Internal Assessment Test II – April 2018

Sub: Finite Element Methods **Code:** 15ME61

Date: 16/ 04 / 2018 **Duration:** 90 mins Max Marks: 50 **Sem:** VI **Branch:** MECH

Note: Answer any **five** questions.

Marks OBE CO RBT

1 a The nodal co-ordinates of a triangular element are shown in fig 1. The x - coordinates of an interior point P is 3.3 and shape function. $N_1 = 0.3$. Determine N_2, N_3 and y - coordinate point P. **04** CO3 L2

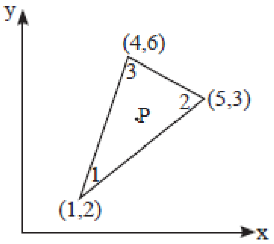


fig 1

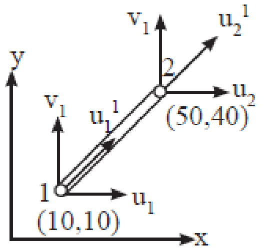


fig 2

b For the truss element shown in below fig 2 (x, y) co-ordinates of the element are indicated near nodes 1, 2. The element displacement dof vector is given by

$[u] = [1.5, 1.0, 2.1, 4.3]^T \times 10^{-2}$ mm Take $E = 300\times10^3$ N/mm^2

A = 100 mm^2 Determine the following

a) Element displacement dof in local co-ordinates

b) Stress in the element

c) Stiffness matrix of the element

06 CO4 L2

2 Derive Shape function and Jacobian matrix for CST element **10** CO3 L3

3 Derive shape functions for QUAD 4 element using Lagrangian element **10** CO3 L3

4 Derive stiffness matrix for 2D truss **10** CO4 L3

5 For the two-bar truss shown in fig 3. Determine the nodal displacements element stresses and support reactions. A force of P = 1000 KN is applied at node 1. Assume E = 210GPa and A = 600mm^2 for each element. **10** CO4 L4

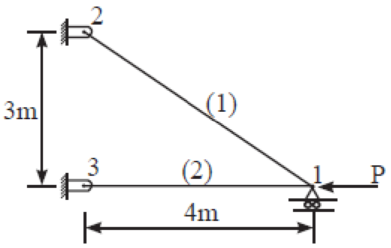


fig 3

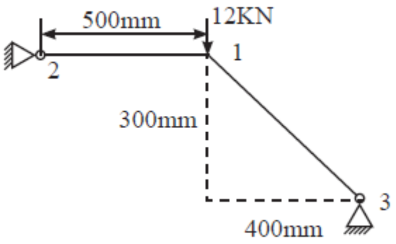
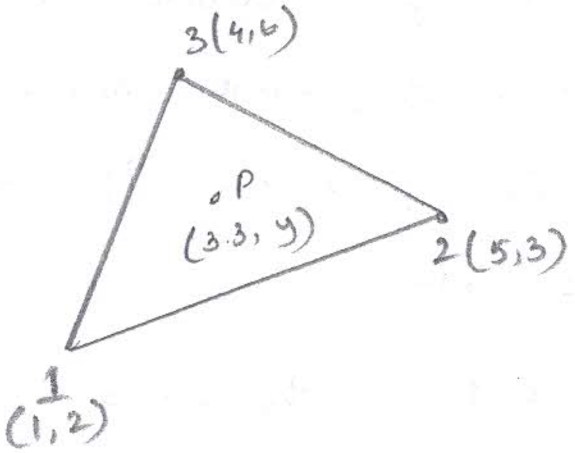


fig 4

6 Determine the unknown displacements and stresses for the structure shown in fig 4. Take $E = 0.7 \times 10^5$ MPa, A = 200mm². **10** CO4 L4

