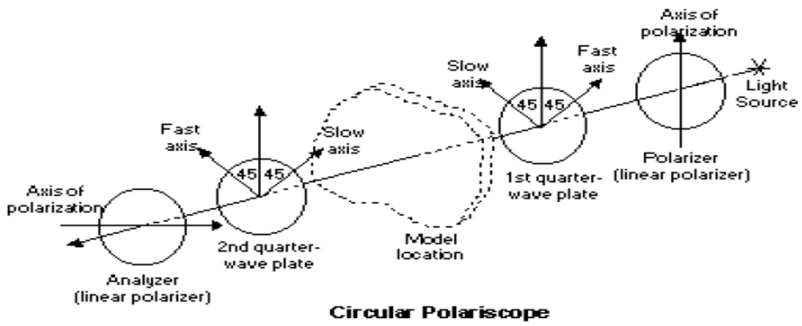
USN											
Sub:	Experimental Stress Analysis					Sub Code:	15ME832	Branch:			
Date:	29.04.19	Duration:	90 min's	Max Marks:	50	Sem / Sec:					

Answer All Questions (Each question carries 10 marks)

ANSWER KEY

1 Derive the expression for effect of stressed model in circular polariscope with a neat sketch

Effect of Stressed Model in Circular Polariscopes under Dark Field Arrangement:

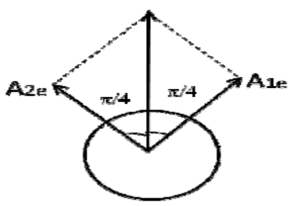


Circular Polariscope

The light vector emerging out of polarizer

$$A = a\cos\omega t$$

Light vector entering first Q.W.P



Ist Q W P

$$A_{1e} = A\cos 45$$

$$A_{2e} = A\cos 45$$

$$A_{1e} = a\cos\omega t\frac{1}{\sqrt{2}}$$

$$A_{2e} = a\cos\omega t\frac{1}{\sqrt{2}}$$

The two vectors travels with a different velocity along the Q.W.P so they will have an angular phase shift of $\pi/2$.

The light vector leavening first Q.W.P

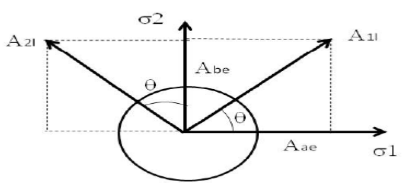
$$A_{1l} = \cos(\omega t + \pi/2)\frac{a}{\sqrt{2}}$$

$$A_{1l} = -\sin(\omega t)\frac{a}{\sqrt{2}}$$

$$A_{2l} = \cos(\omega t)\frac{a}{\sqrt{2}}$$

Here Ist light vector gains the phase shift of $\pi/2$ because it travels faster than the second light vector.

The light vector entering the model



$$A_{ae} = A_{1l}\cos\theta - A_{2l}\sin\theta$$

$$A_{ae} = -\sin(\omega t) * \frac{a}{\sqrt{2}} * \cos\theta - \cos(\omega t) * \frac{a}{\sqrt{2}} * \sin\theta$$

$$A_{ae} = -\frac{a}{\sqrt{2}}\{\sin(\omega t)\cos\theta + \cos(\omega t)\sin\theta\}$$

$$A_{ae} = -\frac{a}{\sqrt{2}}\{\sin(\omega t + \theta)\}$$

$$A_{be} = A_{1l}\sin\theta + A_{2l}\cos\theta$$

$$A_{be} = \sin(\omega t) * \frac{a}{\sqrt{2}} * \sin\theta - \cos(\omega t) * \frac{a}{\sqrt{2}} * \cos\theta$$

$$A_{be} = \frac{a}{\sqrt{2}}\{\sin(\omega t)\sin\theta - \cos(\omega t)\cos\theta\}$$

