Scheme Of Evaluation
Internal Assessment Test 2 – Apr.2019

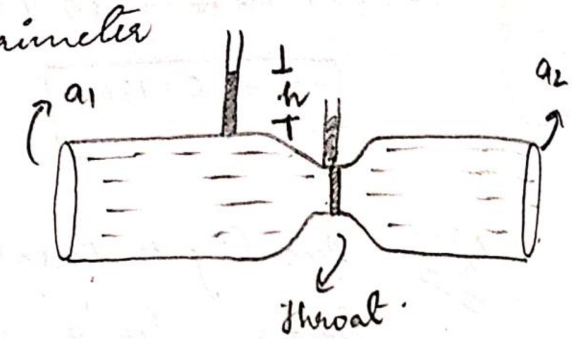
Sub:	Fluid Mechanics							**Code:**	17ME44
Date:	20/04/2019	Duration:	90mins	Max Marks:	50	**Sem:**	IV	**Branch:**	ME

Part A: Answer any 2 questions

Question #		Description	Marks Distribution		Max Marks
1	a)	**Derive expression for continuity equation for three dimensional flow in Cartesian co-ordinates.** • Diagram • Steps • General form of continuity equation	• 2M • 5M • 3M	10 M	10 M
2	a)	**Derive an expression for discharge through a venturimeter.** • Diagram • Derivation	• 3M • 7M	10 M	10 M
3	a)	**Derive Bernoulli's equation from Euler's equation. State the assumptions.** • Diagram • Derivation and expression • Assumptions	• 1M • 7M • 2M	10 M	10 M
Part A: Answer any 3 questions					
4	a)	**A vertical venturimeter has an area ratio 5. It has a throat diameter of 10cm. When oil of specific gravity 0.8 flows through it, the mercury in the differential gauge indicates a difference in height of 12cm. Find the discharge through venturimeter. Take co-efficient as 0.98.** • Data • Steps • Answer	• 2M • 6M • 2M	10 M	10 M
5	a)	**Velocity potential function for a two dimensional fluid flow is given by Φ=x(2y-1). Check the existence of flow. Determine the velocity of flow at a point (2,3) and the stream**		10 M	10 M

		function. • Data • Steps • Answer	• 2M • 6M • 2M		
6	a)	**Find the functional relation of frictional torque T of a disc which depends on diameter D rotating at speed N in a fluid of viscosity μ and density ρ.** • Data • Steps • Answer	• 2M • 6M • 2M	10 M	10 M
7	a)	**The pressure drop dP in a pipe of diameter D and length L depends on the density ρ, viscosity μ of the flowing fluid, mean velocity of flow V and average height of proturberence t, show that the pressure drop can be expressed in the form:** $$dP = \rho V^2 \Phi\left(\frac{L}{D}, \frac{\mu}{\rho V D}, \frac{t}{D}\right)$$ • Data • Steps • Answer	• 2M • 6M • 2M	10 M	10 M

Q2. Discharge through Venturimeter



Let the venturimeter be placed in a horizontal pipe. Let a_1 be the area of opening and a_2 be the area of throat.

We know from Bernoulli's Equation

$$\frac{P_1}{\delta g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\delta g} + \frac{v_2^2}{2g} + Z_2$$

As the venturimeter is at horizontal position

$Z_1 = Z_2$ hence $Z_1 - Z_2 = 0$.

$$\frac{v_1^2}{2g} - \frac{v_2^2}{2g} \Rightarrow \frac{1}{\delta g}\left[P_1 - P_2\right].$$

we know $\dfrac{v_1^2}{2g} - \dfrac{v_2^2}{2g} = h.$

from Continuity equation we have.

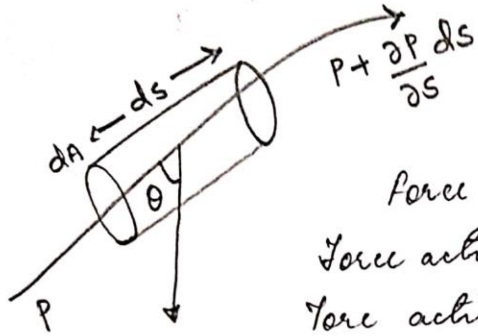
$$A_1 V_1 = a_2 v_2$$

$$v_1 = \frac{a_2 v_2}{a_1}$$

$$v_1^2 - v_2^2 = 2gh.$$

We know discharge $Q = A_1 v_1$.

$$Q = \frac{a_1 \cdot a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}} \Rightarrow \boxed{Q = \frac{a_1 \cdot a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}}$$

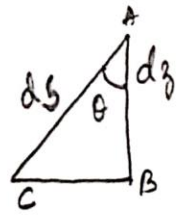


Let the element of water flowing through the cylinder having length ds and area of circular section dA.