Solution of Internal Assessment Test II

1 a



$x = N_1 x_1 + N_2 x_2 + N_3 x_3$

$3.3 = 0.3(1) + \eta(5) + (1-\xi-\eta)4$

$3.3 = 0.3 + 5\eta + 4 - 4\xi - 4\eta$

$-4\xi + \eta = -1.$

$-4(0.3) + \eta = -1$

$\eta = 0.2$

$\therefore N_2 = \eta = 0.2 //$

$N_3 = 1-\xi-\eta = 1-0.3-0.2 = 0.5$

$N_3 = 0.5 //$

<u>y Coordinate</u>

$y = N_1 y_1 + N_2 y_2 + N_3 y_3$

$= 0.3(2) + 0.2(3) + 0.5(6)$

$y = 4.2 //$

1 b

Node Data

Node No	x (mm)	y (mm)
1	10	10
2	50	40

Element Connectivity Table

Element No	Initial Node	Final Node	Length of element	l	m
1	1	2	50	0.8	0.6

$$l_e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(50-10)^2 + (40-10)^2}$$

$$l_e = 50\,\text{mm}$$

$$l = \frac{x_{FN} - x_{IN}}{le} = \frac{40}{50} = 0.8$$

$$m = \frac{y_{FN} - y_{SN}}{le} = \frac{30}{10} = 0.6$$

i) Element displacement dof in local coordinate

$$q' = L\,q$$

$$\begin{bmatrix} q_1' \\ q_2' \end{bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.6 & 0 & 0 \\ 0 & 0 & 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} 1.5 \\ 1 \\ 2.1 \\ 4.3 \end{bmatrix} \times 10^{-2}$$

$$\begin{bmatrix} q_1' \\ q_2' \end{bmatrix} = \begin{bmatrix} 0.018 \\ 0.0426 \end{bmatrix} \text{mm} //$$

ii) Stress $\sigma = E \cdot \frac{1}{le} \begin{bmatrix} -l & -m & l & m \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$

$$= 300 \times 10^{3} \times \frac{1}{50} \begin{bmatrix} -0.8 & -0.6 & 0.8 & 0.6 \end{bmatrix} \begin{bmatrix} 1.5 \\ 1 \\ 2.1 \\ 4.3 \end{bmatrix} \times 10^{-2}$$

$$\sigma = 147.6\ \text{N/mm}^2 //$$

Stiffness matrix

$$K = \frac{EA}{l_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

$$= \frac{300\times10^3\times100}{50}\begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix}$$

$$K = 10^5\begin{bmatrix} 3.84 & 2.88 & -3.84 & -2.88 \\ 2.88 & 2.16 & -2.88 & -2.16 \\ -3.84 & -2.88 & 3.84 & 2.88 \\ -2.88 & -2.16 & 2.88 & 2.16 \end{bmatrix} //$$

2

Let the Shape function N_1 be

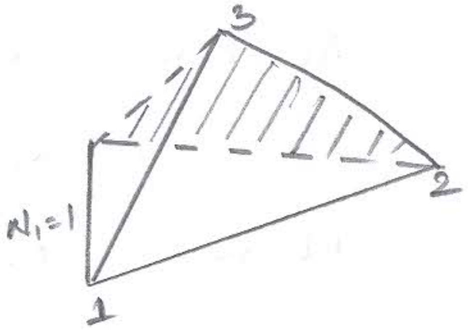
$$N_1 = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta$$