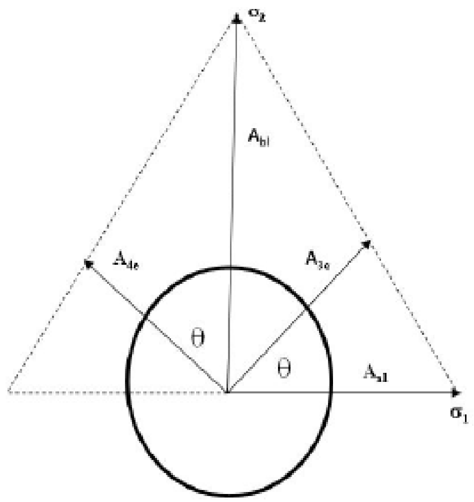
$$A_{be} = \frac{a}{\sqrt{2}}\{\cos(\omega t + \theta)\}$$

The light vector leavening the model will have an angular phase shift Δ

$$A_{al} = -\frac{a}{\sqrt{2}}\{\sin(\omega t + \theta + \Delta)\}$$

$$A_{bl} = +\frac{a}{\sqrt{2}}\{\cos(\omega t + \theta)\}$$

The light vector entering second Q.W.P



$$A_{3e} = A_{al}\cos\theta + A_{bl}\sin\theta$$

$$A_{3e} = -\frac{a}{\sqrt{2}}\{\sin(\omega t + \theta + \Delta)\}\cos\theta + \frac{a}{\sqrt{2}}\{\cos(\omega t + \theta)\}\sin\theta$$

$$A_{4e} = A_{bl}\cos\theta - A_{al}\sin\theta$$

$$A_{4e} = \frac{a}{\sqrt{2}}\{\cos(\omega t + \theta)\}\cos\theta + \frac{a}{\sqrt{2}}\{\sin(\omega t + \theta + \Delta)\}\sin\theta$$

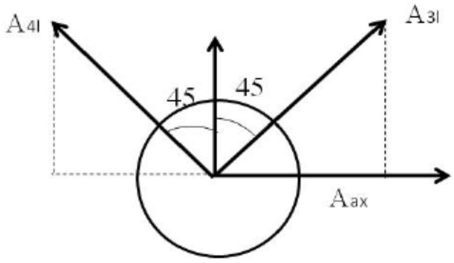
The light vector leaves the second Q.W.P with a phase difference of π/2

$$A_{3l} = -\frac{a}{\sqrt{2}}\{\sin(\omega t + \theta + \Delta)\}\cos\theta + \frac{a}{\sqrt{2}}\{\cos(\omega t + \theta)\}\sin\theta$$

$$A_{4l} = \frac{a}{\sqrt{2}}\{\cos(\omega t + \theta + \ \pi/2)\}\cos\theta + \frac{a}{\sqrt{2}}\{\sin(\omega t + \theta + \Delta + \ \pi/2)\}\sin\theta$$

$$A_{4l} = -\frac{a}{\sqrt{2}}\{\sin(\omega t + \theta)\}\cos\theta + \frac{a}{\sqrt{2}}\{\cos(\omega t + \theta + \Delta)\}\sin\theta$$

