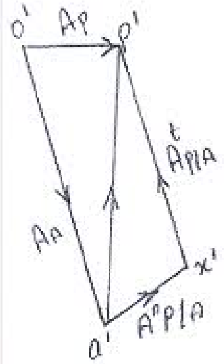
3a. Explain Klien's construction for slider crank mechanism.

Sol A graphical method to find the velocity and acceleration of piston of a reciprocating engine mechanism was given by Prof. Klein.

(i) Velocity diagram.

1. Draw the slider crank mechanism OAP of the given position of crank OA as shown in fig.(a).
2. Draw a perpendicular from O to meet the extension of PA at M.
3. The triangle OAM is known as Klien's velocity diagram.



Fig(c).

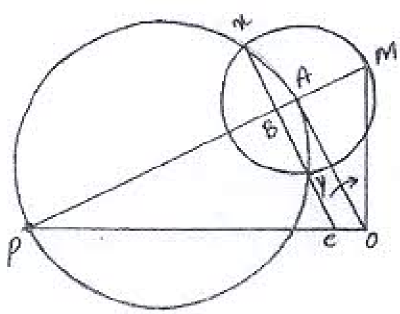
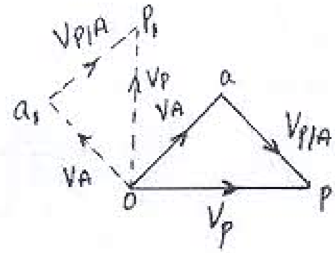


Fig (a)



Fig(b).

4. The Velocity diagram obtained by relative method is shown in fig(b). as triangle Oap which is rotated by 90° and indicated by dotted lines as Oa_1P_1.

5. Since the triangle OAM obtained by Klein's construction, and the triangle Oa_1P_1 obtained by the relative velocity method are similar.

$$\frac{a_1P_1}{AM} = \frac{OP_1}{OM} = \frac{Oa_1}{OA} = \omega.$$

$\therefore$ $a_1P_1 = V_{PA} = \omega . AM$ = Velocity of connecting rod.

$OP_1 = V_P = \omega . OM$ = Velocity of piston.

$Oa_1 = V_A = \omega . OA$ = Velocity of crank.

6. Also, $V_{PA} = \omega_{AP} \times AP \therefore \omega_{PA} \times AP = \omega \times Am$.

$\therefore$ Angular Velocity of connecting rod $\omega_{CR} = \omega_{AP} = \omega . \frac{AM}{AP}$

(ii) Acceleration diagram (Klein's construction).

1. With A as centre, and radius equal to AM draw a circle.
2. With AP as diameter the second circle to intersect the first circle at points x and y.
3. Join x and y cutting AP at B and OP at C.
4. join OABC which forms a quadrilateral and known as klein's acceleration diagram.

. Since the quadrilateral OABC obtained by klein's construction and the quadrilateral shown in fig (c) obtained by the relative acceleration method are similar.

$$\frac{o'a'}{OA} = \frac{a'x'}{AB} = \frac{x'p'}{BC} = \frac{p'o'}{CO} = \omega$$

$\therefore o'a' = \omega^2 \times OA$ = Acceleration of crank OA.

$a'x' = \omega^2 \times AB = A^n_{PA}$ = Normal acceleration of connecting rod

$x'p' = \omega^2 \times BC = A^t_{PA}$ = Tangential acceleration of —"—

$p'o' = \omega^2 \times OC = A_P$ = Acceleration of piston.

$A^t_{PA} = \alpha_{PA} \times AP$

$\therefore \omega^2 \times BC = \alpha_{PA} \times AP$

$\alpha_{PA} = \omega^2 \times \frac{BC}{AP}$ = Angular acceleration of connecting rod.

<u>Proof</u>:-

In order to prove the quadrilateral OABC represents the acceleration diagram, it is necessary to prove that the quadrilateral OABC is similar to the quadrilateral o'a'x'p'. For that, if it satisfies the following two conditions, then the two quadrilaterals are similar.

(i) The sides of quadrilateral OABC must be parallel to the side of quadrilateral o'a'x'p'.

(ii) Ratio of adjacent sides of quadrilateral must be equal to the ratio of corresponding adjacent sides to another quadrilateral.

(a) OC parallel to o'p' by construction

OA parallel to o'a' ——"——

AB Parallel to a'x' ——"——

BC Parallel to x'p' ——"—— Since XY is ⊥ to AP.

Hence it satisfies the first condition.

(b) Join Px and Ax and from the triangle APx and ABx.

Angle AxP = Angle ABx = right angles

Angle PAx = Angle BAx = common angle.

