_2 + u_3 g_3$$

(ii) pb. of undetected error is given by,

$$P_u(E) = \sum_{i=1}^{n} A_i\, p^i (1-p)^{n-i}, \quad \text{here } n=7$$

where,

$A_1 = A_2 = A_5 = A_6 = 0$

$A_0 = A_7 = 1$

$A_3 = A_4 = 7.$

$$P_u(E) = A_3 p^3 (1-p)^4 + A_4 p^4 (1-p)^3 + A_7 p^7 (1-p)^0$$

$$= 7 \cdot p^3 (1-p)^4 + 7 \cdot p^4 (1-p)^3 + p^7$$

$$\boxed{P_u(E) \simeq 7 \times 10^{-6}} \; ; \; p = 10^{-2}$$

(iii). Minimum distance of given linear block code,

$d_{min} \triangleq W_{min}$

where, $W_{min} \rightarrow \{ w(x) : x \in C, x \neq 0 \}$

Here, $W_{min} = 3.$

Hence, $\boxed{d_{min} = 3}$

(OR) $H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ $\Rightarrow$ Sum of smallest no. of columns of $H' = 0$

$\rightarrow \boxed{d_{min} = 3}$

(iv) Error detection capability $\Rightarrow t_d = \frac{d_{min}}{2}$

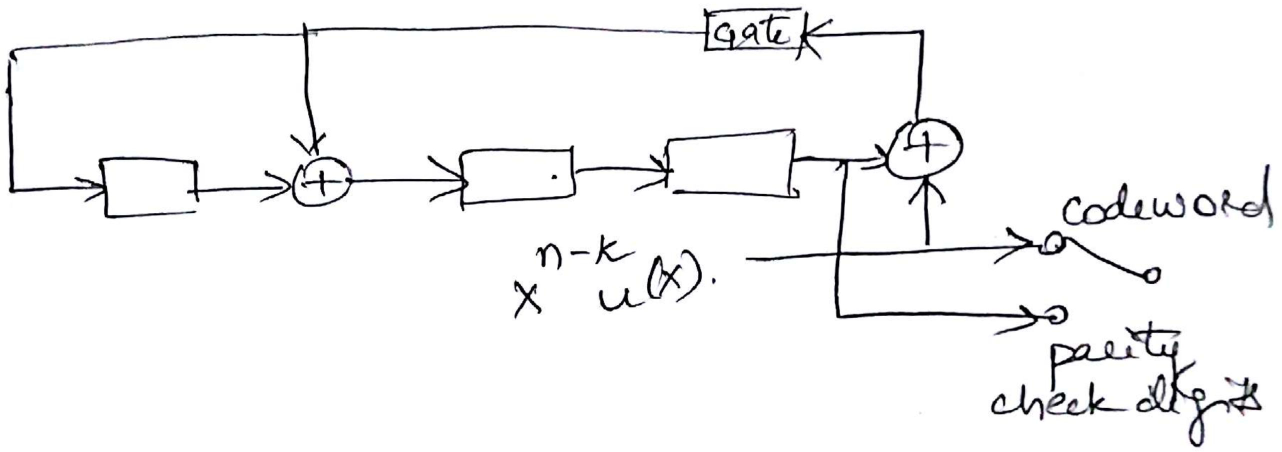
$= \frac{3}{2} \approx \underline{\underline{1}}$

Error correction capability $\Rightarrow t_c = \frac{(d_{min} - 1)}{2}$

$= \frac{2}{2} = \underline{1}$

$\boxed{3Q}$ Given $g(x) = 1 + x + x^3$ generator polynomial in a cyclic code.

