HEAT TRANSFER (15ME63)

INTERNAL ASSESMENT TEST - I SOLUTION

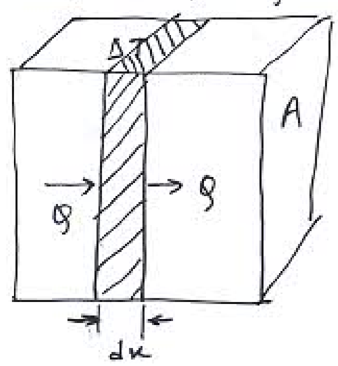
1) a) The three basic modes of heat transfer are:

- → Conduction
- → Convection
- → Radiation.

The laws governing these modes of heat transfer are as follows -

i) <u>Fourier's Law of Conduction:</u>

It states that the rate of heat flow by conduction in any direction is directly proportional to the temp. gradient and the area perpendicular to the flow direction.



$$Q \propto A$$

and $Q \propto \frac{dT}{dx}$

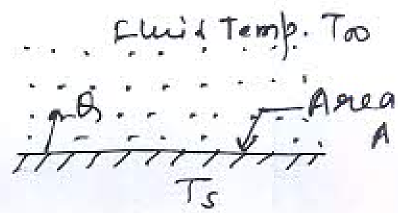
$$\Rightarrow \boxed{Q = -kA\frac{dT}{dx}}$$

$k \rightarrow$ thermal conductivity. (W/mK).

ii) <u>Newton's Law of Cooling:</u>

It states that heat transfer is proportional to the surface area exposed to the fluid and the temp. difference between the solid surface and fluid.

$Q \propto A_s$ and $Q \propto (T_s - T_\infty)$



$$\Rightarrow \boxed{Q = hA_s(T_s - T_\infty)}$$

$h \rightarrow$ heat transfer coefficient (W/m^2K)

iii) Steafan. Boltzman Law for radiation heat transfer:

It states that the heat radiated is proportional to the fourth power of the absolute temperature of the surface and the area available for radiative heat transfer.

$$Q \propto A \quad \text{and} \quad Q \propto T^4$$

$$\Rightarrow \boxed{Q = \epsilon \sigma A T^4}$$

Net rate of heat transfer between a real surface & surroundings is given by

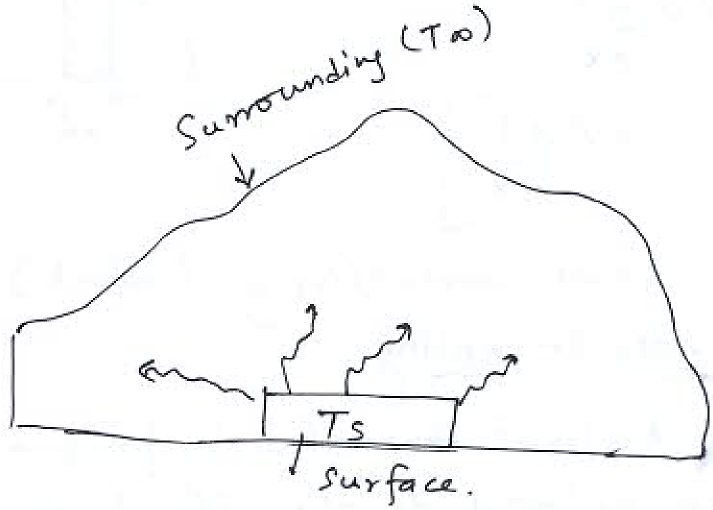
$$Q = \epsilon \sigma A \left(T_s^4 - T_\infty^4\right)$$

T_s → Surface temperature

T_∞ → Surrounding temperature

ϵ → Emissivity of the surface.

σ → Steafan-Boltzman constant.



1) b) The different factors affecting the value of heat transfer coefficient are as follows -

- → Surface Geometry
- → Surface Roughness.
- → Nature of fluid Motion
- → Properties of fluid (ρ, μ, C_p, k)
- → Bulk fluid velocity.

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2) a) <u>Thermal conductivity</u>:

Thermal conductivity is a physical property of the material.

