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Internal Assessment Test 1 – March 2019

Sub:	Heat Transfer	Sub Code:	15ME63	Branch:	ME
Date:	06/03/19	Duration: 90 min	Max Marks: 50	Sem / Sec:	6^{th}/A & B

	➢ **Attempt any 5 questions** ➢ Use of Heat Transfer Data Hand Book is permitted	MARKS	CO	RBT
1 (a)	State the laws governing the three basic modes of heat transfer with suitable diagram.	[06]	CO1	L1
(b)	Explain the terms : (i) Thermal Conductivity (ii) Thermal Resistance	[04]	CO1	L2
2	Derive the general three dimensional heat conduction equation in Cartesian coordinate and state the assumption made.	[10]	CO1	L3
3	A furnace has a composite wall constructed of a refractory material for the inside layer and an insulating material on the outside. The total wall thickness is limited to 60cm. The mean temperature of the gases within the furnace is 850^{o}C, the external temperature is 30^{o}C and the material interface temperature is 500^{o}C. The thermal conductivities of refractory and insulating materials are 2 W/mK and 0.2 W/mK respectively. The combined coefficient of heat transfer by convection and radiation between gases and refractory surface is 200 W/m^{2}K and between outside surface and atmosphere is 40 W/m^{2}K. Find: i) Thickness of each material. (ii) Rate of heat loss to atmosphere. (iii) Temperatures of external and internal surfaces.	[10]	CO1	L3
4	Consider a plane wall of thickness L whose thermal conductivity varies in a specified temperature range as $k=k_o(1+bT^2)$ where k_0 and b are two specified constants. The wall surface at x=0 is maintained at a constant temperature of T_1, while the surface at x=L is maintained at T_2. Assuming steady one-dimensional heat transfer, obtain a relation for the heat transfer rate through the wall.	[10]	CO1	L3
5	A truncated cone like solid has its circumference insulated and heat flows along the axis. The area of section at x is given by $A = \frac{\pi}{4x^3}$ and the faces are at 0.075 m and 0.225 m. The thermal conductivity of the material varies as $k = 0.5\,(1 + 5 \times 10^{-3}\, T)$ W/mK. The surface at x = 0.075 m is at 300°C and the surface at x = 0.225 is at 50°C. Determine the rate of heat flow.	[10]	CO1	L3
6	A 30 mm outside diameter steam pipe is to be covered with two layers of insulation each having a thickness of 25 mm. The average thermal conductivity of one material is five times the other. Determine percentage decrease in heat transfer if better insulating material is next to the pipe than when it forms the outer layer. Assume the outside and inside surface temperature of the composite insulation are fixed.	[10]	CO1	L3
7	A wire of 8mm diameter at a temperature of 60^{o}C is to be insulated by a material having k=0.174W/mK. Heat transfer coefficient h=8W/m^{2}K and ambient temperature is 25^{o}C. For maximum heat loss find the minimum thickness of insulation. Find % increase in heat dissipation due to insulation.	[10]	CO1	L3

1(a) The law governing the three basic modes of heat transfer are —

1) CONDUCTION
2) CONVECTION
3.) RADIATION.

CONDUCTION:

Fourier's Law : This law states that the time rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area, at the right angles to that gradient through which the heat flows.

$$Q = -kA\frac{dT}{dx}$$

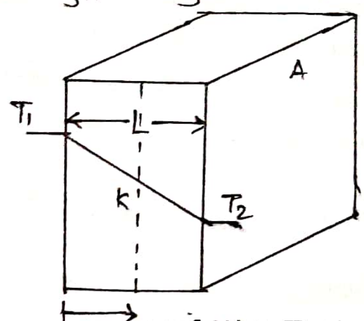


FIG: CONDUCTION

where, k = Thermal conductivity, w/mk
A = Area, m^2
Q = Heat flow, W
$\frac{dT}{dx}$ = Temperature gradient.