The light vector entering the analyzer



$$A_{ax} = A_{3l}\cos 45 - A_{4l}\cos 45$$

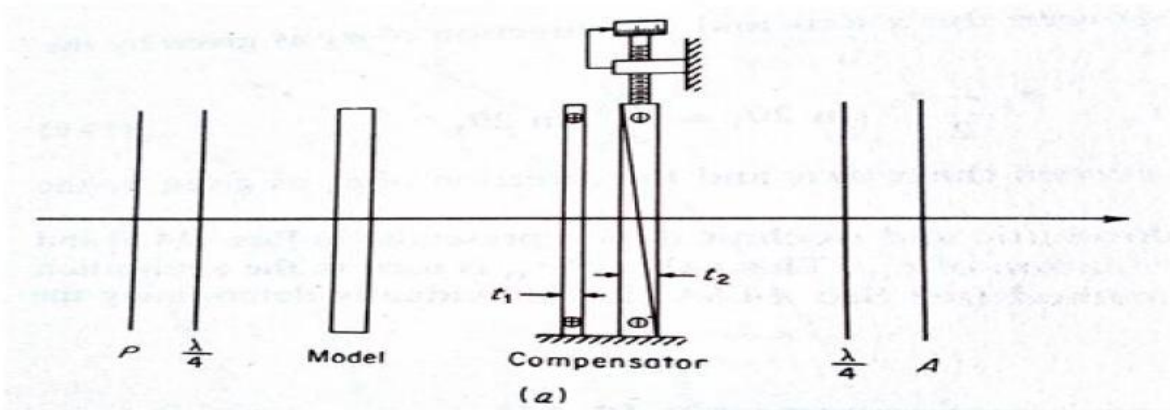
$$A_{ax} = \frac{1}{\sqrt{2}}\Big[-\frac{a}{\sqrt{2}}\{\sin(\omega t + \theta + \Delta)\}\cos\theta + \frac{a}{\sqrt{2}}\{\cos(\omega t + \theta)\}\sin\theta + \frac{a}{\sqrt{2}}\{\sin(\omega t + \theta)\}\cos\theta - \frac{a}{\sqrt{2}}\{\cos(\omega t + \theta + \Delta)\}\sin\theta\Big]$$

$$A_{ax} = \frac{a}{2}[\{\cos(\omega t + \theta)\}\sin\theta + \{\sin(\omega t + \theta)\}\cos\theta - \{\cos(\omega t + \theta + \Delta)\}\sin\theta + \{\sin(\omega t + \theta + \Delta)\}\cos\theta]$$

$$A_{ax} = \frac{a}{2}[\{\sin(\omega t + 2\theta)\} - \{\sin(\omega t + 2\theta + \Delta)\}]$$

2 Explain Babinet Soleil and Friedel method of compensation with suitable sketches.

(1) Babinet-Soleil compensator



Method of compensation

Consider any one point in the model, the relative retardation is between 3λ to 3.5 λ as observed by the bright and dark field setups. Let us assume that the value is 3.36 λ. this is equal to 3.36 fringe order. The decimal part i.e., 0.36 of this value is called the fractional fringe order at that point. The compensation means raising the existing value 3.36 to 4 or reducing the value to 3.0.

3a) **Explain stress optics law with sketch.**

<u>Stress optic law:</u>

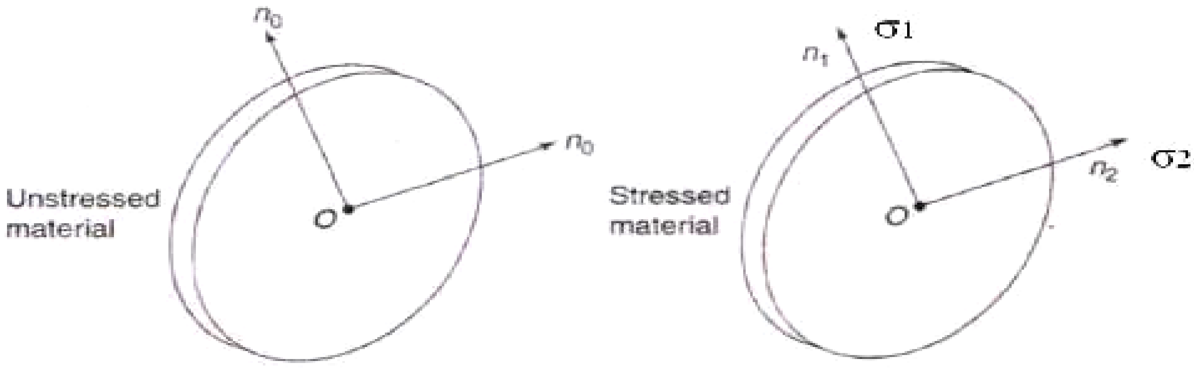


fig 1 shows a unstressed thin disc of a photo-elastic model. The refractive index of this material in any direction is n_0. This thin disc is subjected to loads such that the principal stresses developed at a point o are σ_1 and σ_2 as shown in fig 2. The refractive index of material changes in the direction of σ_1 to the value n_1 and n_2 in the directionσ_2. These changes in refractive idiocies are linearly proportional to the stresses.

