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CMRIT

Internal Assessment Test II – April 2018

Sub:	Heat Transfer							Code:	15ME63
Date:	17/ 04 / 2018	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	MECH

Note: Answer any **five** questions.

		Marks	OBE	
			CO	RBT
1	A wire of 6.5 mm diameter at a temperature of 60°C is to be insulated by a material having k=0.174 W/mK. Convection heat transfer coefficient is 8.722 W/m^{2}K. The ambient temperature is 20°C. For maximum heat loss, what is the minimum thickness of insulation? Also determine heat loss per unit length with & without insulation. Find percentage increase in heat dissipation. **OR** (i) Derive an expression for critical thickness of insulation for a sphere. (ii) It is desired to increase the heat dissipated over the surface of an electronic device of spherical shape of 5 mm radius exposed to convection with h=10W/m^{2}K by encasing it in a spherical sheath of conductivity 0.04 W/mK. Determine the diameter of the sheath for maximum heat flow.	10	CO2	L3
2	Derive an expression for temperature distribution and heat dissipation in a straight short fin losing heat at tip. **OR** A steel rod (k=30 W/mK) with 1 cm diameter and 5 cm long with insulating end is to be used as a spine. It is exposed to surrounding temperature of 65°C and heat transfer coefficient of 50 W/m^{2}K. The temperature of the base is 98°C. Determine (i) Temperature at the end of spine (ii) Heat dissipation from spine (iii) Fin effectiveness (iv) Fin efficiency.	10	CO2	L3
3	Define: (i) Lumped parameter analysis (ii) Thermal time constant (iii) Biot Number (iv) Fourier No(v) Semi Infinite body **OR** Derive an expression for variation of temperature with time using lumped parameter analysis. Also find the instantaneous heat transfer rate and total heat transferred at a given time.	10	CO3	L1/L3
4	A hot cylinder ingot of 50 mm diameter and 200 mm long is taken out from the furnace at 800°C and dipped in water till its temperature falls to 500°C. Then it is directly exposed to air till its temperature falls to 100°C. Find the total time required for the ingot to reach from 800°C to 100°C. (For ingot: k=60 W/mK, c=200J/mK, ρ=800 kg/m^3) (h_{water} = 20 W/m^{2}K, hair=20 W/m^{2}K, temperature of air and water = 30°C) **OR** A long cylindrical bar (k=17.4 W/mK, α=0.019 m^2/h) of radius 80 mm comes out of oven at 830°C throughout and is cooled by quenching it in a large bath of 40°C coolant with h=180 W/m^{2}K. Determine: (i) The time taken by shaft to reach 120°C. (ii) The surface temperature of the shaft when its centre temperature is 120°C. (iii) Temperature gradient at outside surface at the same time.	10	CO3	L3
5	Define: (i)Total emissive power (ii)Spectral emissive power (iii)Black Body (iv)White Body(v)Transparent Body (vi)Emissivity (vii)Stefan Boltzman's Law (viii)Wien's Law (ix)Lambert's Law (x)View Factor **OR** Derive an expression for rate of heat transfer between two infinite gray plates at temperatures T1 & T2 and emissivities ϵ1 & ϵ2 respectively.	10	CO5	L1/L3

1) A wire of 6.5mm diameter at a temperature of 60°c is to be insulated by a material having K = 0.174 W/mk. convection heat transfer co-efficient is 8.722 W/m²k. The ambient temperature is 20°c. For maximum heat loss what is the minimum thickness of insulation? Also determine the heat loss per unit length ~~of wire~~ with and without insulation. Find the percentage increase in heat dissipation.

$$r_i = \frac{6.5\times10^{-3}}{2} = 3.25\times10^{-3} \quad T_1 = 60°c,\ k = 0.174\,W/mK$$

$$h_o = 8.722\,W/m^2K \quad T_o = 20°c$$

(i) Critical thickness of insulation.