CONVECTION:

Newton's Law of cooling states that the rate of heat transfer of a body is directly proportional to the difference in the temperature between the body and its surrounding provided the temperature difference is small and the nature of radiating surface remains the same

$$Q = hA\Delta T$$

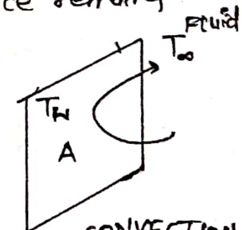


FIG: CONVECTION

where, h = convective heat transfer coefficient, w/m^2k
A = Area, m^2
ΔT = Temperature difference, k

RADIATION:

Stefan-Boltzmann law states that the total radiant energy emitted from a surface is proportional to the fourth power of its absolute temperature.

$$F = \sigma T^4$$

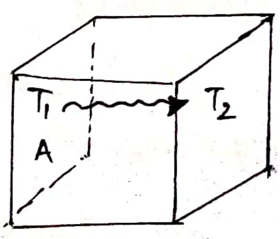


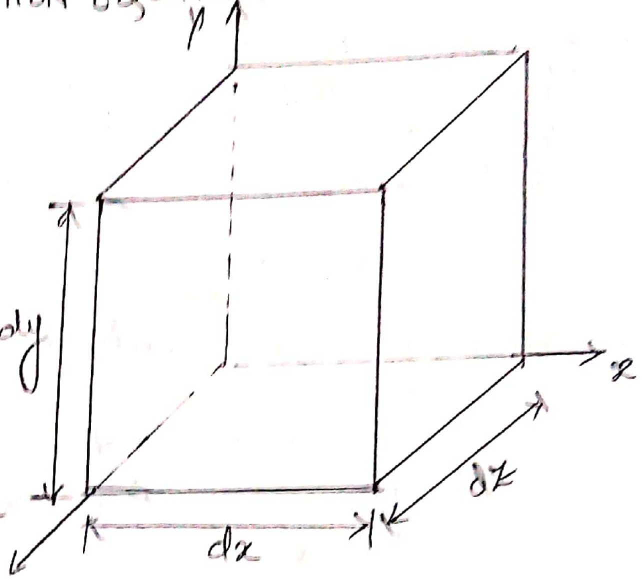
FIG: RADIATION

where, F = energy flux
σ = Stefan Boltzmann constant
$= 5.67 \times 10^{-8}\ w/m^2k^4$
T = Temperature, k.

1(b) i) Thermal conductivity : is the heat energy transferred per unit time and per unit surface area, divided by the temperature difference.

ii) Thermal Resistance : is the heat property and a measurement of a temperature difference by which an object resist heat flow.

2. THE GENERAL 3D HEAT CONDUCTION EQUATION IN CARTESIAN COORDINATE



consider a cuboidal element of dimension $dx \times dy \times dz$ for the element.

Energy Balance equation can be given as —

$$E_{IN} - E_{OUT} + E_{gen} = E_{st}$$

where,

E_{IN} — Energy entering the element

E_{out} — Energy leaving the element

E_{gen} — Energy generated within the element

E_{st} — Energy stored within the element

Now $(E_{IN} - E_{OUT}) = (E_{in} - E_{out})_x + (E_{IN} - E_{out})_y + (E_{IN} - E_{out})_z$

$$(E_{IN} - E_{OUT})_x = q_x - q_{x+dx}$$

$$= q_x - \left(q_x + \frac{\partial}{\partial x} q_x dx\right)$$

$$= -\frac{\partial}{\partial x} q_x dx$$

$$= \frac{-\partial}{\partial x}\left(-k\,(dy \times dz)\frac{\partial T}{\partial x}\right)dx$$

considering, the material is homogeneous and isotropic —

$$(E_{IN} - E_{OUT})_x = k \frac{\partial^2 T}{\partial x^2} dx\,dy\,dz$$

Similarly,

$$(E_{IN} - E_{OUT})_y = k \frac{\partial^2 T}{\partial y^2} dx\,dy\,dz$$

$$\left(E_{IN} - E_{OUT}\right)_z = k \frac{\partial^2 T}{\partial z^2} dx\, dy\, dz$$

$$\therefore E_{IN} - E_{OUT} = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] dx\, dy\, dz$$

$$= k(\nabla^2 T)\, dx\, dy\, dz$$

consider the heat generation per unit volume inside the element to be at rate of $\dot{q}_g$