It is defined as the ability of material to conduct heat through it.

Thermal conductivity of a material is the measure of how fast heat will flow in that material.

It is denoted by 'k' & its SI unit is w/mK.

$$\boxed{K = -\frac{Q}{A\,(dT/dx)}}$$

b) <u>Thermal Diffusivity</u>:

It is the property of a material that represents how fast heat propagates through the material.

It is the ratio of heat conducted to the heat energy stored per unit volume within the material.

$$\boxed{\alpha = \frac{k}{\rho c}}$$

K → Thermal conductivity.

ρc → heat capacity.

c) Thermal Resistance:

Thermal resistance is a heat property & a measurement of the resistance offered by a material to the flow of heat through a material.

Thermal resistance is the reciprocal of thermal conductance.

Its unit is K/W.

The concept of thermal resistance can be utilized to solve steady state heat transfer problems that involve series, parallel or combined series-parallel components.

d) Overall Heat Transfer Coefficient:

Overall heat transfer coefficient represents the intensity of heat transfer when heat is transferred from one fluid to another through a wall separating them. Numerically, it is equal to the quantity of heat passing through unit area of wall surface when temperature difference is equal to unity.

Its unit is W/m^2K.

$$\boxed{U = \frac{Q}{A \Delta T}}$$

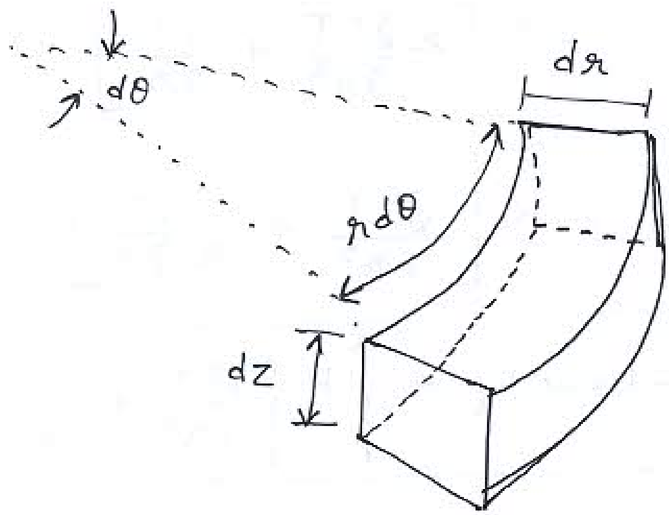
3) General Heat Conduction Equation in Cylindrical Co-ordinates:

Assumptions:

→ Material is homogeneous and isotropic.

→ ρ, C, and K remain constant w.r.t. time.

→ Temp. gradient exists in all the directions.



Consider the above cylindrical element with the mentioned dimensions.

Applying energy balance for the element,

we get,

$$\boxed{E_{in} - E_{out} + E_{gen} = E_{st}} \quad - (1)$$

E_{in} → Rate of energy entering the element.

E_{out} → Rate of energy leaving the element.

E_{gen} → Rate of energy generation within the element.

E_{st} → Rate of energy stored within the element.

Now, $(E_{in} - E_{out})_r = q_r - q_{r+dr}$

$$= -\frac{\partial}{\partial r} q_r \, dr$$

$$= -\frac{\partial}{\partial r}\left[-k\,(r\,d\theta \cdot dz) \cdot \frac{\partial T}{\partial r}\right] dr$$

$$= k\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] d\theta \cdot dz \cdot dr.$$

$$= k\left[r\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\right] d\theta \cdot dr \cdot dz$$

$$= k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] r\,d\theta \cdot dr \cdot dz.$$

or. $$\boxed{(E_{in} - E_{out})_r = k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] dV.}$$

$$(E_{in} - E_{out})_\theta = -\frac{\partial}{\partial \theta} q_\theta \cdot d\theta.$$

$$= -\frac{\partial}{\partial \theta}\left[-k\,(dr \cdot dz)\,\frac{\partial T}{r\,\partial \theta}\right] d\theta$$