- Encoder circuit



- ① for message 1011

message	Register contents
–	0 0 0
1	1 1 0
1	1 0 1
0	1 0 0
1	1 0 0

Code word is

100 1011

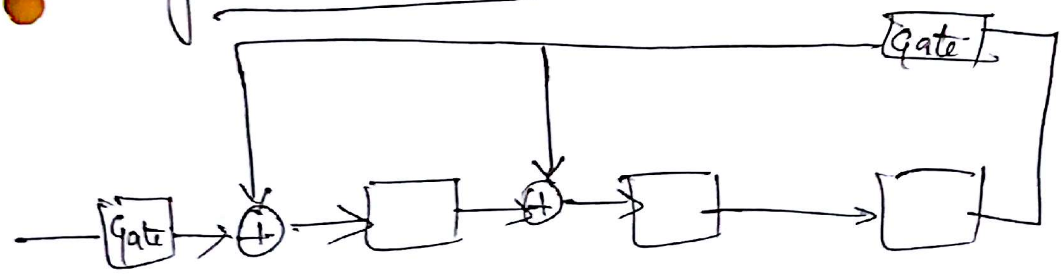
(2) for message 1111

message	Register contents
–	0 0 0
1	1 1 0
1	1 0 1
1	0 1 0
1	1 1 1

codeword is

111 1111.

Syndrome ckt



i/p	shift Register content
–	0 0 0.
1	1 0 0
1	1 1 0
0	0 1 1
1	0 1 1
1	0 1 1
0	1 1 1
1	0 0 1

Syndrome is $\underline{\underline{001}}$

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Proof : i) Suppose $g_0 = 0$, then

$$g(x) = g_1 x + g_2 x^2 + \cdots + g_{n-k} x^{n-k}.$$

$$= x \left(g_1 + g_2 x + \cdots + g_{n-k} x^{n-k-1} \right) \quad \text{(1)}$$

The degree of $g(x) = g_0 x^0 + g_1 x^1 + g_2 x^2 + \cdots g_{n-k} x^{n-k}$ is "$n-k$"

From eqn (1) if we cyclicly shift it by "n-1" places to the right (or to one place by left)

we have a non zero code polynomial

$$= g_1 + g_2 x + \cdots + g_{n-k} x^{n-k-1}.$$

This contradicts to the assumption that $g(x)$ is a nonzero code polynomial with minimum degree "$n-k$"

→ Thus ~~g~~ $g_0 \neq 0$

ii) → Let $g(x)$ be a generator polynomial of an (n,k) cyclic code

→ multipling $g(x)$ by 'x^k' we have, $x^k \cdot g(x)$ of degree 'n'

→ Dividing $x^k \cdot g(x)$ by x^n+1, we obtain

$$x^k g(x) = (x^n+1) + g^{(k)}(x). \longrightarrow (2)$$

Where, $g^{(k)}(x)$ is remainder.

→ W.K.T $x^i v(x) = q(x)(x^n+1) + v^{(i)}(x)$.
Where. $v(x)$ is a codeword & $v^{(i)}(x)$ is also a codeword obtained by cyclicly shifting $v(x)$

→ Thus $g^{(k)}(x)$ is a code polynomial obtained by shifting $g(x)$ cyclicly to the right 'k' times.

$$\therefore g^{(k)}(x) = a(x).g(x) \longrightarrow (3)$$

→ Substiting (3) in (2).

$$x^k g(x) = (x^n+1) + a(x) \cdot g(x).$$

$$x^n+1 = x^k g(x) + a(x)\, g(x)$$

$$= \cancel{(x^k + g(x) + g(x))}$$

$$x^n+1 = g(x)\,(x^k + a(x))$$

Thus $g(x)$ is a factor of x^n+1

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iii) Let $g(x), xg(x), \ldots, x^{k-1}g(x)$ be k - polynomials of degree "n-1" or less.

→ A linear combination of all these 'k' polynomials result in

$$V(x) = a_0 g(x) + a_1 x g(x) + \cdots + a_{k-1} x^{k-1} g(x)$$

$$= (a_0 + a_1 x + \cdots + a_{k-1} x^{k-1}) g(x)$$

Where $V(x)$ is a polynomial of degree "n-1" or less & is a multiple of $g(x)$.

→ Let $v(x) = v_0 + v_1 x + \cdots + v_{n-1} x^{n-1}$ be a code polynomial in this code. Multiplying $v(x)$ by x, we obtain.

$$x v(x) = v_0 x + v_1 x^2 + \cdots + v_{n-2} x^{n-1} + v_{n-1} x^n$$

$$= v_{n-1}(x^n + 1) + (v_{n-1} + v_0 x + \cdots + v_{n-2} x^{n-1})$$

$$= v_{n-1}(x^n + 1) + v^{(1)}(x).$$

Where $v^{(1)}(x)$ is a cyclic shift of $v(x)$.