$$r_c = \frac{k}{h_o} = \frac{0.174}{8.722} = 0.01995m = 19.95mm$$

$$r_o - r_1 = r_c - r_i = 16.7mm$$

(ii) Heat loss per m length

Case(1): with insulation

$$q_{with} = \frac{T_1 - T_o}{\dfrac{\log_e \frac{r_c}{r_i}}{2k\pi l} + \dfrac{1}{2\pi r_c h_o l}}$$

$$= \frac{60-20}{\dfrac{\log \frac{0.01995}{0.00325}}{2\pi\times0.174\times1} + \dfrac{1}{2\pi\times0.01995\times8.722\times1}}$$

$q_{with} = 15.537 \, W/m$

Case (ii) without Insulation

$$q_{without} = \frac{T_i - T_0}{\frac{1}{2\pi r_i h_0 L}} = \frac{60 - 20}{\frac{1}{2\pi \times 0.00325 \times 8.722 \times 1}}$$

$$q_{without} = 7.124 \, W/m$$

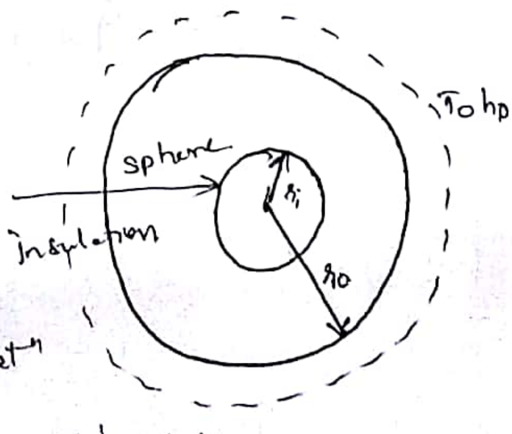
% increase in heat dissipiation

$$= \frac{q_{with} - q_{without}}{q_{without}} \times 100$$

$$= \frac{15.537 - 7.124}{7.124} \times 100 = 118.09\%$$

2) (i) Derive an expression for critical thickness of insulation for a sphere.

(ii) It is desired to increase the heat dissipiated over the surface of an electronic device of spherical shape of 5mm radius exposed to convection with $10 \, W/m^2K$ by encasing it in a spherical sheath of conductivity $0.04 \, W/mK$.. Determine the diameter of sheath for maximum heat flow



Consider a hollow sphere with insulation as shown in fig

r_i = Inside radius of insulation

r_0 = outside radius of insulation

T_1 = Inside surface temp of insulation

T_0 - Ambient temperature

$$Q = \frac{T_1 - T_0}{\frac{r_0 - r_i}{4\pi k r_i r_0} + \frac{1}{4\pi r_0^2 h_0}}$$

For max heat transfer

$$\frac{dR_{net}}{dr_0} = 0$$

$$\frac{d}{dr_0}\left[\frac{r_0 - r_i}{4\pi k r_0 r_i} + \frac{1}{4\pi r_0^2 h_0}\right] = 0$$

$$\frac{d}{dr_0}\left[\frac{1}{4\pi k r_i} - \frac{1}{4\pi k r_0} + \frac{1}{4\pi r_0^2 h_0}\right] = 0$$

$$\frac{1}{4\pi k r_0^2} = \frac{2}{4\pi h_0 r_0^3}$$

$$r_0 = \frac{2k}{h_0} = r_c = \text{critical radius of insulation}$$

Critical thickness of insulation

$$= r_0 - r_i = r_c - r_i$$

$$= \frac{2k}{h_0} - r_i$$

For maximum heat flow

$$r_c = r_0$$

$$r_0 = \frac{2k}{h_0}$$

$$r_0 = \frac{2(0.04)}{10} = 0.008 = 8mm$$

$\boxed{D = 16mm}$

3) Derive an expression for temperature distribution and heat dissipation in a straight short fin losing heat at tip.

