IAT 2 question paper solutions, 2017-18 ①

Sem: VI

Sections: A&B

Subject: DME II

Staff: RPR

1(a) A Bellivellie spring is made of 3 mm sheet steel with an outside diameter of 125 mm and inside diameter of 50 mm. The spring is dished to 5 mm. The maximum stress has to be 500 MPa. Determine

1) Safe load carried by spring
2) Deflection at this load.
3) Stress produced at outer edge.
4) Load for flattening the spring.

Assume $\nu = 0.3$ & $E = 200\,GPa$.

data

$t = 3\,mm$

$d_o = 125\,mm$

$d_i = 50\,mm$

$h = 5\,mm$

$\nu = 0.3$

$E = 200 \times 10^3\ N/mm^2$

Since max. stress is induced at the inner surface,

$\sigma_i = 500\ N/mm^2$

to find

1) F
2) y
3) σ_o
4) F_{max}

2) deflection (y)

$$\sigma_i = \frac{4Ey}{(1-\nu^2)\, d_o^2 M}\left[\left(h-\frac{y}{2}\right)c_1 + c_2 t\right]$$

$\frac{d_o}{d_i} = 2.5$

From Fig 20.3,

$M = 0.75$

$c_1 = 1.32$

$c_2 = 1.54$.

Solving for y

$y = 16.36$ mm & 0.615 mm.

Since y can't be more than h (=5mm)

$y = 0.615$ mm.

2) safe load (F)

$$F = \frac{4Ey}{(1-\nu^2) M d_o^2}\left[(h-y)\left(h-\frac{y}{2}\right)t + t^3\right]$$

$= 4093.66$ N.

3) Stress at the outer edge (σ_o)

$$\sigma_o = \frac{4Ey}{(1-\nu^2) M d_o^2}\left[c_1\left(h-\frac{y}{2}\right) - c_2 t\right]$$

$= 73.706$ N/mm^2

4) Load for flattening the spring (F_{max})

The spring gets flattened when $h = y$

$\therefore$ substituting $y = 5$mm in the expression for F,

$$F = \frac{4Ey}{(1-\nu^2) M d_o^2} t^3.$$

$= 10,127.5$ N.

1(b) A semi elliptical leaf spring has a span length of 1·8 m. The spring seat is midway between the shackles and carries a helical spring upon which an impact energy equal to 2500 N-m is imposed. The laminated spring is composed by 2 full length and 10 graduated leaves, each 5 mm thick and 50 mm wide. The coil spring comprises of 6 effective turns, 15 mm wire diameter and has a mean diameter of 100 mm. Calculate the max. stress induced in each spring. Take E = 210 GPa and G = 84 GPa. ③

Ans:

data

Leaf spring

$2l_1 = 1{\cdot}8\,m$

$= 1800\,mm$

$2l_1 = 2l$ since l_B is not given

$\therefore l = 900\,mm$

$i_f = 2$

$i_g = 10$

$\therefore i = 12$

$h = 5\,mm$

$b' = 50\,mm$

$E = 210 \times 10^3\ N/mm^2$

Helical Spring

$i = 6$

$d = 15\,mm$

$D = 100\,mm$

$C = \frac{D}{d} = 6{\cdot}667$

$G = 84 \times 10^3\ N/mm^2$

to find

1) τ

2) σ_f

Let F_1 & F_2 be the loads shared by helical and leaf springs respectively. Since both springs act as a single unit, the energy absorbed by both the springs is the same.

1) τ for helical spring

$$U = \frac{1}{2} F_1 y$$

where $y = \frac{8 F_1 D^3 i}{G d^4}$.

$$\therefore \quad 2500 \times 10^3 = \frac{1}{2} F_1 \left(\frac{8 F_1 \times 100^3 \times 6}{84 \times 10^3 \times 15^4} \right)$$

$$\Rightarrow F_1 = 21{,}046.82 \text{ N}$$

$$\therefore \tau = \frac{8 F_1 D K}{\pi d^3}.$$

where $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$

$$= 1.224.$$

$$\tau = \frac{8 \times 21{,}046.82 \times 100 \times 1.224}{\pi \times 15^3}$$

$$= 1{,}943.72 \text{ N/mm}^2.$$

2) σ_f for leaf spring

$$U = \frac{1}{2} F_2 \times y$$

where $y = \frac{\beta F l^3}{E i b' h^3}$

where $\beta = \frac{12}{2 + r}$

where $r = \frac{i_f}{i} = \frac{2}{12} = 0.166.$

$\therefore \beta = 5.54$.