$\therefore \angle APx = \angle AxB$

Therefore the two triangles are similar.

$$\therefore \frac{Ax}{AP} = \frac{AB}{Ax}$$

$$\text{i.e., } AB = \frac{Ax^2}{AP} = \frac{AM^2}{AP} \quad (\because Ax = AM)$$

Dividing b.s by OA.

$$\frac{AB}{OA} = \frac{AM^2}{AP.OA} \quad \text{— (1)}$$

$$\text{Now, } \frac{a'x'}{o'a'} = \frac{\text{Normal acceleration of connecting rod AP}}{\text{Absolute acceleration of crank OA}}$$

$$= \frac{V_{PA}^2/AP}{\omega^2 \times OA} = \frac{\omega^2 \times AM^2}{AP.\omega^2 \times OA} \quad (\because V_{PA} = \omega.AM)$$

$$\text{i.e } \frac{a'x'}{o'a'} = \frac{AM^2}{AP.OA} \quad \text{— (2)}$$

Equating (1) and (2), we get.

$$\frac{AB}{OA} = \frac{a'x'}{o'a'}$$

Hence it satisfies the second condition also.

$\therefore$ Quadrilateral OABC is similar to quadrilateral $o'a'x'p'$.

3b. The crank and connecting rod of a reciprocating engine are 200mm and 700mm respectively. The crank is rotating in clockwise direction at 120 rad/sec. Find with the help of Klien's construction:

(i) Velocity and acceleration of Piston.

(ii) Angular Velocity and angular acceleration of the connecting rod,

at the instant when the crank is at 30° to the inner dead centre.

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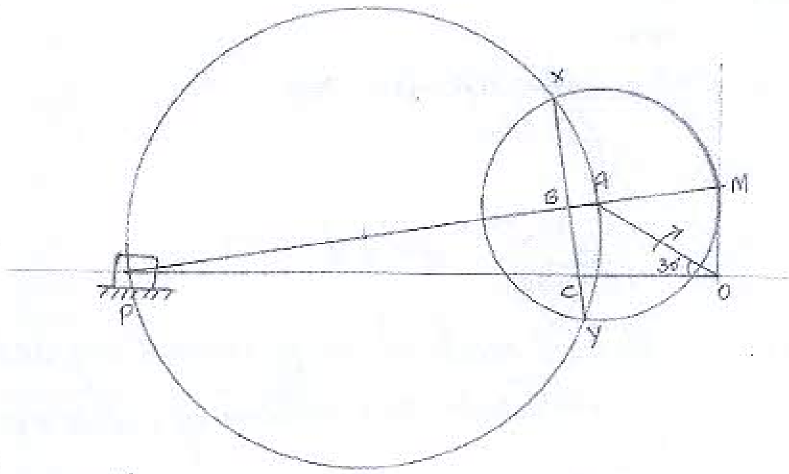
Scale
100mm = 1cm.



Given

$\omega_{OA} = 120 \text{ rad/sec}$.

$OA = 200mm = 0.2m$

$AP = 700mm = 0.7m$.

(i) Velocity of piston, $V_{PO} = \omega \times OM$

$= 120 \times 130 \times 10^{-3}$

$= 15.6 m/s$.

acceleration of piston, $A_p = \omega^2 \times OC$

$= 120^2 \times 200 \times 10^{-3}$

$= 2880 m/sec^2$.

(ii) Angular velocity of connecting rod.

$$\omega_{AP} = \omega \times \frac{AM}{AP}$$

$$= \frac{120 \times 180}{700} = 30.85 \text{ rad/sec}.$$

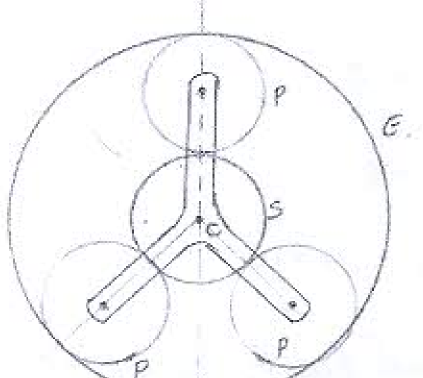
Angular acceleration of connecting rod.

$$\alpha_{AP} = \omega^2 \times \frac{BC}{AP}$$

$$\alpha_{AP} = \frac{120^2 \times 100}{700} = 2057.14 \text{ rad/sec}^2$$

4. An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star shaped carrier C. The sizes of different toothed wheels are such that the planet carrier C rotates at $1/5^{th}$ of the speed of the sun wheel S. The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel S. The minimum number of teeth on any wheel is 100 N-m
(i) Number of teeth on different wheels of the train, and
(ii) Torque necessary to keep the internal gear stationary.