At $x=0$ $T=T_0$

$x=L$, Rate of conduction = Rate of convection

$$-kA_c \left.\frac{d\theta}{dx}\right|_{x=L} = hA_c\theta_L$$

$$\boxed{\theta_L = T_L - T_\infty}$$

Here.

$$\frac{\theta}{\theta_0} = \frac{\cosh[m(l-x)] + \frac{h}{mk}\sinh(m(l-x))}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}$$

$$Q = \frac{-kA_c\theta_0\left[-m\sinh(m(l-x)) + \frac{h}{mk}\cosh(m(l-x)(-m))\right]}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}$$

$$Q = -kA\theta_0\sqrt{\frac{hp}{kA_c}}\left[\frac{\sinh[m(L-x)] + \frac{h}{mk}(\cosh[m(L-x)])}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}\right]$$

$$Q = \sqrt{hPkA_C}\,\theta_0\left[\frac{\sinh[m(L-x)] + \frac{h}{mk}\cosh m(L-x)}{\cosh(mL) + \frac{h}{mk}[\sinh(mL)]}\right]$$

$$Q = \sqrt{hPkA_C}\,\theta_0\left[\frac{\sinh(mL) + \frac{h}{mk}\cosh(mL)}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}\right]$$

4) A steel rod ($k = 30\ W/mK$) with 1cm dia and 5cm long with insulating end and is to be used as a spine. It is exposed to surrounding temp of 65°c and heat transfer co-efficient of 50 W/m^2K. The temperature of the base is 90°c. Determine
(i) Temp at the end of spine (ii) Heat dissipiation from spine
(iii) Fin effectiveness (iv) Fin efficiency.

Temp at the end of the fin ($L = x$)

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh[m(L-x)] + \left(\frac{hL}{mK}\right)\sinh[m(L-x)]}{\cosh(mL) + \left(\frac{hL}{mK}\right)\sinh mL}$$

$$m = \sqrt{\frac{hp}{kA_c}}$$

$h = 50\ W/m^2k$

$p = 2\pi r$

$p = 2\pi \times 0.5 \times 10^{-2}$

$p = 0.0314\ m$

$A_c = \frac{\pi d^2}{4} = \frac{\pi\ 1^2}{4}$

$= 0.785\ cm^2$

$= 0.785 \times 10^{-4}\ m^2$

$$m = \sqrt{\frac{50 \times 0.0314}{30 \times 0.785 \times 10^{-4}}}$$

$m = 25.81$

$mL = 25.81 \times 0.05$

$mL = 1.29 < 3$

∴ It is short fin.

$$\frac{T - 65}{90 - 65} = \frac{\cosh[25.81(0)] + \frac{hL}{mk}\sinh(25.81(0))}{\cosh(1.29) + \frac{hL}{mk}\sin(1.29)}$$

$= 0.484$

$$\boxed{T = 77.1°C}$$

(ii) $$q = (T_b - T_\infty)\frac{\tanh(mL) + (hL/mk)}{1 + (hL/mk)\tanh(mL)}\sqrt{hpkA}$$

~~q = 21.87 W~~ $q = 1.33\ W$.

$$\underline{\text{Fin effectiveness}} \quad \epsilon = \frac{q_{fin}}{q_{w/o\,fin}}$$

$$\epsilon = \frac{1.33}{0.09}$$

$$\epsilon = 13.547$$

$$q_{w/o\,fin} = hA_c\Delta T$$

$$\cancel{= 50 \times 2\pi \times 0.05 \times 5\times10^{-4} \times 25}$$

$$\cancel{= 1.96W}$$

$$= 50 \times \pi \times (0.5\times10^{-3})^2 \times 25$$

$$=$$

$$\underline{\text{Fin efficiency}} = \frac{q_{act}}{q_{max}} = \frac{1.33}{1.96} \times 100$$

$$= 67.85\%.$$

$$q_{max} = hA_s\Delta T$$

$$= 50 \times 2\pi \times 0.5 \times 5\times10^{-4} \times 25$$

$$= 1.96W$$

5) Define

<u>Lumped parameter analysis</u>: If the size of the solid is very small or it has very high k. The temp gradient existing in the solid can be neglected. Hence temp is the function of time only. The analysis of heat transfer under such assumptions is known as lumped parameter analysis.