$$= k\left[\frac{1}{r}\frac{\partial^2 T}{\partial \theta^2}\right] dr \cdot d\theta \cdot dz.$$

$$= k\left[\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right] r\,d\theta \cdot dr \cdot dz$$

or $$\boxed{(E_{in} - E_{out})_\theta = k\left[\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right] dV}$$

$$(E_{in} - E_{out})_z = -\frac{\partial}{\partial z} q_z \, dz$$

$$= \frac{-\partial}{\partial z}\left[-k\,(r\,d\theta . dr)\frac{\partial T}{\partial z}\right] dz$$

$$= k\left[\frac{\partial^2 T}{\partial z^2}\right] r\,d\theta . dr . dz.$$

or $\boxed{(E_{in} - E_{out})_z = k\left[\frac{\partial^2 T}{\partial z^2}\right] dV}$

Now, $E_{in} - E_{out} = (E_{in} - E_{out})_r + (E_{in} - E_{out})_\theta + (E_{in} - E_{out})_z$

$$\Rightarrow \boxed{E_{in} - E_{out} = k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] dV} \quad -2$$

Let, rate of energy generated per unit volume be $\dot{q}_g$

$$\therefore \boxed{E_{gen} = \dot{q}_g \times dV} \quad -3$$

$$E_{st} = mC\frac{dT}{d\tau}$$

$$\Rightarrow E_{st} = (\rho\, dV.)\, C\frac{dT}{d\tau}$$

$$\Rightarrow \boxed{E_{st} = \rho C\frac{dT}{d\tau} . dV} \quad -4$$

from eq^n 1, 2, 3 and 4

we get,

$$k\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] dV + \dot{q}_g\, dV = \rho C\frac{\partial T}{\partial \tau} dV$$

$$\Rightarrow \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\dot{q}_g}{K} = \frac{\rho c}{K}\frac{\partial T}{\partial \tau}$$

$$\Rightarrow \boxed{\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\dot{q}_g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

where, $\alpha = \frac{K}{\rho c}$ & is called thermal diffusivity.

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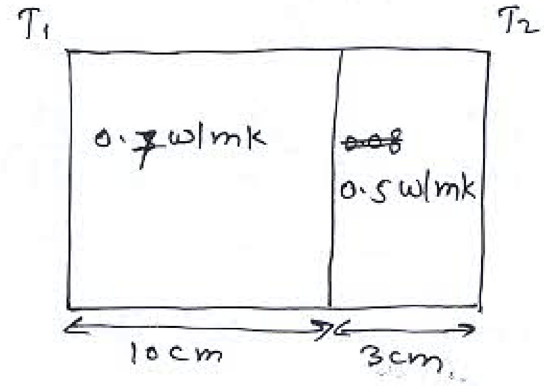
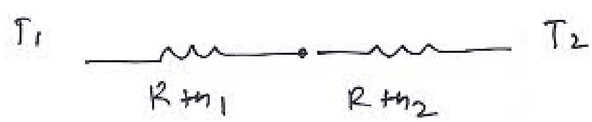
4) Case-I → Wall without insulation.



Given:
$k_1 = 0.7$ w/mk
$L_1 = 0.1$ m
$K_2 = 0.5$ w/mk
$L_2 = 0.03$ m

Let cross sectional area by A. & end temp be T_1 & T_2

∴ Electrical analogy for the given case can be draws as below:—



where, $R_{th_1} = \frac{L_1}{k_1 A}$ & $R_{th_2} = \frac{L_2}{K_2 A}$.