Sol



(i) Number of teeth on different wheels.
As the minimum number of teeth on any wheel is 16, take the number of teeth on sun wheel $T_S = 16$.
Since the pitch circle radius is proportional to number of teeth and the gears have same pitch.

$$r_E = r_S + 2r_P$$

$$\text{i.e } T_E = T_S + 2T_P$$

$$\therefore T_P = \frac{T_E - T_S}{2} \quad -(1)$$

Tabular column :-

Condition of motion.	Planet carrier C	Sun wheel S	Planet wheel P	Internal gear E.
Fix the planet c and give +1 rev to S	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_E}$
Multiply by x	0	x	$-\frac{T_S}{T_P}\cdot x$	$-\frac{T_S}{T_E}\cdot x$
Add y	y	x + y	$y-\frac{T_S}{T_P}\cdot x$	$y-\frac{T_S}{T_E}x.$

Planet carrier C rotates @ 1/5 of the speed of the Sunwheel S
i.e For every 5 revolutions of the sunwheel S, planet carrier C will make 1 revolution.

$$\therefore y = 1 \text{ and } x + y = 5$$
$$\therefore x = 4.$$

Internal gear E is stationary

$$\text{i.e., } y - \frac{T_S}{T_E} x = 0$$
$$\text{i.e, } 1 - \frac{T_S}{T_E} \cdot 4 = 0$$
$$\therefore T_E = 4 T_S = 4 \times 16$$
$$T_E = 64$$

i.e. Number of teeth on internal gear E, $T_E = 64$.

From eqn (1) $T_P = \frac{T_E - T_S}{2} = \frac{64 - 16}{2} = 24$.

i.e Number of teeth on planet wheel P, $T_P = 24$.

(ii) Torque necessary to keep the internal gear stationary

From energy equation

$$\tau_S N_S + \tau_C N_C + \tau_E N_E = 0$$
$$\text{i.e } \tau_S N_S + \tau_C N_C = 0 \quad (\because N_E = 0)$$
$$100 \times 5 + \tau_C \times 1 = 0$$
$$\tau_C = -500 \text{ N-m}$$

$\therefore$ Torque on planet carrier C = -500 N-m.

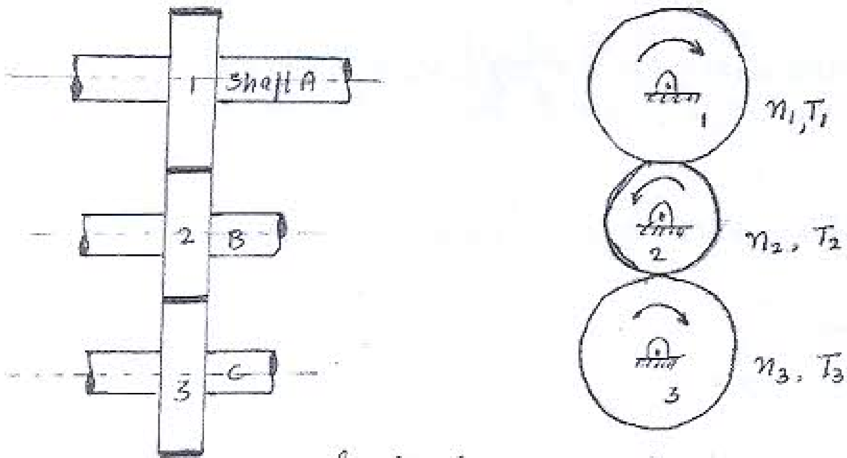
From torque equation,

$$\tau_S + \tau_C + \tau_E = 0$$
$$100 - 500 + \tau_E = 0$$

Torque necessary to keep the internal gear E stationary $\tau_E = 400$ N-m.

5a. What are the types of gear trains? Explain with the help of neat sketches.

Sol: Simple gear train is as shown in fig.



1. In simple gear train each shaft carries only one gear.
2. All the gears revolve about fixed axis.
3. Velocity ratio (V.R):

let n_1, n_2 and n_3 are speeds of gears 1, 2 and 3 respectively. T_1, T_2 & T_3 are number of teeth on gears 1, 2 and 3 respectively.

Consider gear 1 to be the driver.