$n_1 - n_0 = c_1\sigma_1 + c_2\sigma_2$.......(1)

$n_2 - n_0 = c_1\sigma_2 + c_2\sigma_1$.......(2)

Where σ_1 and σ_2 are the principal stresses at a point in a material under plane stress condition. n_0 is refractive index of unstressed model. n_1 n_2 are the refractive index of stressed model associated with the directions of principal stress σ_1 and σ_2. c_1, c_2 are stress optic co-efficient. These changes is refractive index can be used as the basis for a stress measurement technique I.e photo-elasticity.

eq(2)-eq(1)

$$n_2 - n_1 = (c_1 - c_2)(\sigma_2 - \sigma_1)$$

$n_2 - n_1 = c(\sigma_1 - \sigma_2)$.......(3)

4 Explain different methods of Brittle coating

Types of brittle coatings

Resin based coating/ stress coat: this consists of about one third zinc resinate as a base dissolved in about two-third carbon disulphide with a small amount of plasticizer. Dibutyl phthalate is used as a polarizer to vary the degree of brittleness of the coating, which increases with its increase. The strain sensitivity of this coating, varies from 0.0003 to 0.0030. it can be applied to the test specimen by a spraying method. This coating can be used upon 60 deg C and absorbs water and oil. The thickness of this coating can be made varying from 0.1 to .15mm and can be used for macro and micro applications. Stress coat has been employed widely and can be applied to all materials . synthetic resin dissolved in trichloroethylene or benzene and phenolic resin mixed with titanic white and dissolved in a mixture of benzene, toluene and xylene have also been used as brittle lacquers.

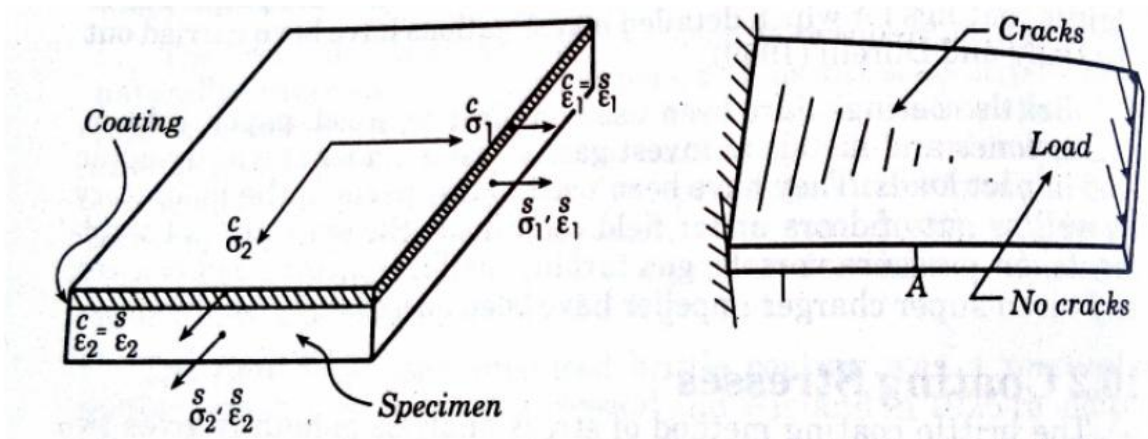
Ceramic based coating: It consists of finely ground ceramic particles suspended in a solvent. It can be sprayed by conventional means onto the specimen. Upon drying at room temperature the coating presents a chalklike appearance and is not suitable for use. In order to make the coating effective, it must be fired at about 540 deg C until the ceramic particles melt and coalesce. When fired, the coating is glasslike in appearance and brown in color. These coatings are relatively insensitive to minor changes in temperature. They can be used upon 370 deg C and are not influenced by the presence of oil and water. Their disadvantages include the high temperature of 537 deg C required to fire the coating which produces detrimental effect on components fabricated from aluminum, magnesium, plastics and highly heat treated steel. Their visual inspection for cracks is not possible and Statiflux method must be resorted to. Strain sensitivity of these coatings range from 0.0002 to 0.002.