→ Since both $xv(x)$ & $x^n + 1$ are divisible by $g(x)$, $v^{(1)}(x)$ is also divisible by $g(x)$

→ Hence $v^{(1)}(x)$ is a code polynomial formed by linear combination of $g(x), xg(x), \ldots, x^{k-1}g(x)$

→ Thus polynomial $g(x)$ generates (n,k) cyclic code

5.5 DECODING OF CYCLIC CODES

Decoding of cyclic codes consists of the same three steps as for decoding linear codes: syndrome computation, association of the syndrome with an error pattern, and error correction. It was shown in Section 5.4 that syndromes for cyclic codes can be computed with a division circuit whose complexity is linearly proportional to the number of parity-check digits (i.e., $n-k$). The error-correction step is simply adding (modulo-2) the error pattern to the received vector. This addition can be performed with a single EXCLUSIVE-OR gate if correction is carried out serially (i.e., one digit at a time); n EXCLUSIVE-OR gates are required if correction is carried out in parallel, as shown in Figure 3.8. The association of the syndrome with an error pattern can be completely specified by a decoding table. A straightforward approach to the design of a decoding circuit is via a combinational logic circuit that implements the table-lookup procedure; however, the limit to this approach is that the complexity of the decoding circuit tends to grow exponentially with the code length and with the number of errors that are going to be corrected. Cyclic codes have considerable algebraic and geometric properties. If these properties are properly used, decoding circuits can be simplified.

The cyclic structure of a cyclic code allows us to decode a received vector $\mathbf{r}(X) = r_0 + r_1X + r_2X^2 + \cdots + r_{n-1}X^{n-1}$ serially. The received digits are decoded one at a time, and each digit is decoded with the same circuitry. As soon as the syndrome has been computed the decoding circuit checks whether the syndrome $\mathbf{s}(X)$ corresponds to a correctable error pattern $\mathbf{e}(X) = e_0 + e_1X + \cdots + e_{n-1}X^{n-1}$ with an error at the highest-order position X^{n-1} (i.e., $e_{n-1} = 1$). If $\mathbf{s}(X)$ does not correspond to an error pattern with $e_{n-1} = 1$, the received polynomial (stored in a buffer register) and the syndrome register are cyclically shifted once simultaneously. Thus, we obtain $\mathbf{r}^{(1)}(X) = r_{n-1} + r_0X + \cdots + r_{n-2}X^{n-1}$, and the new contents in the syndrome register form the syndrome $\mathbf{s}^{(1)}(X)$ of $\mathbf{r}^{(1)}(X)$. Now, the second digit r_{n-2} of $\mathbf{r}(X)$ becomes the first digit of $\mathbf{r}^{(1)}(X)$. The same decoding circuit will check whether $\mathbf{s}^{(1)}(X)$ corresponds to an error pattern with an error at location X^{n-1}.

If the syndrome $\mathbf{s}(X)$ of $\mathbf{r}(X)$ does correspond to an error pattern with an error at location X^{n-1} (i.e., $e_{n-1} = 1$), the first received digit r_{n-1} is an erroneous digit, and it must be corrected. The correction is carried out by taking the sum $r_{n-1} \oplus e_{n-1}$. This correction results in a modified received polynomial, denoted by $\mathbf{r}_1(X) = r_0 + r_1X + \cdots + r_{n-2}X^{n-2} + (r_{n-1} \oplus e_{n-1})X^{n-1}$. The effect of the error digit e_{n-1} on the syndrome is then removed from the syndrome $\mathbf{s}(X)$, by adding the syndrome of $\mathbf{e}'(X) = X^{n-1}$ to $\mathbf{s}(X)$. This sum is the syndrome of the modified received polynomial $\mathbf{r}_1(X)$. Now, $\mathbf{r}_1(X)$ and the syndrome register are cyclically shifted once simultaneously. This shift results in a received polynomial $\mathbf{r}_1^{(1)}(X) = (r_{n-1} \oplus e_{n-1}) + r_0X + \cdots + r_{n-2}X^{n-1}$. The syndrome $\mathbf{s}_1^{(1)}(X)$ of $\mathbf{r}_1^{(1)}(X)$ is the remainder resulting from dividing $X[\mathbf{s}(X) + X^{n-1}]$ by the generator polynomial