<u>Thermal time constant</u>:

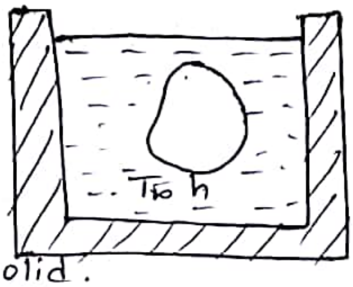
Thermal time constant can be defined as the time in which the temperature difference b/w the body and the surroundings becomes 36.78% of the initial temp difference.

<u>Biot number</u>: Biot number is the ratio of internal resistance due to conduction to the surface resistance due to convection.

<u>Fourier number</u>: Fourier number is defined as the ratio of the rate of heat conduction to the rate of the thermal energy storage in the solid.

<u>Semi-infinite body</u>: A semi-infinite body is one in which at any instant of time there is always a point where the effect of heating (or) cooling at one of its boundaries is not felt at all. At this point the temp remains the same.

6) Derive an expression for variation of temperature with time using lumped parameter analysis. Also find the instantaneous heat transfer rate ~~at~~ and total heat transferred at a given time.



V = Volume of the Solid.

A = Surface ~~of~~ area of the solid.

k = Thermal conductivity of the solid.

ρ = Density of the solid.

T_i = Uniform initial temperature of solid.

T = Temperature of solid at the given time interval.

Assuming ~~temp~~ is a function of time 't' only we have.

Rate of heat flow into the Solid of volume V = Rate of increases in internal energy in the Solid of Volume V.

$$hA(T_\infty - T) = mc\frac{dT}{dt}$$

$$hA(T_\infty - T) = \rho Vc.\frac{dT}{dt}.$$

$$= -\frac{hA}{\rho Vc.}(T - T_\infty)$$

$$\frac{dT}{T - T_\infty} = \frac{-hA}{\rho Vc.}dt$$

Integrating the above expression

$$\log_e(T - T_\infty) = \frac{-hA}{\rho Vc}t + C_1$$

Applying boundary condition

at $t = 0$, $T = T_i$

$$\log_e(T_i - T_\infty) = C_1$$

Substituting the value of C_1 in eqn ①

$$\log_e(T - T_\infty) = \frac{-hAt}{\rho Vc} + \log_e(T_i - T_\infty)$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho Vc}t} = e^{-B_i F_0}$$

where, $B_i = \frac{h L_c}{k}$ $\quad F_0 = \frac{\alpha t}{L_c^2}$

Instantaneous heat flow rate

$$Q_i = \rho Vc \frac{dT}{dt} = \rho Vc \frac{d}{dt}\left[T_\infty + (T_i - T_\infty)e^{-\frac{hA}{\rho Vc}t}\right]$$

$$Q_i = \rho Vc \left[0 + (T_i - T_\infty)\left[\frac{-hA}{\rho Vc}\right] e^{\frac{-hA}{\rho Vc}t}\right]$$

$$= -hA(T_i - T_\infty)\, e^{-B_i F_0}$$

Total heat flow

$$Q_t = \int_0^{t_f} Q_i\,dt = \left[-hA(T_i - T_\infty)\frac{e^{-\frac{hA}{\rho Vc}t}}{-hA/\rho Vc}\right.$$

$$= \rho Vc (T_i - T_\infty)\left[e^{\frac{-hA}{\rho Vc}t_f} - e^{-\frac{hA}{\rho Vc}\cdot 0}\right]$$

$$= \rho Vc (T_i - T_\infty)\left[e^{\frac{-hA}{\rho Vc}t_f} - 1\right]$$

$$Q_t = \rho Vc (T_i - T_\infty)(e^{-B_i F_0} - 1)$$ Joules

7) A hot cylinder ingot of 50mm diameter and 200mm long is tanken out from the furnace at 800°c and dipped in water till its temperature falls to 500°c. Then it is directly exposed to air till its temperature falls to 100°c. Find the total time required for the ingot to reach from 800°c to 100°c

for ingot $k = 60 \, w/mk \cdot c = 200 J/mk$ $\rho = 800 \, kg/m^3$

$h_{water} = 20 \, w/m^2k$. $h_{air} = 20 \, w/m^2k$.. temp of air and water is 80°c.