Force acting on an object ⇒ P· area.

Force acting upwards = P·dA.

Force acting downwards ⇒ $\left[P + \frac{\partial P}{\partial s} ds\right] dA$.



Weight element ⇒ $\rho g \cos\theta \, dA \cdot ds$.

Total Force acting =

$$P \cdot dA - \left[P + \frac{\partial P}{\partial s} ds\right] dA - \rho g \cos\theta \, dA \cdot ds = \rho a_s \, dA \cdot ds.$$

$$\Rightarrow \cancel{P \cdot dA} - \cancel{P \cdot dA} - \frac{\partial P}{\partial s} ds \cdot dA - \rho g \cos\theta \, dA \cdot ds = \rho a_s \, dA \cdot ds$$

$$\Rightarrow -\cancel{dA}\,\cancel{ds}\left[\frac{\partial P}{\partial s} + \rho g \cos\theta\right] = \rho a_s \cancel{dA}\,\cancel{ds}$$

from △ABC

$$\cos\theta = \frac{\partial z}{\partial s}$$

$$\Rightarrow -\left[\frac{\partial P}{\partial s} + \rho g \cos\theta\right] = \rho a_s.$$

$$a_s = \frac{\partial v}{\partial s} \cdot \frac{ds}{dt} + \frac{dv}{dt}$$

$\frac{dv}{dt} = 0$ {Steady flow}

$$a_s = \frac{v \partial v}{\partial s} \quad \left\{\frac{ds}{dt} = v\right\}$$

$$\Rightarrow -\left[\frac{\partial P}{\partial s} + \rho g \cos\theta\right] = \rho v \frac{\partial v}{\partial s}$$

$$\Rightarrow \rho v \frac{\partial v}{\partial s} + \frac{\partial P}{\partial s} + \rho g \cos\theta = 0 \text{ \{constant\}}$$

$$\Rightarrow \rho v \frac{\partial v}{\partial s} + \frac{\partial P}{\partial s} + \rho g \frac{dz}{ds} = 0.$$

As S is the only variable we have.

$$\Rightarrow \rho v \frac{\partial v}{ds} + \frac{dP}{ds} + \rho g \frac{dz}{ds} = 0$$

$$\Rightarrow \rho v\, dv + dP + \rho g\, dz = 0.$$

Dividing By ρg we have.

$$\Rightarrow \frac{\rho v\, dv}{\rho g} + \frac{dP}{\rho g} + dz = 0.$$

$$\Rightarrow \frac{v dv}{g} + \frac{dP}{\rho g} + dz = 0. \quad \text{——— (1)}$$

Integrating above equation

$$\Rightarrow \int \frac{dP}{\rho g} + \int \frac{v dv}{g} + \int dz = 0.$$

$$\Rightarrow \boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + z_1 = 0.}$$

The above is known as Bernoulli's Equation.

Q.3 $\frac{a_1}{a_2} = 5 \quad d_2 = 10cm = 0.1m \quad a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.1)^2$

$\Rightarrow 7.8539 \times 10^{-3} m^2$

$a_1 = 5a_2 = 5 \times 7.5839 \times 10^{-3} = 0.03926 m^3, \quad S_{oil} = 0.8$

$x = 12 cm = 0.12 m \qquad Cd = 0.98$

Pressure head $= h = x\left[\frac{S_m}{S_{oil}} - 1\right] = 0.12\left[\frac{13.6}{0.8} - 1\right] = 1.92 m$

Discharge $= Q = \frac{Cd \cdot a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$

$\Rightarrow Q = 0.04821 m^3/sec$

Q.5. Velocity components

$u = 6y \qquad \partial u/\partial x = 0$

$v = -6x \qquad \partial v/\partial y = 0$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ Hence flow is possible.