At node 1, $N_1 = 1$, $\xi = 1$, $\eta = 0$

$$1 = \alpha_1 + \alpha_2 \rightarrow ①$$

At node 2, $N_1 = 0$, $\xi = 0$, $\eta = 1$

$$0 = \alpha_1 + \alpha_3 \rightarrow ②$$



At node 3, $N_1 = 0$, $\xi = 0$, $\eta = 0$

$$\alpha_1 = 0 \rightarrow ③$$

$\alpha_3 = 0$; $\alpha_2 = 1$

$$\boxed{N_1 = \xi}$$

Let the shape function N_2 be

$$N_2 = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta$$

At node 1, $N_2 = 0$, $\xi = 1$, $\eta = 0$

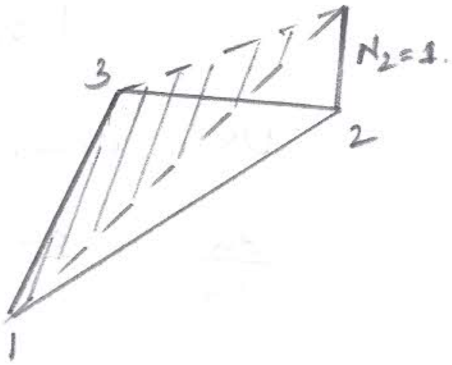
$$0 = \alpha_1 + \alpha_2 \rightarrow ①$$

At node 2, $N_2 = 1$, $\xi = 0$, $\eta = 1$

$$1 = \alpha_1 + \alpha_2 \rightarrow ②$$

At node 3, $N_2 = 0$, $\xi = 0$, $\eta = 0$

$$\alpha_1 = 0 \rightarrow ③$$



$\alpha_2 = 0$, $\alpha_3 = 1$.

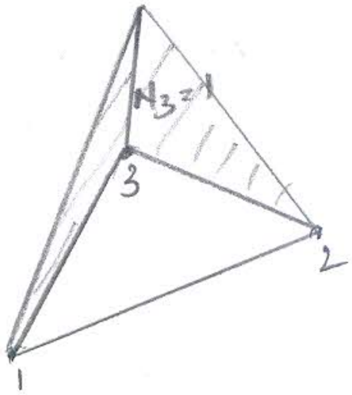
$$\boxed{N_2 = \eta}$$

Let the shape function N_3 be

$$N_3 = \alpha_1 + \alpha_2 \xi + \alpha_3 \eta$$

At node 1, $N_3 = 0$, $\xi = 1$, $\eta = 0$

$$0 = \alpha_1 + \alpha_2 \quad \rightarrow ①$$



At node 2, $N_3 = 0$, $\xi = 0$, $\eta = 1$

$$0 = \alpha_1 + \alpha_3 \quad \rightarrow ②$$

At node 3, $N_3 = 1$, $\xi = 0$, $\eta = 0$

$$\alpha_1 = 1 \quad \rightarrow ③$$

$$\alpha_2 = -1, \qquad \alpha_3 = -1$$

$$\boxed{N_3 = 1 - \xi - \eta}$$

DERIVATION OF STRAIN MATRIX FOR CST

Strain can be expressed as

$$\epsilon = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \partial u / \partial x \\ \partial v / \partial y \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{bmatrix} \quad \rightarrow ①$$

Using chain rule of partial differentiation

$$\frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial \xi} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial \xi}$$

$$\frac{\partial u}{\partial \eta} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial \eta} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial \eta}$$

$$\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \partial x/\partial \xi & \partial y/\partial \xi \\ \partial x/\partial \eta & \partial y/\partial \eta \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{bmatrix} = J \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix} \rightarrow ②$$

Where $J \rightarrow$ Jacobian

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

Jacobian is a scaling matrix which Corelates b/w Cartesian Coordinate & natural Coordinate w.r.t the differential of the displacements.