$$\frac{T - T_\infty}{T_i - T_\infty} = erf\left[\frac{x}{2\sqrt{\alpha t}}\right]$$

$$\frac{500-100}{800-100} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$erf\left[\frac{x}{2\sqrt{\alpha t}}\right] = 0.5714$$

$$erf(z) = 0.5714$$

$$z = 0.56$$

$$\frac{x}{2\sqrt{\alpha t}} = 0.56$$

$$\alpha = \frac{k}{\rho c} = \frac{60}{800 \times 200} = 3.75 \times 10^{-4} \, m/s$$

$$\frac{200 \times 10^{-3}}{2\sqrt{3.75 \times 10^{-4} \, t}} = 0.56$$

$$\boxed{t = 85.03 \text{ sec}}$$

8) A long cylindrical bar ($k = 17.4$ W/mK $\alpha = 0.019$ m²/h) of radius 80mm comes out of oven at 830°C throughout and is cooled by quenching it in large bath of 40°C coolant with $h = 180$ W/m²K Determine:

(i) The time taken by shaft to reach 120°C

(ii) The surface temp of the shaft when its centre temp is 120°C

(iii) Temperature gradient at outside surface at the same time.

(i) $$Bi = \frac{hr}{k} = \frac{180 \times 80 \times 10^{-3}}{k} = 0.8275$$

$$F_0 = \frac{\alpha \tau}{L_c^2} = \frac{\frac{0.019}{3600} \times \tau}{(80 \times 10^{-3})^2} = 8.246 \times 10^{-4}\, \tau$$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = e^{-Bi\,F_0}$$

$$\frac{120 - 40}{830 - 40} = e^{-0.8275 \times 8.246 \times 10^{-4} \tau}$$

$$\tau = 3356.02\,s$$

(ii) For the surface of cylinder

$$r = R_0$$

$$\frac{r}{R_0} = 1.$$

$$\frac{T_{(r/R_0)} - T_\infty}{T_0 - T_\infty} = 0.7$$

$$\frac{T_{r/R_0} - 40.}{120. - 40} = 0.7.$$

$$T_{r/R_0} = 96°C$$

(iii) $$\frac{dT}{dt} = \frac{-hA}{\rho V C}(T - T_\infty)$$

$$= \cancel{180 \times 2\pi \times 80 \times 10^{-3} \times 1}$$

$$= \frac{-1}{3356.028}(120 - 40)$$

$$= 0.023837$$

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Q) Define

(i) Total emissive power : It is defined as the total radiant energy emitted by the Surface in all direction over the entire wavelength range per unit Surface area per unit time

(ii) Spectral emissive power : The variation distribution of the monochromatic emissive power with wavelength.

(iii) Black body : It is assigned to a perfect absorber of radiation.

(iv) White body : A body that reflects all the incident thermal radiations to pass through it is called ~~transparent~~ white body.

(V) Transparent body : A body that allows all the incident radiations to pass through it is called transparent body.