5. Explain the following with neat sketch:

a) Coating Stresses and Crack patterns

Coating stresses

Coating is sprayed over the surface of the specimen until a thickness of 0.1 to 0.25 mm is built up. Then, coating is dried either at room temperature or at an elevated temperature in a hot air oven. After the coating is completely dried or cured, loads are applied on the sample. Since the coating is very thin, it can be safely assumed that surface strain of the specimen are faithfully transmitted from specimen to coating without any magnification or attenuation. From the stresses in specimen, the stresses in the coating can be obtained.



Let us take

σ_1^s, σ_2^s: Principal stress in the specimen

$\varepsilon_1^s, \varepsilon_2^s$: Principal strains in the specimen

σ_1^c, σ_2^c: Principal stress in the coating

$\varepsilon_1^c, \varepsilon_2^c$: Principal strains in the specimen

ϑ_c, ϑ_s: Poisons ratio of coating and specimen.

E_c, E_s: Young's modulus for coating and specimen respectively.

By Hooke's law

$$\varepsilon_1^s = \frac{\sigma_1^s - \vartheta_s \sigma_2^s}{E_s}$$

$$\varepsilon_2^s = \frac{\sigma_2^s - \vartheta_s \sigma_1^s}{E_s}$$

$$\varepsilon_1^c = \frac{\sigma_1^c - \vartheta_c \sigma_2^c}{E_c}$$

$$\varepsilon_2^c = \frac{\sigma_2^c - \vartheta_c \sigma_1^c}{E_c}$$

since there is perfect adhesion between the coating and the surface of the specimen, hence

$$\varepsilon_1^c = \varepsilon_1^s$$

$$\varepsilon_2^c = \varepsilon_2^s$$

Thus, we get

$$\frac{\sigma_1^c - \vartheta_c \sigma_2^c}{E_c} = \frac{\sigma_1^s - \vartheta_s \sigma_2^s}{E_s} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{\sigma_2^c - \vartheta_c \sigma_1^c}{E_c} = \frac{\sigma_2^s - \vartheta_s \sigma_1^s}{E_s} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Thus on solving eq 1 and 2, we get

$$\sigma_1^c = \frac{E_c}{E_s(1 - \vartheta_c{}^2)}[(1 - \vartheta_c \vartheta_s)\sigma_1^s - (\vartheta_c - \vartheta_s)\sigma_2^s]$$

$$\sigma_2^c = \frac{E_c}{E_s(1 - \vartheta_c{}^2)}[(1 - \vartheta_c \vartheta_s)\sigma_2^s - (\vartheta_c - \vartheta_s)\sigma_1^s]$$

$$\sigma_1^c - \sigma_2^c = \frac{E_c}{E_s}\left(\frac{1+\vartheta_s}{1+\vartheta_c}\right)[\sigma_1^s - \sigma_2^s] \ldots\ldots\ldots\ldots\ldots\ldots .(3)$$

From eq 3 we can say that $\sigma_1^c - \sigma_2^c$ always has the same sign as $\sigma_1^s - \sigma_2^s$.