$$\therefore y = \frac{5.54 \times F_2 \times 900^3}{210 \times 10^3 \times 12 \times 50 \times 5^3}$$

$$\Rightarrow F_2 = 4415.77\ N.$$

WKT $\sigma_f = \dfrac{1.5 \alpha F l}{i b' h^2}$

where $\alpha = \beta = 5.54$.

$$\therefore \sigma_f = \frac{1.5 \times 5.54 \times 4415.77 \times 900}{12 \times 50 \times 5^2}$$

$$= 2201.70\ N/mm^2.$$

2. It is required to determine the proportions of a spur gear drive to transmit 7.5 KW from a shaft rotating at 1170 rpm to a low speed shaft with a speed reduction of 3:1. Assume that the teeth are 20° full depth system with 24 teeth on pinion. The pinion is to be SAE 1040 steel and the gear is of cast steel. Assume that the torsional moment at starting is 150% of torsional moment at the rating. Determine the BHN for pinion and gear to check for wear.

Ans: <u>data</u>

$P = 7.5\ KW$

$n_1 = 1170\ rpm$

$n_2 = \dfrac{1170}{3} = 390\ rpm$ $(\because i = 3)$

$\alpha = 20°$ FDI

$z_1 = 24$.

$z_2 = 24 \times 3 = 72$.

Materials: Pinion : SAE1040 Steel ; $\sigma_{01} = 173$ MPa | T23.18 (5)
gear : Cast Steel : $\sigma_{02} = 138$ MPa | P23.71

Since $(M_t)_{max} = 1.5 (M_t)_{mean} \Rightarrow C_s = 1.5$.

1. Design tangential tooth load (F_t)

$$F_t = \frac{9550 \times 10^3 \times P}{n} \times \frac{C_s}{r}$$

$$= \frac{9550 \times 10^3 \times \cancel{1000}\,7.5}{1170} \times \frac{1.5}{\left(\frac{m z_1}{2}\right)}$$

$$= \left(\frac{7652}{m}\right) \text{N}.$$

2. module (m)

$$F_t = \sigma_0 \cdot b \cdot y \cdot p \cdot K_v$$

assuming carefully ~~cut~~ cut teeth for the gears,

$$K_v = \frac{4.5}{4.5 + v_m}$$

where $v_m = \frac{\pi d_1 n_1}{60{,}000} = \frac{\pi (m z_1) n_1}{60{,}000}$

$$= \frac{\pi \times m \times 240 \times 1170}{60{,}000}$$

$$= (1.47m) \text{ m/sec}.$$

$$\therefore K_v = \frac{4.5}{4.5 + (1.47m)}$$

assume $b = 10m$

$$y_1 = 0.154 - \frac{0.912}{24}$$

$$= 0.116.$$

$$F_t = \frac{7652}{3} = 2550\,N$$

$$K_v = \frac{4.5}{4.5 + (1.47 \times 3)} = 0.505.$$

$$b = 10 \times m$$
$$= 10 \times 3$$
$$= 30\,mm.$$

5. gear teeth proportions

Addendum $h_a = 1m = 3\,mm.$

Dedendum $h_f = 1.25m = 3.75\,mm.$

Total depth $h = 2.25m = 6.75\,mm.$

clearance $(c) = 0.25m = 0.75\,mm.$

working depth $(h') = 2m = 6\,mm.$

6. check for dynamic load (F_d)

$$F_d = F_t + \frac{21 v_m (F_t + bC)}{21 v_m + \sqrt{F_t + bC}}.$$

$$v_m = 1.47m = 5.\cancel{88}\,m/sec.$$

For $v_m = 5.88\,m/sec$, $f = 0.06\,mm.$

$$\therefore C = \overset{696}{\cancel{...}}\,N/mm.$$

0.05 — 580

0.06 — ?

$$\frac{0.06}{0.05} \times 580 = 696\,N/mm.$$

$$\therefore F_d = 1913 + \frac{21 \times \overset{5.88}{\cancel{1.47}} (1913 + (38 \times 696))}{21 \times 5.88 + \sqrt{1913 + (38 \times 696)}}$$

$$= 12.62\,KN.$$

$$y_2 = 0.154 - \frac{0.912}{72}$$

$$= 0.141.$$

Strength factors

Pinion : $\sigma_{01} \cdot y_1 = 20.068$ MPa

gear = $\sigma_{02} \cdot y_2 = 19.45$ MPa.