$\therefore$ Total rate of heat generation within the element will equal to

$$E_{gen} = \dot{q}_g \times dx\, dy\, dz$$

$$E_{st} = mc \frac{\partial T}{\partial t} = \rho c \frac{\partial T}{\partial t} dx\, dy\, dz$$

Now,

$$k(\nabla^2 T)\, dx\, dy\, dz + \dot{q}_g\, dx\, dy\, dz = \rho c \frac{\partial T}{\partial t}\, dx\, dy\, dz$$

$$k \nabla^2 T + \dot{q}_g = \rho c \frac{\partial T}{\partial t} \qquad \div k$$

$$\boxed{\nabla^2 T + \frac{\dot{q}_g}{k} = \frac{\rho c}{k} \cdot \frac{\partial T}{\partial t}}$$

- steady state Heat conduction

$$\nabla^2 T + \frac{\dot{q}_g}{k} = 0 \quad \text{——— Poisson's Eq}^n.$$

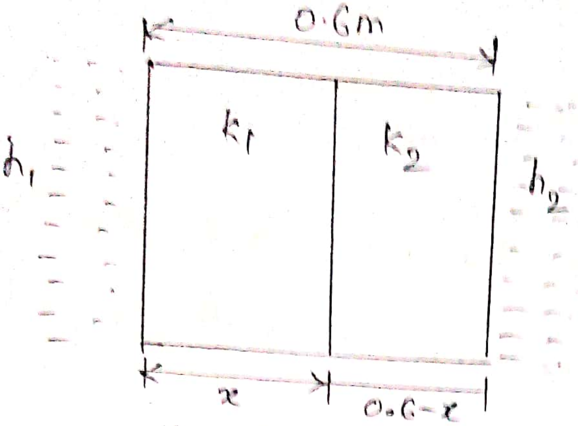
- Heat conduction with no heat generation

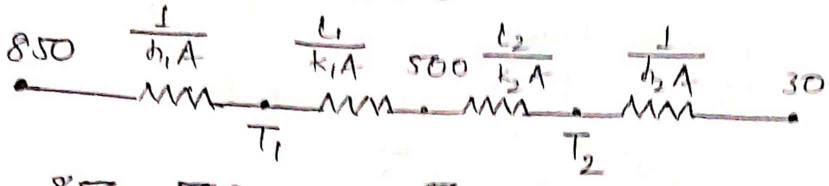
$$\nabla^2 T = \frac{\rho c}{k} \frac{\partial T}{\partial t} \quad \text{——— Fourier's Eq}^n$$

- steady state & Heat conduction with no heat generation

$$\nabla^2 T = 0 \quad \text{——— Laplace eq}^n.$$

3.





1) $$\frac{850-500}{\frac{1}{h_1 \cancel{A}}+\frac{x}{k_1 \cancel{A}}} = \frac{500-30}{\frac{1}{h_2 \cancel{A}}+\frac{0.6-x}{k_2 \cancel{A}}}$$

$$\frac{850-500}{\frac{1}{200}+\frac{x}{2}} = \frac{500-30}{\frac{1}{40}+\frac{0.6-x}{0.2}}$$

$$350\left[\frac{1}{40}+\frac{0.6-x}{0.2}\right] = 470\left[\frac{1}{200}+\frac{x}{2}\right]$$