Here, $(R_{net})_1 = R_{th_1} + R_{th_2}$

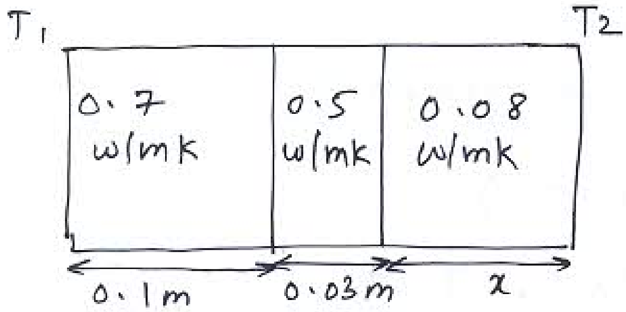
$$= \frac{0.1}{0.7A} + \frac{0.03}{0.5A}$$

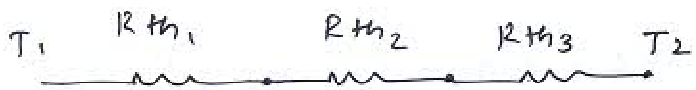
$$= \frac{0.202}{A}$$

$$\therefore \quad Q_1 = \frac{T_1 - T_2}{(R_{net})_1} = \frac{(T_1 - T_2)A}{0.202}$$

Case-II → <u>wall with insulation</u>

Let the thickness of insulation be x





In this case,

$$(R_{net})_2 = R_{th_1} + R_{th_2} + R_{th_3}$$

$$= \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A}$$

$$= \frac{0.1}{0.7A} + \frac{0.03}{0.5A} + \frac{x}{0.08\,A}$$

$$= \frac{0.202 + 12.5x}{A}.$$

$$\therefore \quad Q_2 = \frac{T_1 - T_2}{(R_{net})_2} = \frac{(T_1 - T_2)A}{0.202 + 12.5x}$$

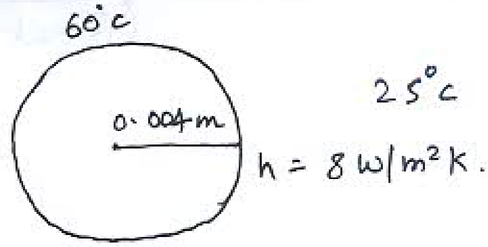
Given: $Q_2 = 0.3\,Q_1$

$$\Rightarrow \frac{(T_1 - T_2)A}{0.202 + 12.5x} = \frac{0.3(T_1 - T_2)A}{0.202}$$

$$\Rightarrow \boxed{x = 0.0378m} = \boxed{3.78\text{ cm}}$$

5) Given: $d_i = 8\ mm$

$\Rightarrow r_i = 4mm = 0.004m.$



$T_1 = 60°C$

$T_2 = 25°C$

$h = 8\ w/m^2 K.$

Case-I $\rightarrow$ Without Insulation.

$$Q_1 = h.A(T_1 - T_2)$$

$$= h(2\pi r l)(T_1 - T_2)$$

Let length of wire be l.