Using Isoparametric formulation

$$x = N_1 x_1 + N_2 x_2 + N_3 x_3$$

$$x = \xi x_1 + \eta x_2 + (1-\xi-\eta) x_3$$

$$\frac{\partial x}{\partial \xi} = x_1 - x_3 = x_{13}$$

$$\frac{\partial x}{\partial \eta} = x_2 - x_3 = x_{23}$$

lly $y = N_1 y_1 + N_2 y_2 + N_3 y_3$

$$y = \xi y_1 + \eta y_2 + (1-\xi-\eta) y_3$$

$$\frac{\partial y}{\partial \xi} = y_1 - y_3 = y_{13}$$

$$\partial y/\partial \eta = y_2 - y_3 = y_{23}$$

$$N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)$$

$i = 1, 2, 3, 4$

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

LAGRANGEAN METHOD

$$N_1(\xi,\eta) = L_1(\xi)\, L_1(\eta)$$

$$L_1(\xi) = \frac{\xi-\xi_2}{\xi_1-\xi_2} = \frac{\xi-1}{-1-1}$$

$$L_1(\xi) = \frac{1-\xi}{2}$$

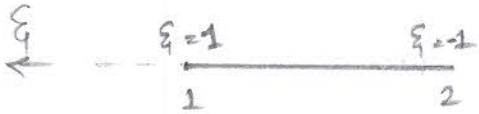
$$L_1(\eta) = \frac{\eta - \eta_4}{\eta_1 - \eta_4} = \frac{\eta - 1}{-1-1} = \frac{1-\eta}{2}$$

$$\therefore \quad N_1 = L_1(\xi)\; L_1(\eta)$$

$$= \frac{1-\xi}{2} \cdot \frac{1-\eta}{2}$$

$$\boxed{N_1 = \frac{1}{4}(1-\xi)(1-\eta)}$$

$$N_2(\xi, \eta) = L_2(\xi)\; L_2(\eta)$$



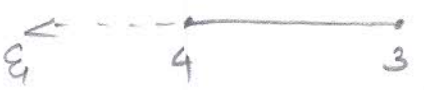
$$L_2(\xi) = \frac{\xi - \xi_1}{\xi_2 - \xi_1} = \frac{\xi - 1}{-1-1} = \frac{1+\xi}{2}$$

$$L_2(\eta) = \frac{\eta - \eta_3}{\eta_2 - \eta_3} = \frac{\eta + 1}{-1-1} = \frac{1-\eta}{2}$$

$$N_2 = \frac{1+\xi}{2} \; \frac{1-\eta}{2}$$

$$\boxed{N_2 = \frac{1}{4}(1+\xi)(1-\eta)}$$

$$N_3(\xi, \eta) = L_3(\xi)\; L_3(\eta)$$



$$L_3(\xi) = \frac{\xi - \xi_4}{\xi_3 - \xi_4} = \frac{\xi + 1}{+1+1} = \frac{1+\xi}{2}$$

$$L_3(\eta) = \frac{\eta - \eta_2}{\eta_3 - \eta_2} = \frac{\eta + 1}{1+1} = \frac{1+\eta}{2}$$

$$\boxed{N_3 = \frac{1}{4}(1+\xi)(1+\eta)}$$

$$N_4(\xi,\eta) = L_4(\xi)\ L_4(\eta)$$

$$L_4(\xi) = \frac{\xi-\xi_3}{\xi_4-\xi_3} = \frac{\xi-1}{-1-1} = \frac{1-\xi}{2}$$

$$L_4(\eta) = \frac{\eta-\eta_1}{\eta_4-\eta_1} = \frac{\eta+1}{1+1} = \frac{1+\eta}{2}$$

$$\boxed{N_4 = \frac{1}{4}(1-\xi)(1+\eta)}$$

DERIVATION OF ELEMENTAL STIFFNESS MATRIX FOR A TRUSS ELEMENT

Relation b/w nodal displacement of a truss element in local coordinates & global coordinate is expressed as

$$q' = L q$$

where $q' = \begin{bmatrix} q_1' \\ q_2' \end{bmatrix}$; $L = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$; $q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$

A truss element in local coordinate is equivalent to 1D bar element having the stiffness matrix

$$K' = \frac{EA}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Strain energy for a truss element in local coordinate is given by

$$U_e = \frac{1}{2} q'^T K' q' \rightarrow ①$$

It is required to determine strain energy of truss element in global coordinates.

$$q' = L q$$

Sub. this in eqn ① we get

$$U_e = \frac{1}{2} q^T L^T K' L q$$

$$U_e = \frac{1}{2} q^T \left(L^T K' L \right) q$$

$$U_e = \frac{1}{2} q^T K_e q$$

Where K_e - elemental stiffness matrix in global coordinates

$$K_e = L^T K' L$$

$$L = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \; ; \quad L^T = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix} \; ; \; K' = \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K_e = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix} \frac{EA}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$$

$$K_e = \frac{EA}{l_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Where l & m are direction cosines.