$L_1 = x = 0.532m = 532\,mm$

$L_2 = 0.6-x = 0.6-0.532 = 0.068m = 68mm$

2) $$\frac{Q}{A} = \frac{850-500}{\frac{1}{h_1}+\frac{L_1}{k_1}} = \frac{850-500}{\frac{1}{200}+\frac{0.532}{2}} = 1291.51\, W/m^2$$

3.) $$\frac{Q}{A} = \frac{850-T_1}{1/h_1} = \frac{T_2-30}{1/h_2}$$

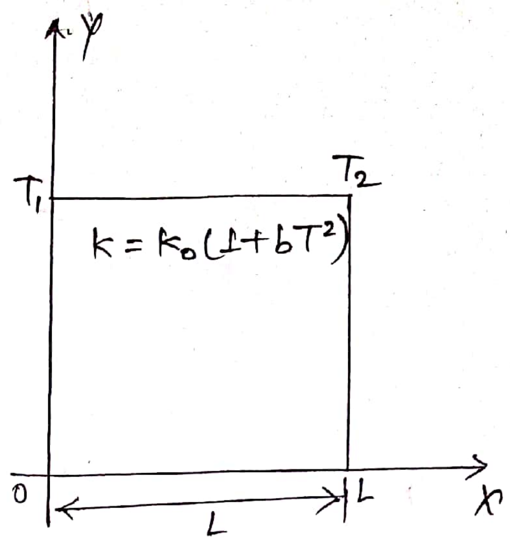
$$1291.51 = \frac{850-T_1}{1/200} \Leftrightarrow T_1 = 843.54\,°C$$

$$1291.51 = \frac{T_2-30}{1/40} \Leftrightarrow T_2 = 62.28\,°C$$

4. Consider a plane wall with surface temperature T_1 & T_2 and thickness 'L' with variable thermal conductivity.

Thermal conductivity of the wall —

$$k = k_o(1+bT^2)$$



$$Q = -kA\frac{dT}{dx}$$

$$Q\,dx = -kA\,dT$$

$$Q\,dx = -k_o(1+bT^2)A\,dT$$

$$Q\int_0^L dx = -k_oA\int_{T_1}^{T_2}(1+bT^2)\,dT$$

$$Q[x]_0^L = -k_oA\left[T+\frac{bT^3}{3}\right]_{T_1}^{T_2}$$

$$Q.L = -k_oA\left[(T_2-T_1)+\frac{b}{3}(T_2^3-T_1^3)\right]$$

$$Q.L = -k_oA\left[(T_2-T_1)+\frac{b}{3}(T_2-T_1)(T_2^2+T_2T_1+T_1^2)\right]$$

$$Q.L = -k_oA\left\{(T_2-T_1)\left[1+\frac{b}{3}(T_1^2+T_1T_2+T_2^2)\right]\right\}$$

$$Q = k_o\left[1+\frac{b}{3}(T_1^2+T_1T_2+T_2^2)\right]A \times \frac{(T_1-T_2)}{L} \quad \text{—— (1)}$$

Let k_m be the mean thermal conductivity of wall material

$$Q = \frac{k_mA(T_1-T_2)}{L}$$

$$k_m = k_o(1+bT_m^2)$$

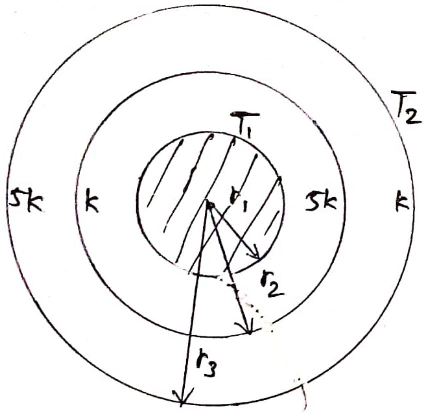
where, T_m = mean temperature.