$\therefore$ V.R for gear 1 and gear 2 is, $\frac{n_1}{n_2} = \frac{T_2}{T_1}$ —(i)

V.R for gear 2 and gear 3 is, $\frac{n_2}{n_3} = \frac{T_3}{T_2}$ —(ii)

Multiplying (i) and (ii), we get

$$\frac{n_1}{n_2} \times \frac{n_2}{n_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$

$$\text{i.e} \quad V.R = \frac{n_1}{n_3} = \frac{T_3}{T_1}$$

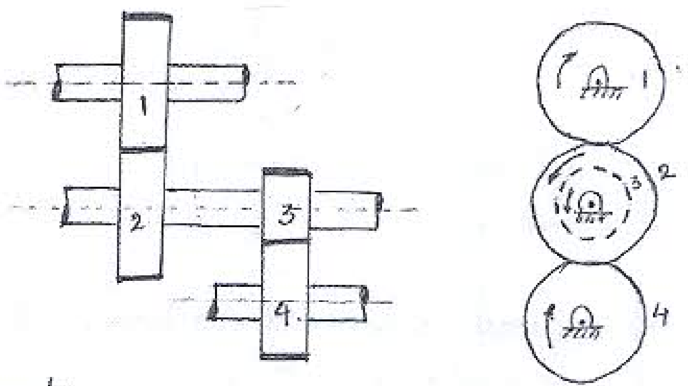
Therefore, the velocity ratio is independent of number of intermediate gears used. Hence Gear wheel 2 is called an idler.

$$\text{Train value} = \frac{n_3}{n_1} = \frac{T_1}{T_3}$$

Compound Gear train

(i) In compound gear train each shaft carries two or more gears except the first and last, one of which acts as a follower and the other as the driver.

(ii) All the gears revolves about a fixed axis.



(iii) Velocity ratio :-

Gear 1 is in mesh with gear 2.

$$\therefore V.R = \frac{n_1}{n_2} = \frac{T_2}{T_1} \quad \text{—(i)}$$

Similarly. VR for gear 3 & gear 4 is

$$\frac{n_3}{n_4} = \frac{T_4}{T_3} \quad \text{—(ii)}$$

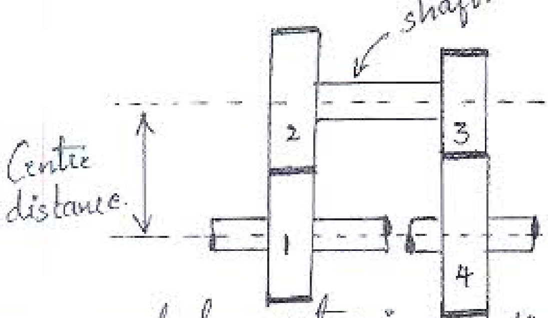
Multiplying (i) & (ii)

$$\frac{n_1}{n_2} \cdot \frac{n_3}{n_4} = \frac{T_2}{T_1} \cdot \frac{T_4}{T_3}$$

$$\text{i.e } V.R = \frac{n_1}{n_4} = \frac{T_2 \cdot T_4}{T_1 \cdot T_3} \qquad (\because n_2 = n_3).$$

$$\therefore T.V = \frac{n_4}{n_1} = \frac{T_1 T_3}{T_2 \cdot T_4} = \frac{\text{Product of number of teeth on driver gears}}{\text{Product of number of teeth on driven gears.}}$$

Reverted gear train



(i) In reverted gear train the first and last gears are on the same axis.

(ii) In reverted gear train, the centre distances of the two pairs of gears must be the same.

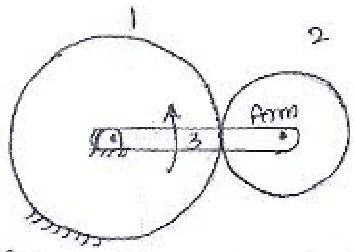
$$\therefore r_1 + r_2 = r_3 + r_4.$$

$$\therefore T_1 + T_2 = T_3 + T_4.$$

(iii) Train value $= \dfrac{T_1 T_3}{T_2 \cdot T_4}$. Since gear 2 and gear 3 are compound gear.

(iv) All the gears revolve about a fixed axis.

Epicyclic gear train



In epicyclic gear train, the axes of rotation of all the wheels are not fixed. In epicyclic gear train, axes of some gears having relative motion with respect to other (or) relative to the frame. The gear 2 revolves about its own axis as well as about the centre of the fixed gear.
Epicyclic gear train is also called planetary gear train.