To calibrate the coating a cantilever calibration strip as shown in fig 2 . Calibration is used to determine the strain sensitivity. The strain at 'A' where the cracks start appearing is called the strain sensitivity of the coating or the threshold strain. The stresses at 'a' in the coating produced by the external load are found by setting $\sigma_2^s = 0$

Thus,

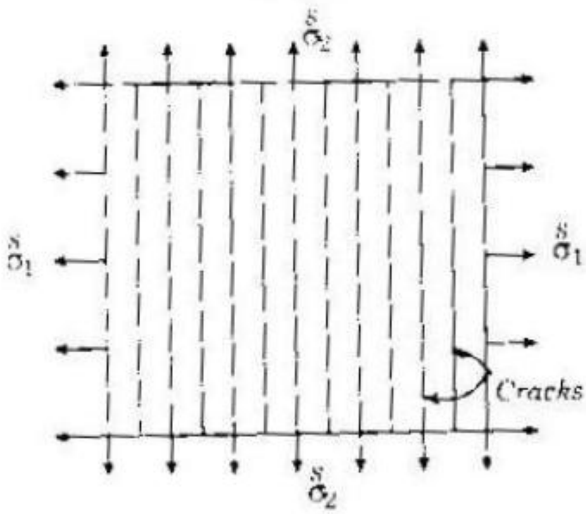
$$\sigma_1^c = \frac{E_c}{E_s(1-\vartheta_c{}^2)}[(1-\vartheta_c\vartheta_s)\sigma_1^s]$$

$$\sigma_2^c = \frac{E_c}{E_s(1-\vartheta_c{}^2)}[(\vartheta_c-\vartheta_s)\sigma_1^s]$$

Crack Patterns

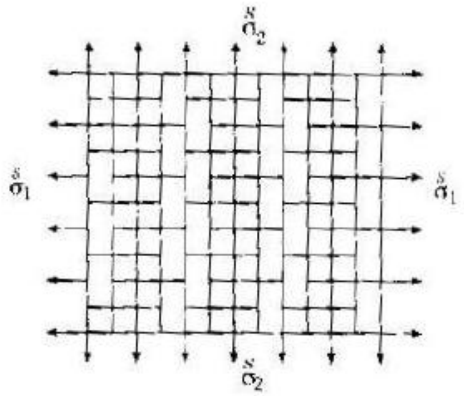
The manner in which a brittle coating fails by cracking depends entirely upon the state of stresses in the specimen to which it adheres. The failure behavior of the coating is determined by the magnitudes of σ_1 andσ_2, the principal stress in the coating. Consider the following special cases when the specimen is subjected to direct loading.

Case 1: $\boldsymbol{\sigma_1} > 0, \boldsymbol{\sigma_2} < 0, \boldsymbol{\sigma_3} = \mathbf{0}$



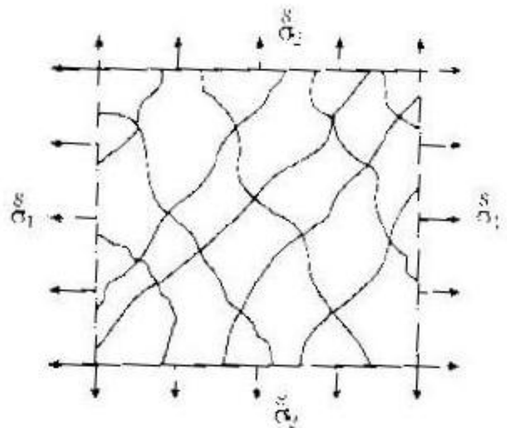
In this case, only one set of cracks forms, and they are perpendicular to σ_1. These cracks indicate the principal stress direction of σ_2, and, as a consequence, the cracks represent stress trajectories, or isostatics.

Case 2: $\sigma_1 > \sigma_2 > 0, \sigma_3 = 0$



For this two families of cracks can form. The first set of cracks due to σ_1 forms perpendicular to σ_1 and parallel to σ_2. When the stress level of σ_2 becomes sufficiently high, a second family of crack will form perpendicular to σ_2 and parallel to σ_1.