$$Q = k_o(1+bT_m)A \times \frac{(T_1-T_2)}{L} \quad \text{—— (2)}$$

On comparing (1) & (2)

$$T_m = \sqrt{\frac{T_1^2+T_1T_2+T_2^2}{3}}$$

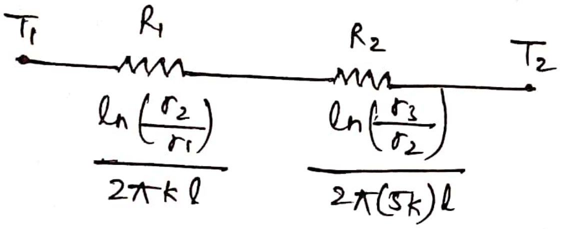
(6) Case-01 Material with higher thermal conductivity is placed to the outside.



$r_1 = \frac{30}{2}\,mm = 15\,mm$

$r_2 = 15 + 25 = 40\,mm$

$r_3 = 40 + 25 = 65\,mm$



$$R_{th(1)} = \frac{\ln\left(\frac{40\times10^{-3}}{15\times10^{-3}}\right)}{2\pi k l} + \frac{\ln\left(\frac{65\times10^{-3}}{40\times10^{-3}}\right)}{2\pi (5k) l}$$

$$= \frac{1}{2\pi k l}\left[\ln\left(\frac{40}{15}\right) + \frac{\ln\left(\frac{65}{40}\right)}{5}\right]$$

$$= \frac{1}{2\pi k l}\left[0.9808 + 0.0971\right]$$

$$= \frac{1.0779}{2\pi k l}$$

Case-02 Material with lower thermal conductivity is placed to the outside

$$R_{th_2} = \frac{\ln\left(\frac{40\times10^{-3}}{15\times10^{-3}}\right)}{2\pi (5k) l} + \frac{\ln\left(\frac{65\times10^{-3}}{40\times10^{-3}}\right)}{2\pi k l}$$

$$= \frac{1}{2\pi k l}\left[\frac{\ln\left(\frac{40}{15}\right)}{5} + \ln\left(\frac{65}{40}\right)\right]$$

$$= \frac{1}{2\pi k l}\left[0.1961 + 0.4855\right] = \frac{1}{2\pi k l} \times (0.6816)$$

So, in first case the R_{th_1} value is more, so heat transfer will be lesser.

$$\% \text{ decrease in } Q = \frac{1.0779 - 0.6816}{0.6816} \times 100 = 58.14\%$$

(7)

$k = 0.174 \ W/mK$

$h = 8 \ W/m^2K$

$$r_1 = \frac{8 \times 10^{-3}}{2} = 4 \times 10^{-3} \ m$$

$$r_c = \frac{k}{h} = \frac{0.174}{8} = 0.0218 \ m$$

$$t_c = r_c - r_1 = 0.0218 - 4 \times 10^{-3} = 0.0178 \ m$$

$$t_c = \underline{\underline{17.8 \ mm}}$$

$$Q_w = \frac{T_1 - T_2}{\frac{1}{2\pi k l} \ln\left(\frac{r_c}{r_1}\right) + \frac{1}{2\pi r_c l \times h}}$$

$$\frac{Q_w}{l} = \frac{60 - 25}{\frac{1}{2\pi 0.174} \ln\left(\frac{0.0218}{4 \times 10^{-3}}\right) + \frac{1}{2\pi(0.0218) \times 8}}$$

$$= \underline{\underline{14.2 \ W/m}}$$

$$\frac{Q_{w/o}}{l} = \frac{T_1 - T_2}{\frac{1}{h(2\pi r_1 l)}} = \frac{60 - 25}{\frac{1}{8(2\pi(4 \times 10^{-3}))}} = \underline{\underline{7.037 \ W/m}}$$

$$\% \text{ increase in heat dissipation} = \frac{14.2 - 7.037}{7.037} \times 100$$

$$= 101.79\%$$

—— x ——

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