5b. An epicyclic train of gears is arranged as shown in the fig. How many revolution does the arm, to which the pinions B and C are attached, make:

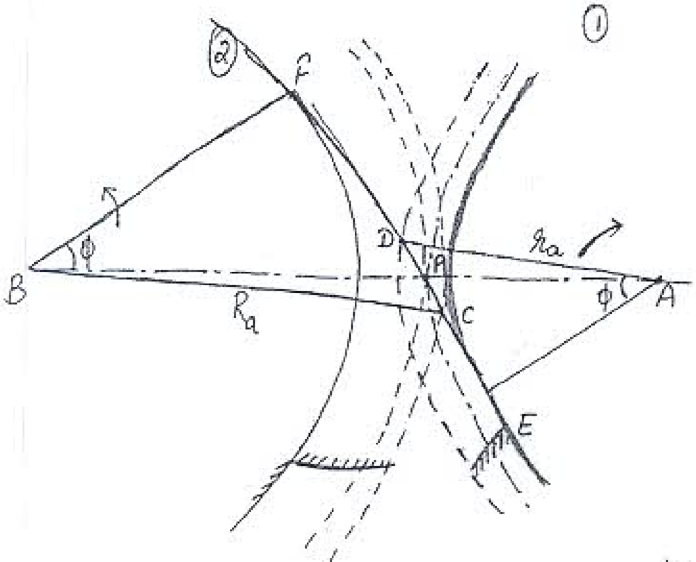
(i) when A makes one revolution clockwise and D makes half a revolution counter clockwise, and

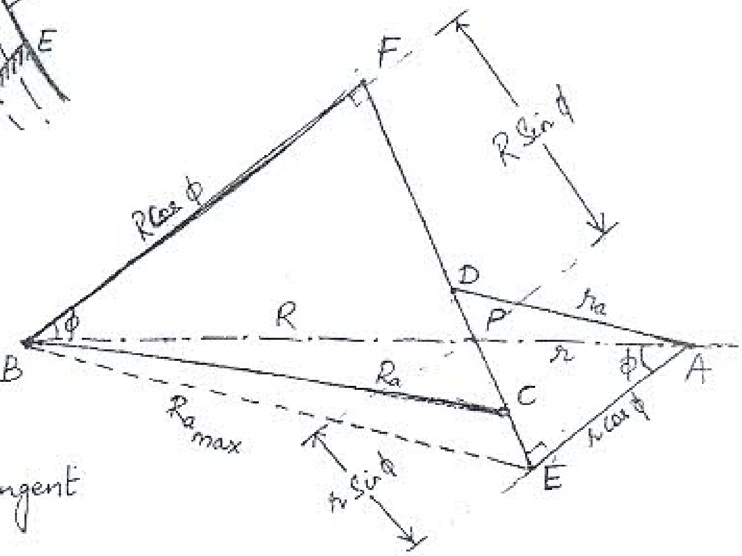
(ii) when A makes one revolution clockwise and D is stationary.

The number of teeth on the gears A and D are 40 & 90 respectively.

4 a.)

LENGTH OF PATH OF CONTACT :-





- The pinion 1 is the driver & is rotating CW. The wheel 2 is driven in CCW direction.
- EF is their common tangent to the base circles.
- Contact of the two teeth is made where the addendum circle of the gear meets the line of action EF, i.e, at C and is broken where the addendum circle of the pinion meets the line of action, i.e, at D. <u>CD is the path of contact.</u>

Let r = Pitch circle radius of pinion
R = Pitch circle radius of gear.
r_a = Addendum circle radius of pinion.
R_a = Addendum circle radius of gear.

Path of contact = Path of approach + path of recess.

$$\Rightarrow CD = CP + PD$$
$$= (CF - PF) + (ED - PE) \quad \text{——— (1)}$$

In ΔBFC; $CB^2 = BF^2 + CF^2$

$$\Rightarrow CF^2 = CB^2 - BF^2$$
$$= R_a^2 - R^2\cos^2\phi$$
$$\Rightarrow \boxed{CF = \sqrt{R_a^2 - R^2\cos^2\phi}}$$

In ΔAED; $AD^2 = ED^2 + EA^2$

$$\Rightarrow ED^2 = AD^2 - EA^2$$
$$= r_a^2 - r^2\cos^2\phi$$
$$\Rightarrow \boxed{ED = \sqrt{r_a^2 - r^2\cos^2\phi}}$$

Putting the values in Eqn (1);
we have

$$CD = \left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right]$$

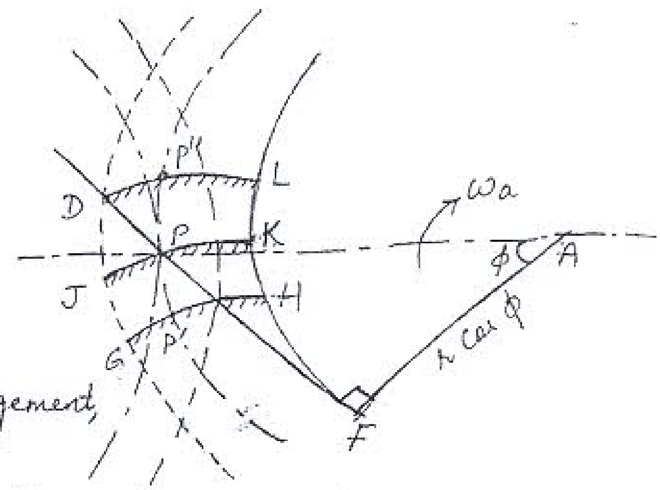
Path of Contact $\Rightarrow$ $\boxed{CD = \sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)(\sin\phi)}$

* Path of approach can be found if the dimensions of the driven wheel or gear are known. Similarly, the path of recess is known from the dimensions of the driving wheel or pinion.