Case 3: $\sigma_1 = \sigma_2 > 0, \sigma_3 = 0$

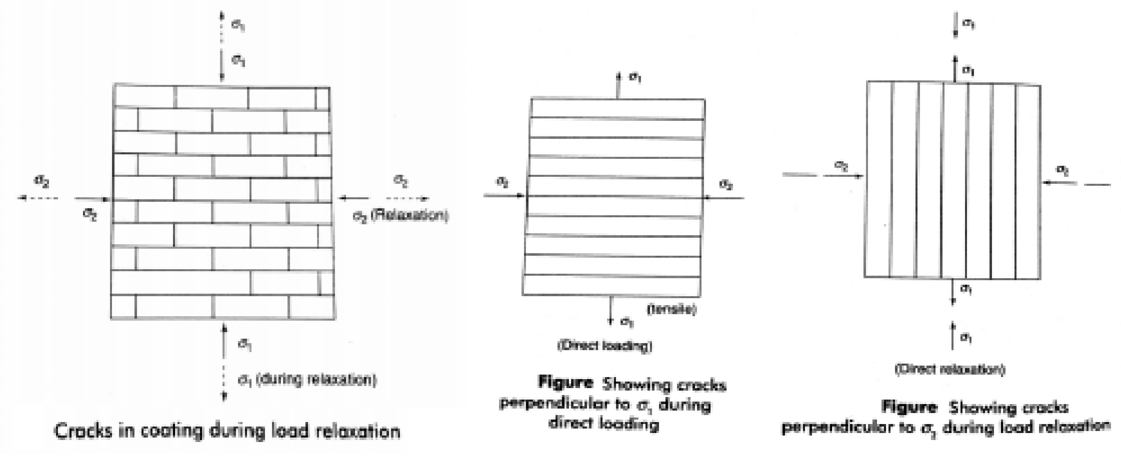


When $\sigma_1 = \sigma_2$ at any point on the body, the stress system is said to be isotropic and every direction is a principal direction. If the values of σ_1 and σ_2 are sufficiently high, the coating will fail; however, the crack pattern produced will be random in character. Crack pattern of this form are often referred to as craze patterns since the crack have no preferential direction.

b) Load relaxation and refrigeration technique.

Load relaxation for compressive stress

- A brittle coating does not respond to compressive loads. In order to overcome this difficulty, a relaxation technique is applied.
- A load is applied to coated specimen before it had opportunity to dry.
- This load is maintained on the coated specimen until drying is complete.
- Under this condition the coating is stress free, while the specimen is highly stressed in compression.
- When load is released, the specimen will stretch, since it was previously compressed and tensile stress will develop in the coating caused the coating cracks.



Cracks in coating during load relaxation

Figure Showing cracks perpendicular to σ_1 during direct loading

Figure Showing cracks perpendicular to σ_1 during load relaxation

Refrigeration technique

- It is possible to obtain coating cracks in low stressed regions by employing refrigeration technique with brittle coating.
- First, the specimen is loaded until the stress in the critical region is just below the yield stress; then, the coating is subjected to a rapid temperature drop while under load.
- The rapid temperature drop produces a state of hydrostatic tension in the coating which is superimposed upon the existing stress in the coating due to load.
- The combined load and thermal stress are used sufficient to produce coating failure and crack pattern.
- The direction of resulting cracks is coincident with one of the principal stress due to the load since the isotropic thermal stresses have no preferential direction.

- This technique is simple to apply and the results are accurate provided the coating stress due to the loads is sufficiently large.