5

Nodal Data

Node No.	x (mm)	y (mm)
1	4000	0
2	0	3000
3	0	0

Element Connectivity Table

Element NO	Initial Node	Final Node	length of element l_e (mm)	l	m
1	2	1	5000	−0.8	−0.6
2	1	3	4000	−1	0

Element Stiffness matrix

$$K = \frac{EA}{l_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

For element 1

$$K_1 = \frac{210\times10^3\times600}{5000}\begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix}$$

$$K_1 = 10^3 \begin{array}{c|cccc|c} & 1 & 2 & 3 & 4 & \\ \hline & 16.128 & -12.096 & -16.12 & 12.096 & 1 \\ & -12.096 & 9.072 & 12.096 & -9.072 & 2 \\ & -16.128 & 12.096 & 16.128 & -12.096 & 3 \\ & 12.096 & -9.072 & -12.096 & 9.072 & 4 \end{array}$$

For element 2

$$K_2 = \frac{210 \times 10^3 \times 600}{4000} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= 10^3 \begin{array}{c|cccc|c} & 1 & 2 & 5 & 6 & \\ \hline & 31.5 & 0 & -31.5 & 0 & 1 \\ & 0 & 0 & 0 & 0 & 2 \\ & -31.5 & 0 & 31.5 & 0 & 5 \\ & 0 & 0 & 0 & 0 & 6 \end{array}$$

Global stiffness matrix

$$K = 10^3 \begin{array}{c|cccccc|c} & 1 & 2 & 3 & 4 & 5 & 6 & \\ \hline & 47.628 & -12.096 & -16.128 & 12.096 & -31.5 & 0 & 1 \\ & -12.096 & 9.072 & 12.096 & -9.072 & 0 & 0 & 2 \\ & -16.128 & 12.096 & 16.128 & -12.096 & 0 & 0 & 3 \\ & 12.096 & -9.072 & -12.096 & 9.072 & 0 & 0 & 4 \\ & -31.5 & 0 & 0 & 0 & 31.5 & 0 & 5 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 6 \end{array}$$

Equilibrium eqn.

$$[K][q] = [F]$$

$$10^3 \begin{bmatrix} 47.628 & -12.096 & -16.128 & 12.096 & -31.5 & 0 \\ 12.096 & 9.072 & 12.096 & -9.072 & 0 & 0 \\ -16.128 & 12.096 & 16.128 & -12.096 & 0 & 0 \\ 12.096 & -9.072 & -12.096 & 9.072 & 0 & 0 \\ -31.5 & 0 & 0 & 0 & 31.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} -10^6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Nodal displacement

$$q_1 = -20.996 \text{ mm} //$$

$$q_2 = q_3 = q_4 = q_5 = q_6 = 0 //$$

Stress

$$\sigma = \frac{E}{l_e} \begin{bmatrix} -l & -m & l & m \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$= \frac{210 \times 10^3}{5000} \begin{bmatrix} 0.8 & -0.6 & -0.8 & 0.6 \end{bmatrix} \begin{bmatrix} -20.996 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sigma_1 = -705.46 \text{ N/mm}^2 //$$

$$\sigma_2 = \frac{210 \times 10^3}{4000} \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -20.996 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= -1102.96 \text{ N/mm}^2$$