LENGTH OF ARC OF CONTACT :⇒

The arc of contact is the distance travelled by a point on either of pitch circle of the two wheels during the period of contact of a pair of teeth.

- In the fig., at the beginning of engagement, the driving involute is shown as GH; when the point of contact is at P, it is shown as JK and when at the end of engagement, it is DL.



- The arc of contact is P'P'' and it consists of the arc of approach P'P and the arc of recess PP''.

Let the time of traverse the arc of approach is t_a

Then;

$$\text{Arc of approach} = P'P = \text{Tangential velocity of } P' \times \text{Time of Approach}$$

$$= (\omega_a \cdot r) \times t_a$$

$$= \omega_a \cdot (r\cos\phi) \times \frac{1}{\cos\phi} \cdot t_a$$

$$= (\text{Tangential vel. of } H) \cdot t_a \cdot \frac{1}{\cos\phi}$$

$$= \frac{\text{Arc } HK}{\cos\phi}$$

$$= \frac{\text{Arc } FK - \text{Arc } FH}{\cos\phi}$$

$$= \frac{FP - FC}{\cos\phi} = \frac{CP}{\cos\phi} = \frac{\text{Path of Approach}}{\cos\phi}$$

⇒ Arc FK is equal to the path FP as the point P is on the generator FP[G] that rolls on the base circle FHK to generate the involute PK.

Similarly Arc FH = Path FC

Arc of recess = PP'' = Tangential velocity of P × Time of recess.

Let t_r = time of recess

$$= (\omega_a \cdot r) \cdot t_r$$

$$= \omega_a \cdot r\cos\phi \cdot \frac{1}{\cos\phi} \cdot t_r$$

$$= (\text{Tangential vel. of } K) \cdot t_r \cdot \frac{1}{\cos\phi}$$

$$= \frac{\text{Arc } KL}{\cos\phi}$$

$$= \frac{\text{Arc } FL - \text{Arc } FK}{\cos\phi}$$

$$PP'' = \frac{FD - FP}{\cos\phi} = \frac{PD}{\cos\phi} = \frac{\text{Path of Recess}}{\cos\phi}$$

$$\therefore \text{Arc of contact} = \frac{CP}{\cos\phi} + \frac{PD}{\cos\phi} = \frac{CP + PD}{\cos\phi} = \frac{CD}{\cos\phi}$$

$$\Rightarrow \boxed{\text{Arc of Contact} = \frac{\text{Path of contact}}{\cos\phi}}$$

CONTACT RATIO :⇒ "Number of pairs of Teeth in Contact.

– The arc of contact is the length of the pitch circle traversed by a point on it during the mating of a pair of teeth. Thus, all the teeth lying in between the arc of contact will be meshing with the teeth on the other wheel.

Therefore; the number of teeth in contact is given as :→

$$n = \frac{\text{Arc of Contact}}{\text{Circular pitch}} = \frac{CD}{\cos\phi} \Big/ p.$$

4 b.) <u>Data</u> :→ $\phi = 20°$

$m = 4$

$T = t = 40$.

Arc of contact = 1.75 × circular pitch.

<u>To find</u> :⇒ addendum (a).

$$R = \frac{mT}{2} = \frac{4 \times 40}{2} = 80 \text{ mm}.$$

$$AoC = 1.75\, p_c = 1.75\, \pi m.$$

$$= 1.75 \times \pi \times 4$$

$$\Rightarrow \boxed{AoC = 21.99 \text{ mm}}$$

$$AoC = PoC / \cos\phi \Rightarrow PoC = AoC \times \cos\phi = 21.99 \times \cos 20°$$

$$\Rightarrow \boxed{PoC = 20.66 \text{ mm}}$$

$$PoC = 2\left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right].$$

$$20.66 = 2\left[\sqrt{R_a^2 - (80)^2(\cos 20)^2} - 80 \times \sin 20\right]$$

$$\Rightarrow \boxed{R_a = 84.09 \text{ mm}}$$

Addendum $\Rightarrow a = R_A - R = 84.09 - 80$.

$$\Rightarrow \boxed{a = 4.09 \text{ mm}}$$ (Ans).