$g(X)$. Because the remainders resulting from dividing $Xs(X)$ and X^n by $g(X)$ are $s^{(1)}(X)$ and 1, respectively, we have

$$s_1^{(1)}(X) = s^{(1)}(X) + 1.$$

Therefore, if 1 is added to the left end of the syndrome register while it is shifted, we obtain $s_1^{(1)}(X)$. The decoding circuitry proceeds to decode the received digit r_{n-2}. The decoding of r_{n-2} and the other received digits is identical to the decoding of r_{n-1}. Whenever an error is detected and corrected, its effect on the syndrome is removed. The decoding stops after a total of n shifts. If $e(X)$ is a correctable error pattern, the contents of the syndrome register should be zero at the end of the decoding operation, and the received vector $r(X)$ has been correctly decoded. If the syndrome register does not contain all 0's at the end of the decoding process, an uncorrectable error pattern has been detected.

A general decoder for an (n, k) cyclic code is shown in Figure 5.8. It consists of three major parts: (1) a syndrome register, (2) an error-pattern detector, and (3) a buffer register to hold the received vector. The received polynomial is shifted into the syndrome register from the left end. To remove the effect of an error digit on the syndrome, we simply feed the error digit into the shift register from the left end through an EXCLUSIVE-OR gate. The decoding operation is as follows:

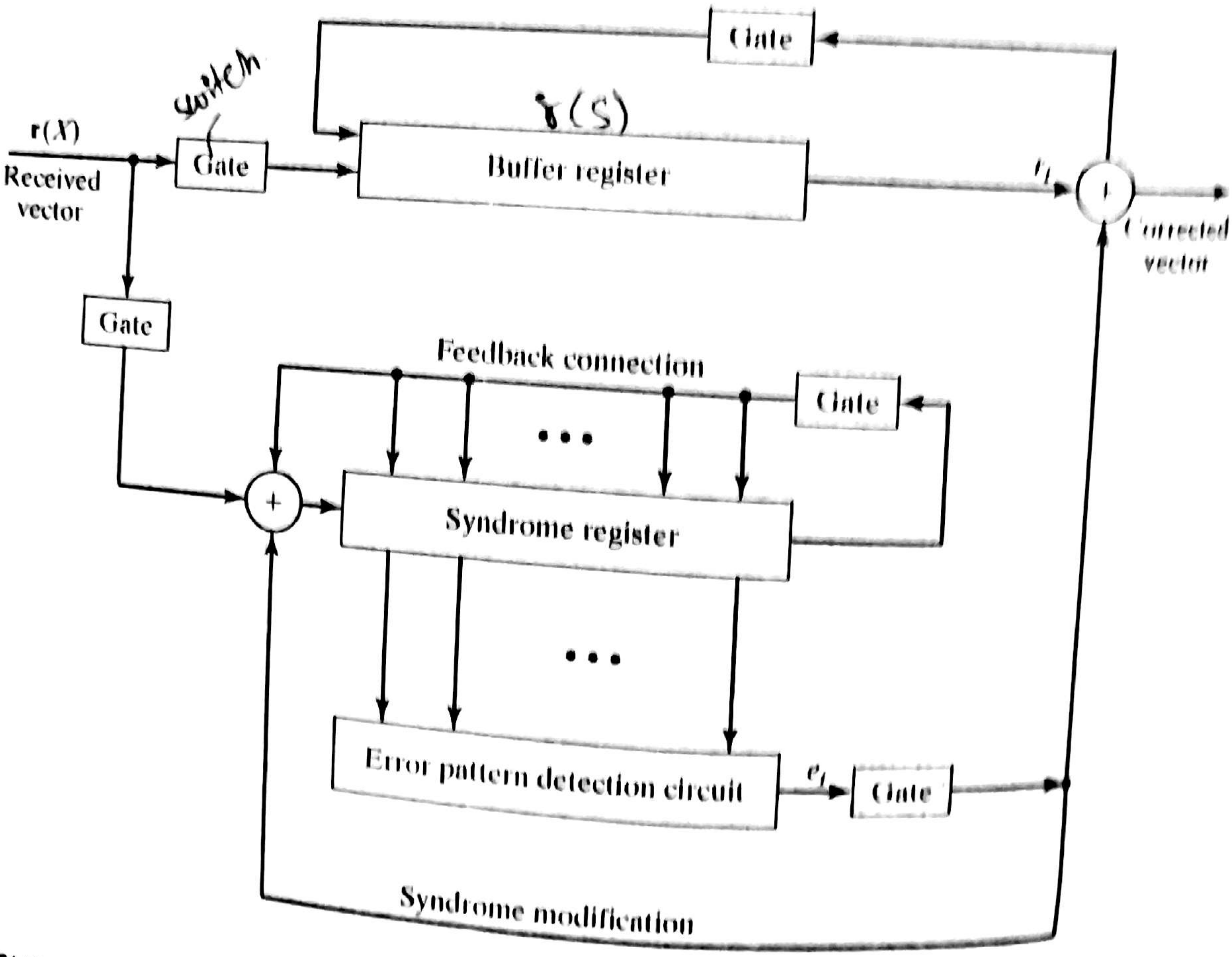

FIGURE 5.8: General cyclic code decoder with received polynomial $r(X)$ shifted into the syndrome register from the left end.

Step 1. The syndrome is formed by shifting the entire received vector into the syndrome register. The received vector is simultaneously stored in the buffer register.

Step 2. The syndrome is read into the detector and is tested for the corresponding error pattern. The detector is a combinational logic circuit that is designed in such a way that its output is 1 if and only if the syndrome in the syndrome register corresponds to a correctable error pattern with an error at the highest-order position X^{n-1}. That is, if a 1 appears at the output of the detector, the received symbol in the rightmost stage of the buffer register is assumed to be erroneous and must be corrected; if a 0 appears at the output of the detector, the received symbol at the rightmost stage of the buffer register is assumed to be error-free, and no correction is necessary. Thus, the output of the detector is the estimated error value for the symbol to come out of the buffer.

Step 3. The first received symbol is read out of the buffer. At the same time, the syndrome register is shifted once. If the first received symbol is detected to be an erroneous symbol, it is then corrected by the output of the detector. The output of the detector is also fed back to the syndrome register to modify the syndrome (i.e., to remove the error effect from the syndrome). This operation results in a new syndrome, which corresponds to the altered received vector shifted one place to the right.

Step 4. The new syndrome formed in step 3 is used to detect whether the second received symbol (now at the rightmost stage of the buffer register) is an erroneous symbol. The decoder repeats steps 2 and 3. The second received symbol is corrected in exactly the same manner as the first received symbol was corrected.

Step 5. The decoder decodes the received vector symbol by symbol in the manner outlined until the entire received vector is read out of the buffer register.

The preceding decoder is known as a Meggitt decoder [11], which applies in principle to any cyclic code. But whether it is practical depends entirely on its error-pattern detection circuit. In some cases the error-pattern detection circuits are simple. Several of these cases are discussed in subsequent chapters.

... gate 3 and clone gate 4

Fire codes

-> Fire Codes are systematic codes for burst error correction

-> Consider a polynomial $p(x)$ of degree m over GF(2)

and ρ is the smallest +ve integer. such that

$p(x)$ divides $x^\rho + 1$

Then the generator polynomial will consist of.

$$g(x) = (x^{2l-1} + 1)\, p(x)$$

where $l \leq m$.

$$n = \mathrm{LCM}(2l-1, \rho)$$

so we say Fire code of (n, k) where k is the degree of k generator polynomial.

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Q7

Step1: move all the content of the contents of the received vector into syndrome register and buffer register. Once the content is moved to syndrome register syndrome S(x) is calculated.

Step2: If the (n-k-l) is all zero before the (n-k) iteration of the shift register then the parity bits contain error if All zeros are not present then the burst error is present in. not in the parity bits of r(x).