∴ gear is weaker.

Applying Lewis eqn to gear,

$$\frac{7652}{m} = 138 \times (10m) \times 0.141 \times (\pi \times m) \times \frac{4.5}{4.5 + 1.47m}$$

$$= \frac{2751\, m^2}{(4.5 + 1.47m)}$$

$$\Rightarrow \quad 2751 m^3 - 11248m - 34{,}434 = 0.$$

Solving for x,

$x_1 = 2.89$ mm.

$x_2 = -1.44$ mm

$x_3 = -1.44$ mm.

∴ Select $m = 3$ mm.

3. Pitch circle dia (d_1, d_2)

$d_1 = mz_1$
$= 3 \times 24$
$= 72$ mm

$d_2 = mz_2$
$= 3 \times 72$
$= 216$ mm.

4. Facewidth (b)

Endurance Strength (F_f) = $\sigma_{ef} \cdot b \cdot y_2 \cdot m$.

σ_{ef} for cast steel = 290 MPa $\left(\frac{T23.33}{P23.76}\right)$.

$\therefore F_f = 290 \times 30 \times 0.442 \times 3$

$= 11.53 KN$.

Since $F_f < F_d$, the design is not safe from strength point of view.

7. Check for wear

$F_W = d_1 \cdot b \cdot Q \cdot K$.

For safe design,

$F_W \geq F_d$.

ie $d_1 \cdot b \cdot Q \cdot K \geq F_d$.

$\geq 12.62 \times 10^3$

$72 \times 30 \times 1.5 \times K \geq 12.62 \times 10^3$

$\Rightarrow K \geq 3.89 MPa$.

$\therefore$ BHN for pinion = 450
BHN for gear = 450 $\Big\} \frac{T23.37B}{P23.80}$,

3 ~~10~~. A pair of helical gears is to transmit 15 KW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion has 80 mm pitch diameter and operates at 10,000 rpm. The gear has 320 mm pitch diameter. If the gears are made of cast steel ($\sigma_0 = 100$ MPa), determine suitable module and face width. The pinion is heat treated to a Brinell of 300 and gear has a Brinell hardness of 200. Check the design for dynamic and wear loads. (10)

Ans: data

$P = 15$ KW.

$\alpha_t = 20^\circ$ stub

$\beta = 45^\circ$.

$d_1 = 80$ mm.

$n_1 = 10{,}000$ rpm.

$d_2 = 320$ mm.

$i = \frac{320}{80} = 4.$

$\therefore n_2 = \frac{n_1}{i} = \frac{1000}{4} = 2500$ rpm.

Materials: pinion : cast steel $\sigma_{01} = 100$ MPa.

gear : " " $= 100$ MPa.

BHN for pinion = 300

BHN for gear = 200

1. Design tangential tooth load (F_t)

$$F_t = \frac{9550 \times 10^3 \times P}{n} \times \frac{C_S}{r_2}$$

$$= \frac{9550 \times 10^3 \times 1000}{10,000} \times \frac{1.5}{40} \quad (\text{assuming } C_s = 1.5)$$

$$= 537 \text{ N}$$

2. module (m_t)

$$F_t = \frac{\sigma_0 \cdot b \cdot y \cdot P_t \, K_v \cos\beta}{C_W}$$

Since pressure angle is measured in diametral plane, m_t should be a std. value

$$v_m = \frac{\pi \times d_1 \times n_1}{60,000} = 41.88 \text{ m/sec}$$

$$K_v = \frac{5.6}{5.6 + \sqrt{v_m}} = 0.46.$$

assume $b = 10 m_t$

$$z_{v1} = \frac{z_1}{\cos^3\beta} = \frac{d_1}{m_t} \times \frac{1}{\cos^3\beta} = \frac{80}{m_t} \times \frac{1}{\cos^3 45^\circ}$$

$$= \frac{226}{m_t}$$

$$z_{v2} = \frac{z_2}{\cos^3\beta} = \frac{d_2}{m_t} \times \frac{1}{\cos^3\beta} = \frac{320}{m_t} \times \frac{1}{\cos^3 45^\circ}$$

$$= \left(\frac{905}{m_t}\right)$$

$$y_1 = 0.17 - \frac{0.915}{\left(\frac{226}{m_t}\right)}$$

$$= \left(0.17 - 4.05 \times 10^{-3} m_t\right).$$

$$y_2 = 0.17 - \left(1.05 \times 10^{-3} m_t\right).$$

Since both the pinion & gear are made of the same material, pinion is weaker.