5 a.) Meshing of two non-conjugate or non-involute teeth is known as <u>interference</u> because the two teeth do not slide properly and thus rough action and binding occurs.

Methods to avoid interference

① Height of the teeth may be reduced.

② The radial flank of the pinion may be cut back, known as under-cutting.

③ Centre distance may be increased. It leads to increase in pressure angle.

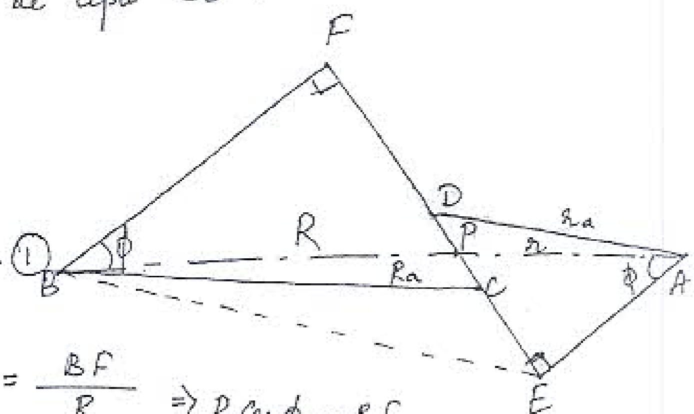
④ By tooth correction.

* To avoid interference, the limiting value of the addendum of the gear is GE whereas that of the pinion is HF.

5 b.) MINIMUM NUMBER OF TEETH TO AVOID ~~I~~NTERFERENCE :⇒

The maximum value of the addendum radius of the wheel to avoid interference can be upto BE.

Referring to the fig.;



from ΔBEF

$$BE^2 = BF^2 + FE^2$$

$$= BF^2 + (FP + PE)^2 \quad \text{—①}$$

⇒ In ΔBFP $\quad \cos\phi = \frac{BF}{BP} = \frac{BF}{R} \Rightarrow R\cos\phi = BF$

$$\Rightarrow \boxed{BF = R\cos\phi}$$

$$\sin\phi = \frac{FP}{BP} = \frac{FP}{R} \Rightarrow \boxed{FP = R\sin\phi}$$

⇒ In ΔAEP

$$\sin\phi = \frac{PE}{AP} = \frac{PE}{r} \Rightarrow \boxed{PE = r\sin\phi}$$

Substituting these values in eqn ①;

$$BE^2 = BF^2 + (FP + PE)^2$$

$$BE^2 = (R\cos\phi)^2 + (R\sin\phi + r\sin\phi)^2$$

$$= R^2\cos^2\phi + R^2\sin^2\phi + r^2\sin^2\phi + 2rR\sin^2\phi$$

$$= R^2(\sin^2\phi + \cos^2\phi) + r\sin^2\phi(r + 2R)$$

$$= R^2 + \sin^2\phi(r^2 + 2rR)$$

$$= R^2\left[1 + \frac{1}{R^2}(r^2 + 2rR)\sin^2\phi\right]$$

$$= R^2\left[1 + \left(\frac{r^2}{R^2} + \frac{2r}{R}\right)\sin^2\phi\right]$$

$$= R^2\left[1 + \left(\left(\frac{r}{R} + 2\right)\frac{r}{R}\right)\sin^2\phi\right]$$

$$\Rightarrow BE = R \times \sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi}$$

$\therefore$ Therefore, the max^m value of the addendum of the wheel can be equal to (BE − Pitch circle radius)

or.
$$a_{w_{max}} = BE - R$$

$$= R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi} - R$$

$$= R\left[\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi} - 1\right]$$

Let t = no. of teeth on pinion

T = " " " " gear wheel.

Now ; Pitch diameter = mT ⇒ Pitch radius = $\frac{mT}{2}$

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$$R = \frac{mT}{2} \quad ; \quad r = \frac{mt}{2} \quad \& \quad G = \frac{T}{t} = \text{Gear ratio} = \frac{R}{r}$$

$$\therefore \quad a_{w_{max}} = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right] \quad \therefore \frac{r}{R} = \frac{1}{G}$$

$$= \frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]$$

Let the adopted value of the addendum in some case be a_w times the module of teeth.

$\therefore$ this adopted value of the addendum must be less than the max^m value of the addendum to avoid interference.

$$\therefore \quad \frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right] \geq a_w \cdot m.$$

$$\Rightarrow \quad T \geq \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

In the limit;

$$T = \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

This gives the minimum number of teeth on the gear for the given values of the gear ratio, the pressure angle & the addendum coefficient, a_w.