As per stream f(n)

$\frac{\partial \psi}{\partial x} = v = -6x \qquad \frac{\partial \psi}{\partial y} = -u = -6y$

Integrating both sides w.r.t x

$\psi = \frac{-6x^2}{2} + C = -3x^2 + C\,(f(y))$

Differentiate the previous eqn

$\frac{\partial \psi}{\partial y} = 0 + f'(y) = f'(y) \qquad f'(y) = -6y$

$f(y) = -3y^2$

Subs in eqn

$\psi = -3x^2 - 3y^2 + \text{constant}$

$\psi = -3(x^2+y^2)$

Q10. $\phi = (2y-1)x$

The velocity components in dir^n of x and y are

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x}\left[x(2y-1)\right] = 1-2y$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y}\left[x(2y-1)\right] = -2x$$

$P(2,3) \Rightarrow$ at $x=2,\ y=3$

$u = 1-2(3) = -5 \quad v = -2(2) = -4$

Resultant velocity $= \sqrt{4^2+5^2} \Rightarrow 6.403$ units/sec.

Value of Stream Function at P

we know that $\partial\psi/\partial y = -u = -(1-2y) = 2y-1$

$\partial\psi/\partial x = -2x$

Integrating w.r.t y.

$$\int d\psi = \int (2y-1)\,dy = \psi = \frac{2y^2}{2} - y$$

+ constant of integration

$\psi = y^2 - y + k$

Differentiating w.r.t x

$$\frac{\partial\psi}{\partial x} = \frac{\partial k}{\partial x} \qquad \text{But } \frac{\partial\psi}{\partial x} = -2x.$$

$$K = \int -2x\,dx = -\frac{2x^2}{2} = -x^2$$

$\psi = y^2 - y - x^2 \quad (P - 2,3) \quad \psi \Rightarrow (3)^2 - (3) - (2)^2$

$\psi = 2$ units/sec.

Q. The functional relationship between dependent and independent variables is given by,

$\Delta p = f(\rho, v, L, D, \mu, t) = 0$ or constant

Dimensions of different variables are,

$\Delta p = [ML^{-1}T^{-2}]$ $\rho = [ML^{-3}]$ $V = [LT^{-1}]$ $L = [L]$ $D = [L]$ $\mu = [ML^{-1}T^{-1}]$
$t = [L]$

- Total number of variables $n = 7$

number of primary dimensions $m = 3$

number of Π terms $= n - m = 7 - 3 = 4$ $(\Pi_1, \Pi_2, \Pi_3, \Pi_4)$

$\therefore f(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0$ or constant

As $m = 3$, the repeating variables are 3

As per Bukingham Π theoram the Π terms can be written as, $\Pi_1 = D^{x_1} V^{y_1} \rho^{z_1} \Delta p$ $\quad \Pi_4 = D^{x_4} V^{y_4} \rho^{z_4} t$

$\Pi_2 = D^{x_2} V^{y_2} \rho^{z_2} \Delta L$.

$\Pi_3 = D^{x_3} V^{y_3} \rho^{z_3} \mu$

consider equation 'i' and substitute the dimensions

$[M^0 L^0 T^0] = [L]^{x} [LT^{-1}]^{y_1} [ML^{-3}]^{z_1} [ML^{-1}T^{-2}]$

Equating powers $M = z_1 + 1$ $\quad z_1 = -1$

For $L \Rightarrow 0 = x_1 + y_1 - 3z_1 - 1$ $\quad x_1 + y_1 = -2$

For $T = 0 = -y_1 - 2$ $\quad y_1 = -2$ and $x_1 = 0$.

Substituting back

$$\Pi_1 = \frac{\Delta p}{\rho V^2}, \quad \Pi_2 = \frac{L}{D}$$

consider equation $M = z_3 + 1$ $\quad z_3 = -1$

$L \Rightarrow 0 = x_3 + y_3 - 3z_3 + 1$ $\quad x_3 + y_3 = -2$.

$T \Rightarrow 0 = -y_3 - 1$ $\quad y_3 = -1$ $\quad x_3 = -1$.

Substitute these values $\Pi_3 = D^{-1} V^{-1} \rho^{-1} \mu$. $\Pi_3 = \dfrac{\mu}{\rho V D}$.

Simillarly $\Pi_4 \Rightarrow \dfrac{t}{D}$ putting values $\Rightarrow \Delta p = \rho V^2 \phi \left[\dfrac{L}{D}, \dfrac{\mu}{\rho V D}, \dfrac{t}{D} \right]$