1) A 30cm x 15cm venturimeter is provided in a vertical pipe line carrying oil of specific gravity 0.9, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 30cm. The differential U-tube mercury manometer shows a gauge deflection of 25cm. Calculate i) the discharge of oil ii) the pressure difference between the entrance section and the throat section. Take coefficient of discharge as 0.98 and specific gravity of mercury as 13.6.

$d_1 = 30cm = 0.3m$

$d_2 = 15cm = 0.15m$

$sg_o = 0.9$

$Z_1 = 0$

$Z_2 = 30cm = 0.3$

$Z_1 - Z_2 = -0.3 m$

$x = 25cm = 0.25m$

$C_d = 0.98$

$sg_m = 13.6$

$$h = x\left[\frac{S_m}{S_o} - 1\right]$$

$$= 0.25\left[\frac{13.6}{0.9} - 1\right]$$

$$\underline{h = 3.527 \text{ m}}$$

$$Q = \frac{C_d a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= \frac{0.98 \times \frac{\pi(0.3)^2}{4} \times \frac{\pi(0.15)^2}{4} \times \sqrt{2 \times 9.8 \times 3.527}}{\sqrt{\left(\frac{\pi(0.3)^2}{4}\right)^2 - \left(\frac{\pi(0.15)^2}{4}\right)^2}}$$

$$\underline{Q = 0.148 \text{ m}^3/\text{sec}}$$

$$h = \frac{P_1 - P_2}{\rho g} + (Z - Z_2)$$

$$\frac{P_1 - P_2}{\rho g} = h - (Z - Z_2)$$

$$P_1 - P_2 = 9.8 \times 1000\left[3.527 - (-0.3)\right]$$

$$P_1 - P_2 = 37542.8 \text{ N/m}^2$$

$$= 37.542 \text{ kN/m}^2$$

$$\boxed{P_1 - P_2 = 37.54 \text{ kPa}}$$

2) Given the velocity field, $V = 5x^3 i - 15x^2 yj$, obtain the equation of streamlines. For the above given velocity field, check the continuity and irrotationality.

$V = 5x^3 i - 15x^2 y j \rightarrow V = ui + vj$

$u = 5x^3$

$v = -15x^2 y$

continuity eqn

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$\frac{\partial u}{\partial x} = 15x^2$

$\frac{\partial v}{\partial y} = -15x^2$

$15x^2 - 15x^2 = 0$

$-30xy$

$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

$\frac{\partial v}{\partial x} = -30xy$

$= \frac{1}{2}\left[-30xy - 0\right]$

$\frac{\partial u}{\partial y} = 0$

$\omega_z = -15xy$

$\omega_z \neq 0 \quad \therefore$ the eqn is not irrotational

$\frac{dy}{dx} = \frac{v}{u}$

$u = -\frac{\partial \psi}{\partial y} \qquad v = \frac{\partial \psi}{\partial x}$

$\cancel{\frac{\partial \psi}{\partial y} = -15} \quad u = \frac{\partial \psi}{\partial y} = 5x^3$

$\psi = -5x^3 y + f(x)$

$v = \frac{\partial \psi}{\partial x} = -15x^2 y$

$\psi = -5x^3 y + g(y)$

$$\boxed{\therefore \psi = -5x^3 y}$$

4) 215 litres of gasoline (specific gravity = 0.82) flow per second upwards in an inclined venturimeter fitted to a 300 mm diameter pipe. The venturimeter is inclined at 60° to the vertical and its 150mm diameter throat is 1.2m from the entrance along its length. Pressure gauges inserted at entrance and throat show pressures of 0.141 N/mm2 and 0.077N/mm2 respectively. Calculate the coefficient of discharge of venturimeter. If instead of pressure gauges, the entrance and throat of the venturimeter are connected to a differential manometer, determine its reading in mm of mercury column.

$Q = 215 \times 10^{-3}\ m^3/s$

$d_1 = 300mm$

$d_2 = 150mm$

$\theta = 60^\circ$

$a_1 = \frac{\pi d_1^2}{4} = \frac{3.14(0.3)^2}{4} = 0.0707\ m^2$

$a_2 = \frac{\pi(0.15)^2}{4} = 0.0177\ m^2$

$Z_1 = 0 \quad Z_2 = 1.2 \sin 30$

$Z_2 = 0.66m$

$P_1 = 0.141\ N/mm^2 = 0.141 \times 10^6\ \frac{N}{m^2}$

$P_2 = 0.077 \times 10^6\ N/m^2$

$$h = \frac{P_1 - P_2}{\rho g} + (Z_1 - Z_2)$$

$$= \frac{0.141 \times 10^6 - 0.077 \times 10^6}{9.8 \times 1000 \times 0.82} + (0 - 0.66)$$

$$\boxed{h = 7.304\ m}$$

$$Q = \frac{C_d\, a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}} \rightarrow C_d = \frac{Q\sqrt{a_1^2\ a_2^2}}{a_1 a_2 \sqrt{2gh}} =$$

$$= \frac{215 \times 10^{-3} \times \sqrt{0.0707^2 - 0.0177^2}}{0.0707 \times 0.0177 \sqrt{2 \times 9.8 \times 7.304}}$$

$$\boxed{C_d = 0.98}$$

$$h = x\left(\frac{S_m}{S_o} - 1\right)$$

$$7.304 = x\left(\frac{13.6}{0.82} - 1\right)$$

$$\boxed{x = 468\ mm}$$

3) For a two dimensional potential flow, the velocity function is given by *Φ=4x(3y-4)*, Find i) the velocity at the point (2,3) and ii) the stream function at the point (2,3).

(i)

$\phi = 4x(3y-4) \Rightarrow \phi = 12xy - 16x$

(2,3)

$\frac{\partial \phi}{\partial x} = -u$

$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x}(12xy - 16x)$

$= -(12y - 16)$

$u = -(12(3) - 16)$

$\underline{u = -20 \text{ units}}$

$\frac{\partial \phi}{\partial y} = -v$

$v = -\frac{\partial \phi}{\partial y} = -(12x - 0) = -(12 \times 2)$

$v = -24 \text{ units}$

$V = \sqrt{u^2 + v^2}$

$= \sqrt{(-20)^2 + (-24)^2}$

$\boxed{V = 31.24 \text{ units}}$