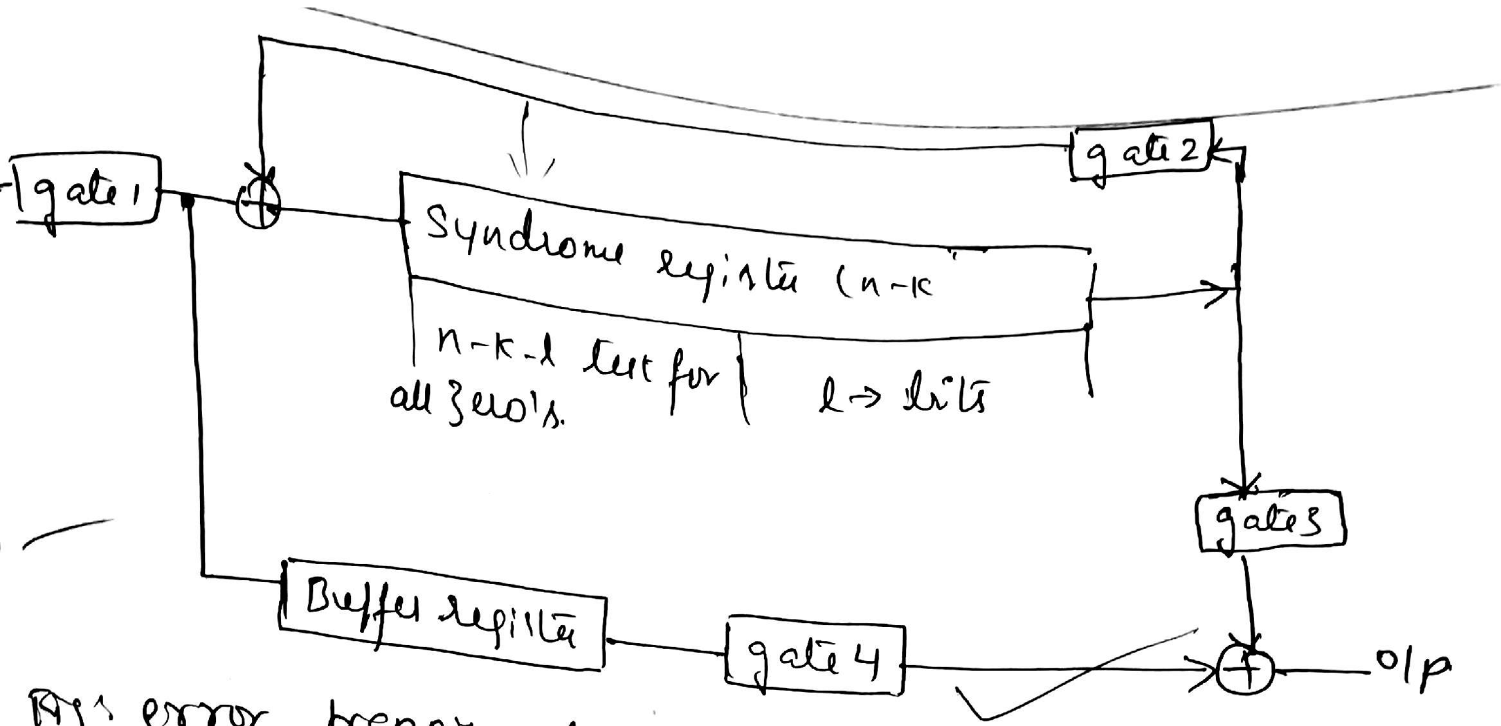

Fig: error trapping decoder structure for burst error correcting codes.

Step 3: If the (n-k-l) is all zero's after the (n-k-l+i) shift then the burst error is present in the polynomial at position equal = $X^{n-i} \ldots X^{n-1}, X^0 \ldots X^{l-i}$

Step 4: If the (n-k-l) bits are all zero then the parity bit of the r(x) contain the burst error

Step 5: If the (n-k-l) is not at all zero even after k shifts then the content of the r(x) is read out from Buffer register by activating gate 4. and simultaneously the gate 2 is activated and contents are shifted ~~into~~ syndrome register. If the content at (n-k-l) becomes zero then.