(Vi) Emissivity : It is the ratio of emissive power of the surface to the emissive power of black surface at the same temperature

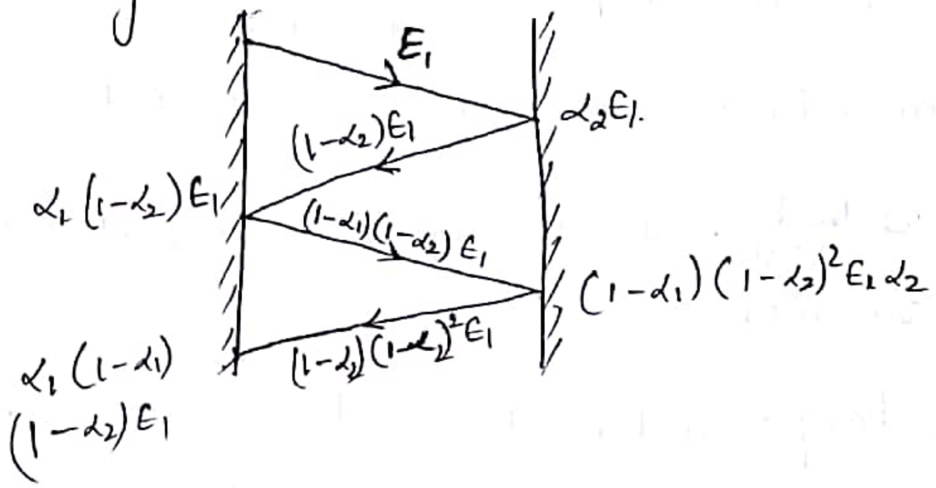
(Vii) Stefan Boltzmann's ~~con~~ law : It states that the heat radiated is proportional to fourth power of the absolute temperature of the surface and the surface area.

(Viii) Wien's law :- It states that the product of maximum wavelength and absolute temp is a constant and is equal to 0.002 9mK.

(IX) Lambert's law :- It states that the intensity of radiation in a direction θ from the normal to the black emitter is proportional to cosine of the angle θ.

(x) View factor: It is defined as the fraction of radiation energy that is diffused from one surface element and strikes the other surface directly with no intervening reflection.

10. Derive an expression for rate of heat transfer between two infinite grey plates at temp T_1 and T_2 and emissivities ε_1 & ε_2 respectively.



α_1 and α_2 – absorptivity of plate 1 and 2.
ε_1 and ε_2 – emissivity of plate 1 and 2

The amount of radiant energy which left surface (1) per unit time.

$$q_1 = E_1 - \Big[\alpha_1(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^2E_1 + \cdots\Big]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\Big[1 + (1-\alpha_1)(1-\alpha_2) + (1-\alpha_1)^2(1-\alpha_2)^2 + \cdots\Big]$$

Let $p = (1-\alpha_1)(1-\alpha_2)$

$$Q = E_1 - \alpha_1(1-\alpha_1)E_1(1+p+p^2+\cdots)$$

Since p is less than unity, the series $1+p+p^2\ldots$ when extended to infinity gives $\dfrac{1}{1-p}$.

$$Q_1 = E_1 - \alpha_1 (1-\alpha_2) E_1 \left(\frac{1}{1-P}\right)$$

$$= E_1 \left[1 - \frac{\alpha_1(1-\alpha_2)}{1-P}\right]$$

$$= E_1 \left[1 - \frac{\alpha_1(1-\alpha_2)}{1-(1-\alpha_1)(1-\alpha_2)}\right]$$

From kirchoff's law, emissivity and absorptivity of a surface are equal, i.e $\alpha_1 = \varepsilon_1$

$$Q_1 = E_1 \left[1 - \frac{\varepsilon_1(1-\varepsilon_2)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\right]$$

$$Q_1 = E_1 \left[\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}\right]$$

$$Q_2 = E_2 \left[\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}\right]$$

$$Q_{12} = Q_1 - Q_2$$

$$Q_{12} = \frac{E_1 \varepsilon_2 - E_2 \varepsilon_1}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

$$\because E_1 = \varepsilon_1 \sigma T_1^4$$

$$Q_{12} = \varepsilon_1 \sigma T_1^4 \varepsilon_2 - \varepsilon_2 \sigma T_2^4 \varepsilon_1$$

$$= F_{12}\, \sigma \cdot (T_1^4 - T_2^4)$$

$$F_{12} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$