Reaction

$$R_2 = K_{21} q_1 = -12.096 \times 10^3 \times -20.996 = 253.96 \text{ kN}$$

$$R_3 = K_{31} q_1 = -16.126 \times 10^3 \times -20.996 = 338.62 \text{ kN}$$

$$R_4 = K_{41} q_1 = -253.96 \text{ kN}$$

$$R_5 = K_{51} q_1 = -31.5 \times 10^3 \times -20.996 = 661.37 \text{ kN}$$

6

Node Data

Node No.	x(mm)	y(mm)
1	500	300
2	0	300
3	900	300

Element Connectivity Table

Element No	Initial Node	Final Node	Length of element 'l_e'	l	m
1	1	2	500	−1	0
2	1	3	500	0.8	−0.6

$$l = \frac{x_{F.N} - x_{I.N}}{le} = \frac{0-500}{500} = -1$$

$$m = \frac{y_{FN} - y_{IN}}{le} = \frac{300-300}{500} = 0$$

Stiffness matrix

$$K = \frac{EA}{le}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

For element 1

$$K_1 = \frac{0.7\times10^5\times200}{500}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_1 = 10^3 \begin{array}{cccccc} & 1 & 2 & 3 & 4 & \\ \left[\right. & 28 & 0 & -28 & 0 & \left.\right] 1 \\ & 0 & 0 & 0 & 0 & 2 \\ & -28 & 0 & 28 & 0 & 3 \\ & 0 & 0 & 0 & 0 & 4 \end{array}$$

For <u>element 2</u>

$$K_2 = \frac{0.7 \times 10^5 \times 200}{500} \begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix}$$

$$= 10^3 \begin{array}{cccccc} & 1 & 2 & 5 & 6 & \\ & 17.92 & -13.44 & -17.92 & 13.44 & 1 \\ & -13.44 & 10.08 & 13.44 & -10.08 & 2 \\ & -17.92 & 13.44 & 17.92 & -13.44 & 5 \\ & 13.44 & -10.08 & -13.44 & 10.08 & 6 \end{array}$$

Global

$$K = 10^3 \begin{array}{cccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & \\ & 45.92 & -13.44 & -28 & 0 & 0 & 0 & 1 \\ & -13.44 & 10.08 & 0 & 0 & 13.44 & -10.08 & 2 \\ & -28 & 0 & 28 & 0 & 0 & 0 & 3 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ & 0 & 13.44 & 0 & 0 & 17.92 & -13.44 & 5 \\ & 0 & -10.08 & 0 & 0 & -13.44 & 10.08 & 6 \end{array}$$

$$[K][q] = [F]$$

$$10^3 \begin{bmatrix} 45.92 & -13.44 & -28 & 0 & 0 & 0 \\ -13.44 & 10.08 & 0 & 0 & 13.44 & -10.08 \\ -28 & 0 & 28 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 13.44 & 0 & 0 & 17.92 & -13.44 \\ 0 & -10.08 & 0 & 0 & -13.44 & 10.08 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} 0 \\ -12 \times 10^3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$q_3 = q_4 = q_5 = q_6 = 0$$

$$q_1 = -0.57 \text{ mm}, \quad q_2 = -1.95 \text{ mm}$$

<u>Stresses:</u>

$$\sigma_1 = E \cdot \frac{1}{l_e} \begin{bmatrix} -l & -m & l & m \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$= 0.7 \times 10^5 \times \frac{1}{500} \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -0.57 \\ -1.95 \\ 0 \\ 0 \end{bmatrix}$$

$$= -79.8 \text{ N/mm}^2$$

$$\sigma_2 = E \cdot \frac{1}{l_e} \begin{bmatrix} -l & -m & l & m \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_5 \\ q_6 \end{bmatrix}$$

$$= 0.7 \times 10^5 \times \frac{1}{500} \begin{bmatrix} -0.8 & 0.6 & 0.8 & -0.6 \end{bmatrix} \begin{bmatrix} -0.57 \\ -1.95 \\ 0 \\ 0 \end{bmatrix}$$

$\sigma_2 = -99.96 \ N/mm^2$

Displacements

$q_1 = -0.57 \ mm$

$q_2 = -1.95 \ mm$

$q_3 = 0$

$q_4 = 0$

$q_5 = 0$

$q_6 = 0$

Stress

$\sigma_1 = -79.8 \ N/mm^2$

$\sigma_2 = -99.96 \ N/mm^2$