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The minimum number of teeth on the pinion is given by :→

$$G = \frac{T}{t} \Rightarrow \boxed{t = \frac{T}{G}}$$

* For $a_w = 1$, i e, when the addendum is equal to one module.

$$T \geq \frac{2}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$$

* For equal number of teeth on the pinion & the gear wheel, $G = 1$ &

$$T_{min} = \frac{2}{\sqrt{1 + 3\sin^2\phi} - 1}$$

* For pressure angle of 20°; i e; $\phi = 20°$

$$T_{min} = \frac{2}{\sqrt{1 + 3\sin^2 20} - 1} = 12.31 \approx 13$$

$$\rightarrow \boxed{T_{min} = 13}$$

5 c.) Data :→

$\phi = 20°$

$m = 10 mm$

$a = 1m = 10 mm.$

$a_w = 1$

$t = 13$

$T = 52.$

$$G = \frac{T}{t} = \frac{52}{13} = 4.$$

$$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

$$= \frac{2\times 1}{\sqrt{1+\frac{1}{4}\left(\frac{1}{4}+2\right)\times 0.117}-1}$$

$$= \frac{2}{0.0324} = 61.76.$$

Interference occurs as 61.76 is the minimum number of teeth to avoid interference and the number of teeth on the given gear is 52.

Again; $52 = \dfrac{2}{\sqrt{1+0.5625\sin^2\phi}-1}$

$\Rightarrow \sqrt{1+0.5625\sin^2\phi} = 1.0385.$

$\Rightarrow 1+0.5625\sin^2\phi = 1.0784.$

$\Rightarrow \sin^2\phi = 0.1394.$

$\Rightarrow \sin\phi = 0.3733.$

$\Rightarrow \boxed{\phi = 21.92^\circ}$

(Ans).

6. $\phi = 20^\circ$

$t = 20$

$G = 2$

~~Rad~~ $m = 5\,mm$

$v_p = 1.2\,m/s$

$a = 1m = 5\,mm$

$G = \frac{T}{t} \Rightarrow T = Gt = 2 \times 20 = 40$

(i) Angle turned by pinion when one pair of teeth is in mesh.

$$r = \frac{mt}{2} = \frac{5 \times 20}{2} = 50\,mm$$

$$R = \frac{mT}{2} \times \frac{5 \times 40}{2} = 100\,mm$$

$$r_A = r + a = 50 + 5 = 55\,mm$$

$$R_A = R + a = 100 + 5 = 105\,mm$$

$$PoA = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(105)^2 - (100)^2(\cos 20^\circ)^2} - 100\sin 20^\circ$$

$$= 46.85 - 34.2$$

$$\Rightarrow \boxed{PoA = 12.65\,mm}$$

$$PoR = \sqrt{r_n^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(55)^2 - (50)^2(\cos 20^\circ)^2} - 50\sin 20^\circ$$

$$= 28.6 - 17.1$$

$$\Rightarrow \boxed{PoR = 11.5\,mm}$$

$$PoC = PoA + PoR$$

$$= 12.65 + 11.5 \Rightarrow \boxed{PoC = 24.15\,mm}$$

$$AoC = \frac{PoC}{\cos\phi} = \frac{24.15}{\cos 20^\circ} = 25.7\,mm$$

$$\Rightarrow \boxed{AoC = 25.7\,mm}$$

$$\text{Angle turned by the pinion} = \frac{AoC \times 360^\circ}{\text{Circumference of Pinion}}$$

$$= \frac{25.7 \times 360^\circ}{2\pi r} = \frac{25.7 \times 360^\circ}{2\pi \times 50} = 29.45^\circ$$

$$\Rightarrow \boxed{\text{Angle turned by pinion} = 29.45^\circ} \text{ (Ans.)}$$

(ii) <u>Maximum velocity of Sliding:</u>

ω_1 = Angular speed of pinion.

ω_2 = Angular speed of Gear.

$$v_p = \omega_1 r = \omega_2 R$$

$$\omega_1 = \frac{v_p}{r} = \frac{1200}{50} = 24 \text{ rad/s.}$$

$$\omega_2 = \frac{v_p}{R} = \frac{1200}{100} = 12 \text{ rad/s.}$$

Maximum velocity of sliding (v_s) $= (\omega_1 + \omega_2) \times P_0A$

$(\because P_0A > P_0R) \Rightarrow v_s = (24 + 12)\, 12.65$

$$\Rightarrow \boxed{v_s = 455.4 \text{ mm/s}} \text{ (Ans)}$$

——— x ——— x ———