$$\therefore Q_1 = 8 \times 2 \times 3.14 \times 0.004 \times (60-25)\ l$$

$$\Rightarrow \boxed{Q_1 = 7.03\ l}$$

Now, for max. heat transfer, radius of insulation,

$$r_o = r_c = \frac{k}{h} \quad ; \quad \text{Given : } k = 0.174\ w/mk$$

$$\Rightarrow r_c = \frac{0.174}{8}$$

$$\Rightarrow \boxed{r_c = 0.02175\ m}$$

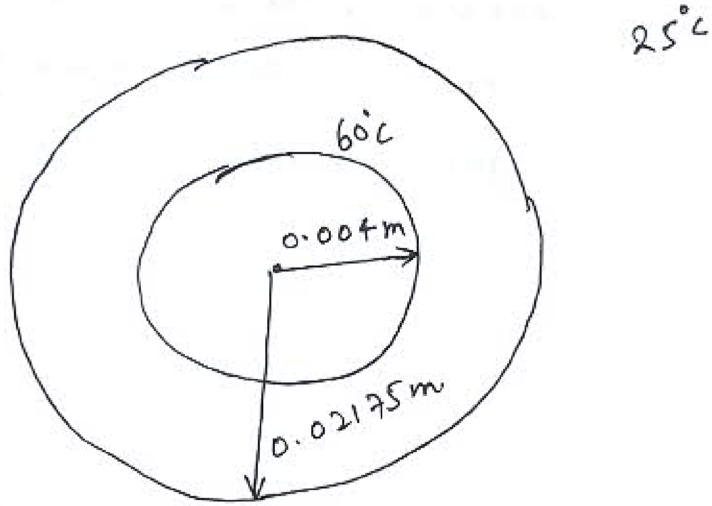
Thus thickness of insulation

$$t_c = r_c - r_i = 0.02175 - 0.004$$

$$\Rightarrow \boxed{t_c = 0.01775\ m}$$

$$\Rightarrow \boxed{t_c = 17.75\ mm}$$

Case - II with Insulation



Here, $Q_2 = \dfrac{\Delta T}{R_{net}}$

$$= \frac{60-25}{\dfrac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi K l} + \dfrac{1}{(2\pi r_o l)h}}$$

$$= \frac{60-25}{\dfrac{\ln\left(\frac{0.02175}{0.004}\right)}{2\pi(0.174)l} + \dfrac{1}{2\pi \times 0.02175 \times 8 \times l}}$$

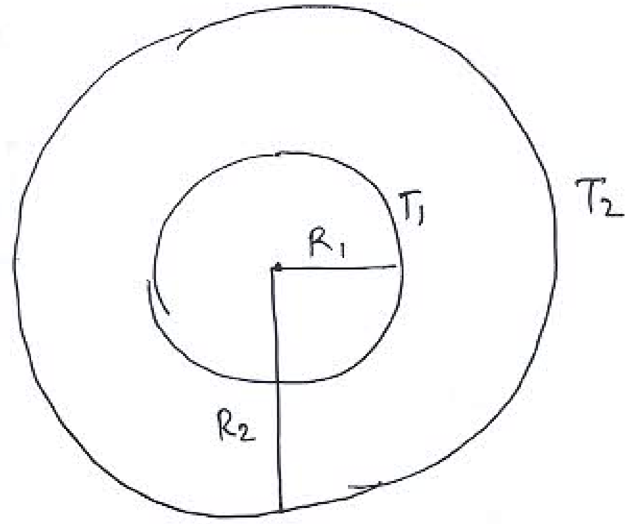
$$\Rightarrow \boxed{Q_2 = 14.2\, l}$$

$\therefore$ % increase in heat transfer rate $= \dfrac{Q_2 - Q_1}{Q_1} \times 100$

$$= \frac{14.2\,l - 7.03\,l}{7.03\,l} \times 100$$

$$= \boxed{101.99\,\%}$$

6. Consider a hollow sphere.
The inner & outer surface of sphere are at temp T_1 & T_2 respectively & have radii R_1 & R_2 respectively



Assuming that only radial heat conduction takes place at steady state and there is no heat generation.

The heat conduction eqn is reduced to -

$$\frac{k}{r^2}\left(\frac{d}{dr} r^2 \frac{dT}{dr}\right) = 0$$

$$\Rightarrow \quad r^2 \frac{dT}{dr} = C_1$$

$$\Rightarrow \quad dT = \frac{C_1}{r^2} dr$$

$$\Rightarrow \quad \boxed{T = \frac{-C_1}{r} + C_2}$$

Now,
at $r = R_1$, $T = T_1$
and $r = R_2$, $T = T_2$

$\therefore$ we get, $T_1 = \dfrac{-C_1}{R_1} + C_2 \quad -①$

$$T_2 = \frac{-C_1}{R_2} + C_2 \quad -②$$

Solving ① + ②

we get,

$$C_1 = -\frac{(T_1 - T_2) R_1 R_2}{R_2 - R_1}$$

and $C_2 = \dfrac{T_2 R_2 - T_1 R_1}{R_2 - R_1}$

Replacing the values of C_1 and C_2, we get -

$$T = \frac{(T_1 - T_2) R_1 R_2}{r (R_2 - R_1)} + \frac{T_2 R_2 - T_1 R_1}{(R_2 - R_1)}$$

or $\boxed{\dfrac{T_1 - T}{T_1 - T_2} = \dfrac{\frac{1}{R_1} - \frac{1}{r}}{\frac{1}{R_1} - \frac{1}{R_2}}}$