Applying Lewis eqn to pinion,

$$537 = \frac{100 \times (10 m_t) \times \left[0.17 - 4.05 \times 10^{-3} m_t\right] \times (\pi m_t) \times 0.46 \times \cos 45^{0}}{1.25}$$

assuming $C_w = 1.25$ for scant lubrication.

Solving for m_t, m_t = 41.96 mm, 2.01 mm, and −1.92 mm.

$\therefore$ Select m_t = ~~2 mm~~ ~~2.5~~ 2.0 mm.

3. No. of teeth (z_1, z_2).

$$z_{v1} = \frac{226}{2.0} = \cancel{90.4}\ 113$$

$$\text{Now } z_{v1} = \frac{z_1}{\cos^3\beta} \Rightarrow z_1 = \cancel{31.91}\ 39.95 \text{ say } \cancel{32}\ 40.$$

$$\therefore z_2 = 40 \times 4 = 160.$$

$\Rightarrow z_2 = 32 \times 4 = 128.$

4. Face width (b)

$$b = 10 m_t$$
$$= 10 \times 2$$
$$= 20 mm.$$

but $$b_{min} = \frac{1.15 \pi m_t}{\tan\beta}$$
$$= \frac{1.15 \times \pi \times 2}{\tan 45^\circ}$$
$$= 7.22 mm.$$

$\therefore$ Select $b = 20 mm.$

5. gear teeth proportions

$$m_n = m_t \cos\beta$$
$$= 2 \times \cos 45^\circ$$
$$= 1.414 mm.$$

$\therefore h_a = 1 m_n = 1.414 mm$

$h_f = 1.25 m_n = 1.767 mm.$

$h = 2.25 m_n = 3.181 mm.$

$h' = 2 m_n = 2.828 mm$

$c = 0.25 m_n = 0.353 mm.$

6. check for dynamic load (F_d)

$$F_d = F_t + \frac{21 v \left(F_t + bc \cos^2\beta\right) \cos\beta}{21 v + \sqrt{F_t + bc \cos^2\beta}}$$

For $v_m = 41.88$ m/sec, $f = 0.01$ mm. $\frac{(T\ 23.35a)}{P\ 23.35}$

$\therefore$ $C = 1501 \cdot N/mm.$ (T23.32).

$$\therefore F_d = 537 + \frac{21 \times 41.88 \left[537 + 20 \times 1501 \times \cos^2 45^\circ\right] / \cos 45^\circ}{21 \times 41.88 + \sqrt{537 + 20 \times 150.1 \times \cos^2 45^\circ}}$$

$= 1.917 KN.$

$F_f = \sigma_f \cdot b \cdot y \cdot m_t \cos\beta.$

$Y = \pi y_1$

$= \pi\left[0.17 - 4.05 \times 10^{-3} \times 2\right]$

$= 0.508$

where $y = \pi y$

σ_f for cast steel $= 290 MPa$ $\left(\frac{T23.33}{P23.76}\right)$

$\therefore F_f = 290 \times 20 \times$ [illegible] $\times 0.508 \times 2.0 \times \cos 45^\circ$

$= 4.16 KN.$

Since $F_f > F_d$, The design is safe.

7. Check for wear (F_W)

Wear load $F_W = \frac{d_1 \cdot b \cdot Q \cdot K}{\cos^2\beta}.$

where $Q = \frac{2Z_2}{Z_1 + Z_2} = \frac{2 \times 128}{(32 + 128)}$

$= 1.6.$

Now For BHN $= 300$ & 200

$K = 0.9035$ $\left(\frac{T23.37B}{P23.80}\right).$

$$\therefore F_W = \frac{80 \times 20 \times 1.6 \times 0.9035}{\cos^2 45^\circ}$$

$= 4.62 KN.$

Since $F_W > F_d$, design is safe for wear.