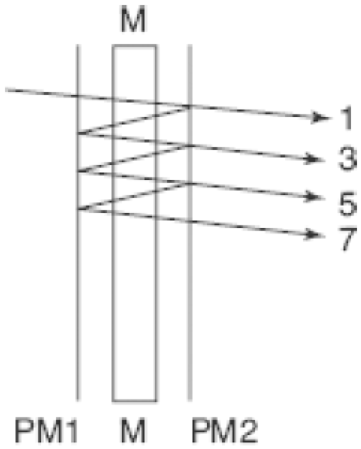
In order to reduce the temperature of coating two methods are employed.

- Ice water is sponged over the area of the coating which has not previously responded. This method is not very much successful.
- A stream of compressed air is passed through a box containing dry ice and it is directed into the surface of the coated model. The stream of much cooled air can be accurately directed and the resulting crack patterns can be closely controlled.

c) Fringe Sharpening and Fringe Multiplication Technique.

Fringe Sharpening by Partial Mirror

To decrease the band width of Isochromatics fringes ordinary circular polariscopes is modified by inserting partial mirrors on both side of the model and both are parallel to the model.



We know that intensity of light

$$I = a^2 \sin^2\frac{\Delta}{2} = k\sin^2\frac{\Delta}{2}$$

The intensity of light is modified by the transmittance (T) and refractance (R)

For ray 1, the intensity of light is lost by reflection from the partial mirror and resultant I will become

$$I_1 = kT^2\,\mathrm{R}^0\sin^2\frac{\Delta}{2} = k\,T^2\sin^2\frac{\Delta}{2}$$

Similarly Ray 3 passes through the model 3 times and combined to yield intensity I_3

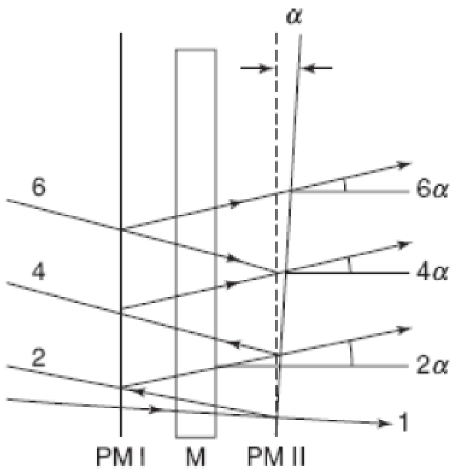
$$I_3 = kT^2\,\mathrm{R}^2\sin^2\left(\frac{3\Delta}{2}\right)$$

For mth ray

$$I_m = kT^2\,\mathrm{R}^{(m-1)}\sin^2\left(\frac{m\Delta}{2}\right)$$

Fringe Multiplication by Partial Mirror

Fringe multiplication is concerned important since standard method of compensation used to evaluate the fraction fringe order are time consuming and in some case also inaccurate. Fringe multiplication is a compensation technique where the fractional orders of the fringe can be determined simultaneously at all points on the model.



Partial mirror can be used to multiply the number of Isochromatics fringes if one partial mirror is placed parallel to the photo elastic model and the other partial mirror is placed slightly inclined

with the plane of photo elastic model as shown in fig. partial mirror PM I is parallel to model M but partial mirror PM II is slightly inclined with the model at an angle α Ray 1 emerges from partial mirror II, traversing two times the partial mirrors and one time through the model, intensity of light for ray 1 is

$$I_1 = k\,T^2 \sin^2 \frac{\Delta}{2}$$

$$I_3 = kT^2\, \mathrm{R}^2 \sin^2 \left(\frac{3\Delta}{2}\right)$$

$$I_5 = kT^2\, \mathrm{R}^4 \sin^2 \left(\frac{5\Delta}{2}\right)$$

$$I_m = kT^2\, \mathrm{R}^{\mathrm{m}-1} \sin^2 \left(\frac{m\Delta}{2}\right)$$

Different rays are inclined with the axis of polariscopes, at angle 0, 2α, 4α, 6α, as shown, each ray gets isolated and passes out from different points of the model. Any of these rays can be observed at the proper image point of focal field lens.